NURBS Curve Shape Modification and Fairness Evaluation for Computer Aided Aesthetic Design

TETSUZO KURAGANO AND AKIRA YAMAGUCHI
Graduate School of Information Science, Meisei University
2-590 Nagabuchi, Ome-City, Tokyo, 198-8655, JAPAN

Abstract: - A curve with a monotone radius of curvature distribution is considered as a fair curve in the area of Computer Aided Aesthetic Design (CAAD). But no official standards have been established. Therefore, a criterion for a fair curve is proposed. A quintic NURBS curve, the first derivative of a quintic NURBS curve, curvature vector, curvature, and radius of curvature are expressed. The concept of radius of curvature specification to modify the shape of a NURBS curve is illustrated. The difference between the NURBS curve radius of curvature and the specified radius of curvature is minimized by introducing the least-squares method to modify the shape of the NURBS curve. Algebraic functions such as linear, quadratic, cubic, quartic, quintic, and six degrees are applied to the radius of curvature distribution of the designed curve as the specified radius of curvature. The radius of curvature distributions given by these six algebraic functions are considered monotone, because the independent variable of these algebraic functions is monotone to the corresponding dependent variable of these functions. Similarity is evaluated using the radius of curvature distribution according to six algebraic functions as references and the radius of curvature distribution of the designed curves as matches by using correlation matching. Curve shape similarity evaluation is tried using an example. Considering that a curve with a monotone variation of radius of curvature distribution is fair, the similarity of the designed curve to a fair curve is evaluated. This measured similarity expresses fairness to the fair curve. Using this technique, the fairness of a curve is evaluated by using the similarity of the radius of curvature distribution.

Key-Words: - curve shape modification, fair curve, radius of curvature specification, correlation matching, fairness evaluation

1 Introduction

In Computer Aided Aesthetic Design (CAAD) [1], designers evaluate the quality of a designed curve by looking at its curvature or radius of curvature plots. If the quality of a designed curve does not meet designer's demands, they usually modify the control points of the curve interactively. If the variation of the radius of curvature of the curve is monotone, this curve is assumed to be a fair curve [2]. But the definition of a fair curve is ambiguous and no official standards are given. Therefore, in this paper we have tried to establish criterion for a fair curve.

A NURBS curve, which is commonly used in the field of CAD・CAM and Computer Graphics, is used as an expression of a freeform curve. A quadratic NURBS curve is used as an expression of a quadratic curve using its weights. In this study, a quadratic curve is not used to express the shape of a curve. Therefore, the weights of a NURBS curve are not used. A cubic NURBS curve is widely used, but in this study, radius of curvature of multi segments of a NURBS curve are modified based on the specified radius of curvature. So, a smooth radius of curvature continuity is needed. Therefore, a quintic NURBS curve is used in this study.

Positions and gradients are given to the NURBS curve equations and first derivative equations of the NURBS curve respectively. Then, a NURBS curve is generated. Afterwards, if necessary, the shape of this NURBS curve is modified according to the specified radius of curvature distribution.

Fair curve expression and evaluation of fairness are described. As a measure of curve fairness evaluation, radius of curvature distribution is used as an alternative characteristic of a curve. Evaluation of whether the designed curve is fair or not is accomplished by comparing the designed curve to a curve whose radius of curvature is monotone.

The radius of curvature distributions of six NURBS curves are modified to follow algebraic functions such as linear, quadratic, cubic, quartic, quintic, and six degrees to specify the radius of curvature distribution. Then, by introducing the correlation matching, the similarities of the designed curve to these six predefined curves are examined. Among these predefined curves, the highest similarity curve to the designed curve is selected as an ideal fair NURBS curve. Then, the fairness of the designed curve to its ideal fair curve is evaluated.

Fair curve generation algorithms related to curvature by modifying the control points have been published. These make monotone curvature [3], use a clothoidal curve for specifying the curvature [4], and automate a curve fairing algorithm for B-spline curves [5, 6]. Fair curve generation algorithms related to energy function have been published. These are smoothing of cubic parametric splines by energy function [7], finding the unfair portion of a curve using energy function [8], and introducing a low-pass filter to energy function [9]. Fair curve generation algorithms related to curvature by specifying curvature distribution have also been published [10].

There are many related works for evaluating similarities of polygons in two dimensional space, especially in the area of image processing. Methods for evaluating similarities, which are based on the distances of corresponding points on polygonal curves, have been reported [11-14]. If the distances are close, it will be determined that the two polygonal curves are similar. Methods using Fourier descriptors for evaluating similar polygons have been developed and implemented [15, 16]. One is to retrieve the image files using Fourier descriptors. The other is to classify the characters expressed by polygonal curves.

