STRUCTUAL APPROACH TO INSTABILITY AND CHAOS IN NON-LINEAR CAUSAL SYSTEMS

JOSEF HRUSAK[1], DANIEL MAYER[2], MILAN STORK[1]
Department of Applied Electronics and Telecommunications[1], Theory of Electrical Engineering[2]
University of West Bohemia
P.O. Box 314, 30614 Plzen
Czech Republic

Abstract: - The paper deals with structural properties of a class of strictly causal systems. It is shown that a special physically correct internal structure of a given system representation caled dissipation normal form can be derived as a natural consequence of strict causality, dissipativity, minimality and asymptotic stability requirements. A generalization of classic Tellegen's theorem together with bi-orthonormal basis of the state velocity space expressing the signal energy conservation law for abstract system state space representations have been used. It is demonstrated by examples that the resulting structure represents a unifying tool for analysis and synthesis of a relatively general class of linear as well as nonlinear causal systems.

Key-Words: - energy-metric function, bi-orthonormal basis, dissipation normal form, Tellegen's principle, non-linear phenomena, instability, chaotic behavior

1 Introduction

Almost in any field of science and technology some sort of stability problem can appear. Instability and chaos are certainly the most important phenomena which should be treated before any other aspect of reality will be attacked. Hence it is not very surprising that a broad variety of approaches to the problem of stability, instability and analysis of chaotic phenomena exists. Many of the most popular techniques in the field of stability and chaos are in a certain sense related to the work of A.M.Lyapunov and can be seen as energy oriented.

Tellegen's theorem is one of the well known forms of energy conservation statement in the field of electrical engineering [1]. The most important feature of Tellegen's approach is the fact that the energy conservation principle holds without any regard to physical nature of constituent network elements. This is the key idea of the proposed approach to problems of dissipativity and chaos [2, 3].

2 Physical correctness and classical Tellegen's theorem

Certainly, any realizable system has to fulfil some causality and energy conservation requirements. Recall that existence of an abstract state space representation is necessary for a system to be causal. On the other hand causality does not imply energy conservation. In electrical engineering Kirchhoff's laws are known to be necessary and sufficient for physical correctness of any electrical network.

Definition 1: (Physical correctness of electrical circuit)
Electrical circuit is physically correct if it is not in contradiction with both the voltage and the current Kirchhof's laws.

Tellegen's theorem, which is known to be one of the most powerful tools of electrical network theory, has proven to be very elegant abstract form of energy conservation principle for a class physically correct system representations, in which voltages and currents have been chosen as state variables [4 - 10].

Let us briefly summarize the essential features of the classical version of Tellegen's theorem. Assume that an arbitrary connected electrical network of b components is given. Let us disregard the specific nature of the network components and represent the network structure by an oriented graph with n vertices and b branches. Let the set of Kirchhoff laws constraints is given

$$\boldsymbol{A}\,\boldsymbol{i}(t) = 0 \qquad \boldsymbol{B}\,\boldsymbol{v}(t) = 0 \tag{1}$$

where A is a *node incidence matrix*, B is *loop incidence matrix*, and **i** and **v** are:

$$\mathbf{i} = [i_1, i_2, \dots, i_n]^T, \quad \mathbf{v} = [v_1, v_2, \dots v_n]^T \tag{2}$$

Let J be the set of all current vectors **i** and V be the set of all voltage vectors **v** such that **i** and **v** satisfy (1). Then the classical Tellegen's principle follows:

Theorem 1. (Classical Tellegen's theorem)
If $\mathbf{i} \in J$ and $\mathbf{v} \in V$ then it holds:

$$\left\langle \mathbf{i}(t), \mathbf{v}(t) \right\rangle = 0 \tag{3}$$

3 Abstract form of energy conservation principle

The *arbitrariness in the choice of state coordinates* motivates introducing a group of state transformations on which a *generalization of the classical Tellegen's principle* can be based. Let S is a nonlinear system and $\Re\{S\}$ is a *discrete-time* finite dimensional strictly causal state space representation of *S:*

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{f}\left[\mathbf{x}(k)\right] + \mathbf{w}(k), \\ \mathbf{w}(k) &= \mathbf{B}\,\mathbf{u}(k), \quad \mathbf{y}(k) = \mathbf{C}\,\mathbf{x}(k) \end{aligned} \tag{4}$$

If an input $\mathbf{u}(k)$ and a state value $\mathbf{x}(k)$ will be chosen then the next state value $\mathbf{x}(k+1)$ will be known, and the *state difference vector* $\Delta\mathbf{x}(k)$ can be defined as

$$\Delta\mathbf{x}(k) = \mathbf{x}(k+1) - \mathbf{x}(k) \equiv \Delta\mathbf{x}_k, \ k \in \{0, 1, 2, ..\} \tag{5}$$

together with a row vector $\eta(k)$, *("discrete-time gradient vector"),* defined by:

$$\eta(\mathrm{k}) = \frac{1}{2}[x(k+1) + x(k)]^T \equiv \eta_k, \ k \in \{0,1,2,\ldots\} \tag{6}$$

The vector $\boldsymbol{\eta}_\mathrm{k}$ is a generalization of standard gradient vector. The discrete-time generalization of Tellegen's principle is then given by the inner product:