Section 2 of this paper describes a quintic NURBS curve, the first derivative of a quintic NURBS curve, curvature vector, curvature, and radius of curvature. In section 3, generation of a quintic NURBS curve which passes through given point sequence and generation of a quintic NURBS curve using the given points and gradients are described. In section 4, NURBS curve shape modification based on the specified radius of curvature is described. Section 5 describes correlation matching to evaluate the similarity of the NURBS curves. Section 6 describes fair curve expression and fairness evaluation giving examples. A criterion for a fair curve is proposed as fairness.

2 NURBS Curve Expression

A quintic NURBS curve is used in this study. The objective of freeform curve design is to design the framework of surface patches. Surface patches are defined as tensor products, which are bi-variate and normally defined by u and v. In other words, one knot sequence in u direction, and another knot sequence in v direction are defined despite the complexity of the surface patches. Therefore, knot spacing is fixed in this study.

A quintic NURBS curve consists of $n-5$ segments $(n \geq 6)$, is composed of n control points such as $\boldsymbol{q_0}, \boldsymbol{q_1}, \cdots, \boldsymbol{q_{n-1}}$ and n weights such as $\omega_0, \omega_1, \cdots, \omega_{n-1}$ as in Eq.(1).

$$\boldsymbol{R}(t) = \frac{\sum_{i=0}^{n-1} N_{i,6}(t) \cdot \omega_i \cdot \boldsymbol{q}_i}{\sum_{i=0}^{n-1} N_{i,6}(t) \cdot \omega_i} \tag{1}$$

where $N_{i,6}(t)$ $(i = 0,1,\cdots,n-1)$ are NURBS basis functions.

These functions are recursively defined by knot sequence $t_0, t_1, \cdots, t_{n+5}$ as in Eq.(2).

$$\left.\begin{aligned} N_{i,1}(t) &= \begin{cases} 1 & (t_i \leq t < t_{i+1}) \\ 0 & \text{otherwise} \end{cases} \\ N_{i,M}(t) &= \frac{t - t_i}{t_{i+M-1} - t_i} N_{i,M-1}(t) + \frac{t_{i+M} - t}{t_{i+M} - t_{i+1}} N_{i+1,M-1}(t) \end{aligned}\right\} \tag{2}$$

where $i = 0,1,\cdots,n-1$ and $M = 2,3,\cdots,6$.

The basis functions are defined by the de Boor-Cox [17] recursion formulas. If the knot vector contains a sufficient number of repeated knot values, then a division of the form $N_{i,M-1}(t)/(t_{i+M-1} - t_i) = 0/0$ (for some i) may be encountered during the execution of the recursion. Whenever this occurs, it is assumed that $0/0 = 0$ [18]. A quintic NURBS curve with knot vector $\{-5,-4,-3,-2,-1, 0, 1, 2, 3, 4, 5, 6\}$ is expressed as in Eq.(3).

$$\begin{aligned} \boldsymbol{R}(t) = \frac{1}{120}\Big\{ & (1-t)^5 \boldsymbol{q}_0 \\ & + (5t^5 - 20t^4 + 20t^3 + 20t^2 - 50t + 26)\boldsymbol{q}_1 \\ & + (-10t^5 + 30t^4 - 60t^2 + 66)\boldsymbol{q}_2 \\ & + (10t^5 - 20t^4 - 20t^3 + 20t^2 + 50t + 26)\boldsymbol{q}_3 \\ & + (-5t^5 + 5t^4 + 10t^3 + 10t^2 + 5t + 1)\boldsymbol{q}_4 \\ & + t^5 \boldsymbol{q}_5 \Big\} \end{aligned} \tag{3}$$

The first derivative of a quintic NURBS curve shown in Eq.(3) is expressed as in Eq.(4).

$$\begin{aligned} \frac{d\boldsymbol{R}(t)}{dt} = \frac{1}{120}\Big\{ & -5(1-t)^4 \boldsymbol{q}_0 \\ & + (25t^4 - 80t^3 + 60t^2 + 40t - 50)\boldsymbol{q}_1 \\ & + (-50t^4 + 120t^3 - 120t)\boldsymbol{q}_2 \\ & + (50t^4 - 80t^3 - 60t^2 + 40t + 50)\boldsymbol{q}_3 \\ & + (-25t^4 + 20t^3 + 30t^2 + 20t + 5)\boldsymbol{q}_4 \\ & + 5t^4 \boldsymbol{q}_5 \Big\} \end{aligned} \tag{4}$$

Curvature vector is expressed by Eq.(5).

$$\boldsymbol{\kappa}(t)=\frac{\left(\dot{\boldsymbol{R}}(t)\times\ddot{\boldsymbol{R}}(t)\right)\times\dot{\boldsymbol{R}}(t)}{\left(\dot{\boldsymbol{R}}(t)\right)^4} \tag{5}$$

where $\dot{\boldsymbol{R}}(t)$ is the first derivative of a NURBS curve, and $\ddot{\boldsymbol{R}}(t)$ is the second derivative of a NURBS curve.

Curvature is the magnitude of the curvature vector, therefore curvature is expressed as in Eq.(6).

$$\kappa(t)=\left|\boldsymbol{\kappa}(t)\right| \tag{6}$$

By definition, the curvature of a plane curve is nonnegative. However, in many cases it is useful to ascribe a sign to the curve [19]. The choosing of the sign is commonly connected with the tangent rotation (in moving along the curve in the direction of the increasing parameter): The curvature of the curve is positive when its tangent rotates counter-clockwise, the curvature of the curve is negative when its tangent rotates clockwise.

Radius of curvature is the reciprocal number of curvature, therefore, radius of curvature is expressed as in Eq.(7).

$$\rho(t)=\frac{1}{\kappa(t)} \tag{7}$$

3 Generation of a NURBS Curve

In this section, a method to generate a quintic NURBS curve which passes through given points in sequence is shown. Another method to generate a quintic NURBS curve using the given points in sequence with gradients is described.

3.1 Generation of a Quintic NURBS Curve which Passes through Given Point Sequence

Putting zero to the parameter of Eq.(3), Eq.(3) is expressed as Eq.(8) by defining the geometrical knot position corresponding to the knot of the knot vector.

$$\boldsymbol{R}_i=\frac{1}{120}(\boldsymbol{q}_i+26\boldsymbol{q}_{i+1}+66\boldsymbol{q}_{i+2}+26\boldsymbol{q}_{i+3}+\boldsymbol{q}_{i+4}) \tag{8}$$

$$(i=0,1,2,3,\cdots,m-1)$$

Where m is the number of the given points, and $\boldsymbol{P}_0$, $\boldsymbol{P}_1$, $\boldsymbol{P}_2$, $\boldsymbol{P}_3$, $\cdots$, $\boldsymbol{P}_{m-2}$, $\boldsymbol{P}_{m-1}$ are the positional vectors of the given points to be assigned to $\boldsymbol{R}_i(i=0,1,2,3,\cdots,m-1)$ in Eq.(8), and $\boldsymbol{q}_0$, $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, $\boldsymbol{q}_3$, $\cdots$, $\boldsymbol{q}_{m+2}$, $\boldsymbol{q}_{m+3}$ are the control points of a quintic NURBS curve.

When the control points of a NURBS curve are calculated using Eq.(8), the number of unknowns, which are the positions of the control points, are four more than the number of equations which are expressed by Eq.(8). In this case, by setting the second derivative of the NURBS curve to zero, and setting the fourth derivative of the NURBS curve to zero, unknown variables become known. Therefore, the number of equations will be equal to the number of unknowns. That is, a linear system is determined [20]. Then a NURBS curve is generated by solving this determined system.

In this study, in addition to the given point sequence, gradient at the given points is defined.

Eq.(9) is applied to the gradients by setting the parameter of Eq.(4) as zero.

$$\frac{d\boldsymbol{R}_i}{dt}=\frac{1}{24}(-\boldsymbol{q}_i-10\boldsymbol{q}_{i+1}+10\boldsymbol{q}_{i+3}+\boldsymbol{q}_{i+4}) \tag{9}$$

$$(i=0,1,2,3,\cdots,n-1)$$

Where n is the number of given gradients. The i shown in Eq.(9) corresponds to the i in Eq.(8) and is determined situationally. As a magnitude of the first derivative, one third value of the distance of adjacent given points is assigned.

The defined gradients are located at the beginning given point and it's adjacent point, and at the end given point and it's adjacent point in general. In this case, the i are determined as 0, 1, $n-2$, $n-1$ respectively. Using given point sequence and four location specified gradients, a linear system becomes determined. That is, the number of unknowns is equal to the number of equations. The concept of a quintic NURBS curve generation using the given point sequence and four location specified gradients are illustrated in Fig.1. $\boldsymbol{P}_0,\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_3,\cdots,\boldsymbol{P}_{m-2},\boldsymbol{P}_{m-1}$ are given points. $\boldsymbol{d}_0,\boldsymbol{d}_1,\boldsymbol{d}_{n-2}$, and $\boldsymbol{d}_{n-1}$ are the four location specified gradients.

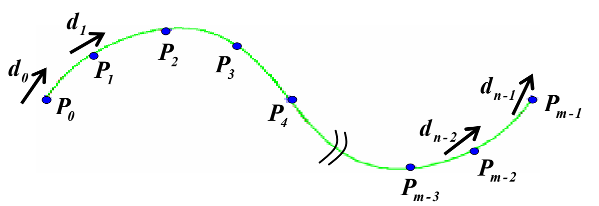


Fig.1 Illustration for a quintic NURBS curve using the given point sequence and four location specified gradients

3.2 Generation of a Quintic NURBS Curve using the Given Points and Gradients

In this sub-section, a NURBS curve generation using the given points with gradients is described.

The concept of generation of a NURBS curve using the given points with gradients is illustrated in Fig.2.

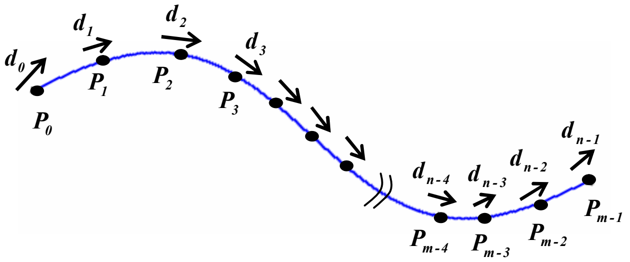


Fig.2 Concept of generation of a NURBS curve using the given points with gradients

$P_0, P_1, P_2, P_3 \cdots, P_{m-2}, P_{m-1}$ is the given point sequence. $d_0, d_1, d_2, d_3, \cdots, d_{n-2}, d_{n-1}$ are gradients assigned to the given points in sequence. A NURBS curve which passes through the given points and has the first derivatives at these given points is generated.

A NURBS curve is generated by solving Eq.(8) and Eq.(9) simultaneously by making m in Eq.(8) equal to n in Eq.(9). In this case, the i in Eq.(8) corresponds to the i in Eq.(9). If the number of given points with gradients is 4, the number of NURBS curve equations (Eq.(8)) is 4 and the number of first derivative equations (Eq.(9)) is 4. As a linear system, the total number of equations is 8, whereas the total number of control points of a NURBS curve is 8. Therefore, this linear system is determined. That is, the rank of a coefficient matrix of a linear system is equal to the number of unknowns. The solution to this linear system is exact.

However, in case the number of given points with gradients is 3, the number of equations (Eq.(8)) which pass through the given points is 3, and the number of equations of the first derivative (Eq.(9)) is 3. In this case, as a linear system, the total number of equations is 6, whereas the number of control points of the NURBS curve is 7. That is, the number of equations is less than the number of unknowns. Therefore, this linear system is underdetermined [21].

For an underdetermined system, while setting auxiliary function, the linear system is solved under the constraint condition by selecting one solution from infinite number of exact solutions using Lagrange's method of indeterminate multipliers.

In case the number of given points with gradients is 5, the number of equations (Eq.(8)) is 5, and the number of equations of the first derivative (Eq.(9)) is 5. In this case, as a linear system, the total number of equations is 10, whereas the number of control points of the NURBS curve is 9. That is, the number of equations exceeds the number of unknowns. Therefore, this linear system is overdetermined [22].

For an overdetermined system, the differences between the right and left sides of all the equations of the system are minimized. The control points calculated are approximations.

For a system where the number of given points with gradients is more than 5, the linear system is overdetermined. For these systems, in accordance with the increments of the differences between the number of equations and the number of unknowns, the status of the approximation worsens.

4 Curve Shape Modification based on the Specified Radius of Curvature

In this section, a method to modify a NURBS curve shape according to the specified radius of curvature distribution to realize an aesthetically pleasing freeform curve is described.

The concept of radius of curvature specification and NURBS curve shape modification based on the specified radius of curvature distribution is shown in Fig.3. A NURBS curve and its radius of curvature plots are shown in Fig.3(a).

A method to modify the shape of the NURBS curve shown in Fig.3(a) to the curve shown in Fig.3(b) is examined.

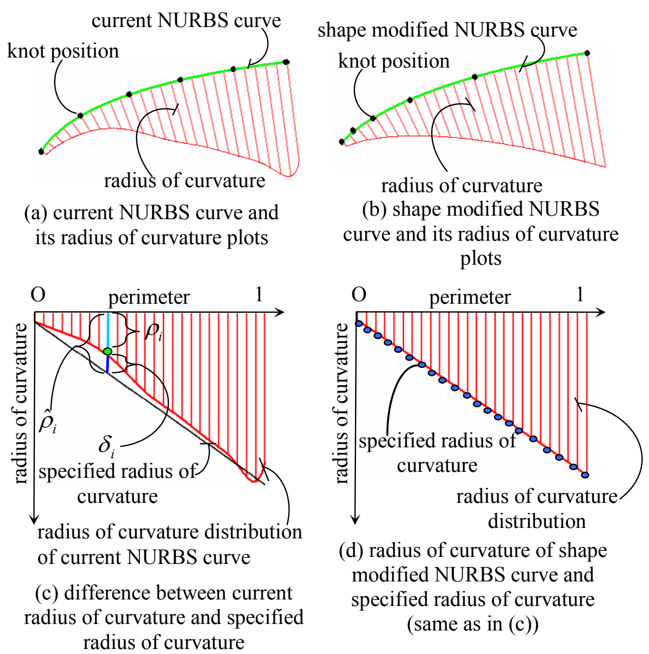


Fig.3 Concept of radius of curvature specification and NURBS curve shape modification based on the specified radius of curvature distribution

Considering the parameter of the NURBS curve is different from the perimeter of the curve, the perimeter of a NURBS curve as a straight line is set to the horizontal axis, and the radius of curvature is set to the vertical axis as shown in Fig.3(c). Then, the radius of curvature distribution to the perimeter is drawn. After this, specified radius of curvature is superimposed on the current radius of curvature distribution. Linear, quadratic, cubic, quartic, quintic, and six degree algebraic functions are applied as specified radius of curvature to the current radius of curvature distribution to modify the shape of the NURBS curve. To be more in detail, coefficients of the algebraic function are calculated by introducing the least-squares method using the current radius of curvature distribution. Then, the radius of curvature is specified by the determined algebraic function.

As an example, the linear algebraic function as a specified radius of curvature specification is shown in Fig.3(c). The i th of radius of curvature distribution of a perimetrically represented NURBS curve is denoted as ρ_i, the specified radius of curvature at the same spot is denoted as $\hat{\rho}_i$, the difference δ_i is shown by Eq.(10) and is illustrated in Fig.3(c).

$$\delta_i = \rho_i(q_1^x,\ \cdots,\ q_{n-2}^x,\ q_1^y,\ \cdots,\ q_{n-2}^y) - \hat{\rho}_i \tag{10}$$

Where $i = 0,1,2,\cdots,m-1$, m is the number of specified radius of curvature, and n is the number of NURBS curve segments plus 5, which is the degree of the curve.

$S(q_1^x,\ \cdots,\ q_{n-2}^x,\ q_1^y,\ \cdots, q_{n-2}^y)$ which is the sum of the squared differences for all specified radius of curvatures in Eq.(11) is minimized by introducing the least-squares method. The radius of curvature expression is non-linear. Therefore, by Taylor's theorem, Eq.(11) is linearlized as in Eq.(12).

$$\begin{aligned} &S(q_1^x,\ \cdots,\ q_{n-2}^x,\ q_1^y,\ \cdots,\ q_{n-2}^y) \\ &= \sum_{i=0}^{m-1}\left[\rho_i(q_1^x,\ \cdots,\ q_{n-2}^x,\ q_1^y,\ \cdots,\ q_{n-2}^y) - \hat{\rho}_i\right]^2 \end{aligned} \tag{11}$$

$$\begin{aligned} &S(q_1^x + \Delta q_1^x,\ \cdots,\ q_{n-2}^x + \Delta q_{n-2}^x,\ q_1^y + \Delta q_1^y,\ \cdots,\ q_{n-2}^y + \Delta q_{n-2}^y) \\ &= \sum_{i=0}^{m-1}\left[\rho_i(q_1^x,\ \cdots,\ q_{n-2}^x,\ q_1^y,\ \cdots,\ q_{n-2}^y) + \frac{\partial \rho_i}{\partial q_1^x}\Delta q_1^x + \right. \\ &\left. \cdots + \frac{\partial \rho_i}{\partial q_{n-2}^x}\Delta q_{n-2}^x + \frac{\partial \rho_i}{\partial q_1^y}\Delta q_1^y + \cdots + \frac{\partial \rho_i}{\partial q_{n-2}^y}\Delta q_{n-2}^y - \hat{\rho}_i\right]^2 \end{aligned} \tag{12}$$

Eq.(12) is minimized by equating to zero all the partial derivatives of $S(q_1^x + \Delta q_1^x, \cdots, q_{n-2}^x + \Delta q_{n-2}^x,\ q_1^y + \Delta q_1^y, \cdots, q_{n-2}^y + \Delta q_{n-2}^y)$ with respect to Δq_r^x and Δq_r^y $(r = 1,\ 2,\ \cdots,\ n-2)$ as in Eq.(13).

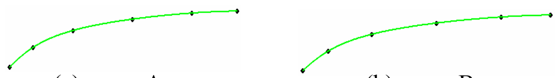
$$\left.\begin{aligned} \frac{\partial S}{\partial \Delta q_r^x} = 0 \quad (r = 1,2,\cdots,n-2) \\ \frac{\partial S}{\partial \Delta q_r^y} = 0 \quad (r = 1,2,\cdots,n-2) \end{aligned}\right\} \tag{13}$$

Using these simultaneous linear equations, Δq_r^x and Δq_r^y $(r = 1,2,\cdots,n-2)$ are calculated. Then, q_r^x and q_r^y are determined.

5 Correlation Matching for Similarity Evaluation

In this section, correlation matching for similarity evaluation is described. Two NURBS curves are shown in Fig.4(a) and 4(b). These curves can hardly be distinguished by just looking at their graphs. But if the radius of curvature plots are drawn for both, the difference between the two curves is recognized immediately as shown in Fig.5(a) and 5(b).

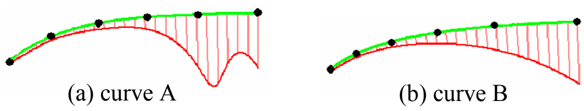
Radius of curvature is plotted using straight lines drawn outward from and perpendicular to the curve, with the line length proportional to the amount of radius of curvature at that spot. Curve shape is judged by looking at the lines coming out from the curve and seeing how their lengths change along the path, not along the parameter. Therefore, radius of curvature to the perimeter is drawn to evaluate the similarity of the curve shape as shown in Fig.6(a) and 6(b).



(a) curve A (b) curve B

• Point marks indicate knot position.

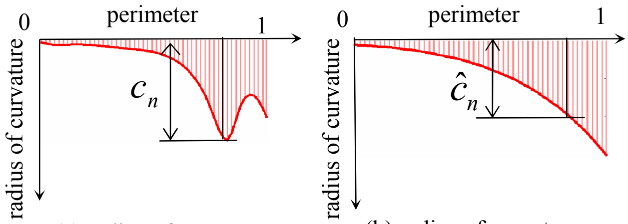
Fig.4 Two NURBS curves



(a) curve A (b) curve B

• Point marks indicate knot position.

Fig.5 Two NURBS curves with radius of curvature plots



(a) radius of curvature distribution for curve A (b) radius of curvature distribution for curve B

Fig.6 Radius of curvature distribution of two NURBS curves

Radius of curvature distribution is used as an alternative characteristic of the shape of the curve to evaluate the quality of a designed curve. To adjust the various lengths of the curve perimeters, the total length of the perimeters and radius of curvature are

rescaled as 1. A perimeter must be calculated according to the knot sequence of the knot vector.
Discrete values $c_n (n = 1, 2, 3, \cdots, m)$ shown in Fig.6(a) which are radius of curvature to the perimeter are considered as the components of m dimensional vector for curve A, denoting $\mathbf{a}$. In the same manner, discrete values $\hat{c}_n$ shown in Fig.6(b) are considered as the components of m dimensional vector for curve B, denoting $\mathbf{b}$.
Similarity between curve A and curve B is evaluated by Eq.(14).

$$S = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \tag{14}$$

Because curve perimeter and radius of curvature are rescaled as 1, and perimeter is calculated according to the knot sequence of the knot vector, the similarity is evaluated independent of location, orientation such as rotation and reflection, and the size of the curves. The evaluated similarity between curve A and curve B shown in Fig.6 is 0.996.

6 Fair Curve Expression and Evaluation of Fairness

A curve with a monotone radius of curvature distribution is considered as a fair curve in the area of Computer Aided Aesthetic Design. But no official standards are given. Therefore, criterion for a fair curve is proposed tentatively.
The shape of a NURBS curve is defined by the number, the location of its control points, and the knot sequence of the knot vector. The designed curve is considered fair if the variation of radius of curvature is monotone for the same number of control points and knot sequence of the knot vector.
In this section, fair curve expression and evaluation of fairness are described. As a measure of curve fairness evaluation, radius of curvature distribution is used as an alternative characteristic of a curve. First, radius of curvature distribution of the designed curve is evaluated to examine the fairness of the curve. Algebraic functions such as linear, quadratic, cubic, quartic, quintic, and six degrees are applied to the radius of curvature distribution of the designed curve as the specified radius of curvature. Then applying the curve shape modification algorithm based on the specified radius of curvature distribution, the radius of curvature distribution is modified according to these six algebraic functions respectively.
These radius of curvature distributions given by these six algebraic functions are considered monotone, because the independent variable of these algebraic functions is monotone to the corresponding dependent variable of these functions. Therefore, curves designed in this manner are considered fair. These judgements are performed by evaluating the similarity technique described in the previous section. This similarity is evaluated by using the radius of curvature distribution according to six algebraic functions used as references and the radius of curvature distribution of the designed curve as a match.
It is considered that the highest similarity reveals that this curve is designed to have this radius of curvature distribution, and the similarity measured is considered as the fairness of the designed curve.
As an example of a fair curve generation and fairness evaluation, a NURBS curve and its radius of curvature distribution to the perimeter are shown in Fig.7.

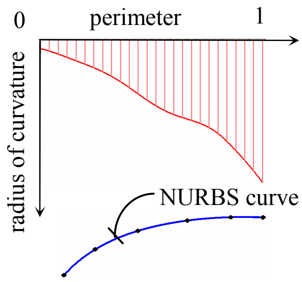


Fig.7 Designed curve and its radius of curvature distribution

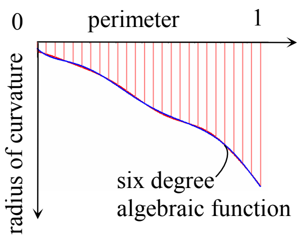


Fig.8 Radius of curvature distribution and six degree algebraic function to specify radius of curvature

Algebraic functions mentioned above are applied to the radius of curvature distribution shown in Fig.7 by using the least-squares method. These six functions are determined as the specified radius of curvature. These are shown in Fig.8 together with the radius of curvature distribution shown in Fig.7. Applying the curve shape modification algorithm based on these six algebraic functions to the designed curve, the shape of the curve is modified. Afterwards, setting these six radius of curvature distributions as references and the radius of curvature distribution of the designed curve as a match, six similarities are evaluated. The similarities evaluated are summarized in Table 1. In Table 1, similarity expresses the fairness of the curve. From Table 1, the designed curve whose radius of curvature is shown in Fig.7, is judged to be designed so that the radius of curvature distribution will be six degree. And this measured similarity 0.99993 expresses the fairness of the designed curve.

Table 1 Fairness of the designed curve

algebraic function	linear	quadratic	cubic	quartic	quintic	six
radius of curvature shown in Fig.7	0.99772	0.99888	0.99905	0.99974	0.99981	0.99993

7 Concluding Remarks

A quintic NURBS curve, the first derivative of a quintic NURBS curve, curvature vector, curvature, and radius of curvature are expressed.

The relation of curvature and radius of curvature are inverted. Therefore, even if the radius of curvature to the perimeter is linear, the curvature distribution to the perimeter is non-linear. The radius of curvature is useful to understand the quality of the designed curve visually.

The concept of radius of curvature specification to modify the shape of a NURBS curve is illustrated. The difference between the NURBS curve radius of curvature and the specified radius of curvature is minimized by introducing the least-squares method to modify the shape of the NURBS curve.

Algebraic functions such as linear, quadratic, cubic, quartic, quintic, and six degrees are applied to the radius of curvature distribution of the designed curve as the specified radius of curvature. The radius of curvature distributions given by these six algebraic functions are considered monotone, because the independent variable of these algebraic functions is monotone to the corresponding dependent variable of these functions.

Similarity is evaluated using the radius of curvature distribution according to six algebraic functions as references and the radius of curvature distribution of the designed curves as matches by using correlation matching. The values of radius of curvature to the perimeter are considered as the components of a multi dimensional vector for the curve. Similarity between two curves is expressed by normalizing the dot product of two vectors. Curve shape similarity evaluation is tried using an example.

Considering that a curve with a monotone variation of radius of curvature distribution is fair, the similarity of the designed curve to a fair curve is evaluated. This measured similarity expresses the fairness of the designed curve. Using this technique, the fairness of a curve is evaluated by using the similarity of the radius of curvature distribution.

In the future, we are planning to establish a definition of a fair curve using a lot of curve data that will be gathered.

References:

[1] C. Werner Dankwort and Gerd Podehl, "A New Aesthetic Design Workflow-Results from the European Project FIORES", *CAD Tools and Algorithms for Product Design*, Springer-Verlag Berlin (2000), pp.16-30.

[2] H. G. Burchard, J.A. Ayers, W.H. Frey, and N.S. Sapidis, "Approximation with Aesthetic Constraints", Designing Fair Curve and Surfaces, SIAM, (1994), pp.3-28.

[3] W.H. Frey, D.A. Field, "Designing Bézier conic segments with monotone curvature", Computer Aided Geometric Design, 17, (2000), pp.457-483.

[4] M. Kuroda, M. Higashi, T. Saitoh, Y. Watanabe, T. Kuragano, "Interpolating curve with B-spline curvature function", Mathematical Methods for Curves and Surfaces Ⅱ, Vanderbilt University Press, Nashville, TN, (1998), pp.303-310.

[5] H. Guan, T. Torii, "Modifying curvatures at design points for convex B-spline curves", IEEE, (1997), pp.223-229.

[6] J. Poliakoff, Y. Wong, P. Thomas, "An automated curve fairing algorithm for cubic B-spline curves", Journal of Computational and Applied Mathematics, 102, (1999), pp.73-85.

[7]J.A.P. Kjellander, "Smoothing of cubic parametric splines", Computer Aided Design, 15, 3, (1983), pp.175-179.

[8] C. Zhang, P. Zhang, F. Cheng, "Fairing spline curve and surface by minimizing energy", Computer Aided Design, 33, (2001), pp.913-923.

[9] X. Yang, G. Wang, "Planar point set fairing and fitting by arc splines", Computer Aided Design, 33, (2001), pp.35-43.

[10] W. Li, S Xu, J. Zheng, G. Zhao, "Target curvature driven fairing algorithm for planar cubic B-spline curves", Computer Aided Geometric Design, 21, (2004), pp.499-513.

[11] M.J. Atallah, "A linear time algorithm for the Hausdorff distance between convex polygons," Information Processing Letters, vol.17, (1983), pp.207-209.

[12] D.P. Huttenlocher and K. Kedem, "Computing the minimum Hausdorff distance for point sets under translation," Proceedings of the sixth annual symposium on computational geometry. ACM Press, (1990), pp.340-349.

[13] H. Alt and M. Godau, "Computing the Fréchet distance between two polygonal curves," International Journal of Computational Geometry and Applications. vol.5, Nos. 1 and 2, World Scientific Publishing Company, (1995), pp.75-91.

[14] H. Alt, C. Knauer and C. Wenk, "Matching polygonal curves with respect to the Fréchet distance," Proceedings 18th International Symposium on Theoretical Aspects of Computer Science, (2001), pp.63-74.

[15] D.J. Lee, S. Antani and L.R. Long, "Similarity measurement using polygon curve representation and Fourier Descriptors for shape-based vertebral image retrieval," Proceeding of SPII Medical Imaging: Image Processing, San Diego, CA, SPIE,vol.5032, Part 3, (2003), pp.1283-1291.

[16] E. Persoon and K.S. Fu, "Shape discrimination using Fourier Descriptors," IEEE Transactions on Systems, Man, and Cybernetics, vol.SMC-7, no.3, (1977), pp.170-179.

[17] C. de Boor, "On calculating with B-spline", Japprox. Theory, 6(1), (1972), pp.50-62.

[18] D. Marsh, "Applied Geometry for Computer Graphics and CAD", Springer-Verlag, (2005), pp.188.

[19] Eugene V. Shikin.: Hand Book and Atlas of CURVES. CRC Press Boca Raton, Florida, (1995), pp.29.

[20] T. Kuragano, Y. Arimitsu, A. Yamaguchi, "A method to generate a fair curve with specified curvature distribution for Computer Aided Aesthetic Design", The 10th World Multi-Conference on Systemics, Cybernetics and Informatics, Vol.III, (2006), pp.249-256.

[21] W. Boehm, and H. Prautzsch, "Geometric Concepts for Geometric Design", (1994), pp.26-27.

[22] G. Farin, and D. Hansford, "Practical Linear Algebra –A Geometry Toolbox－", (2004), pp.253-254.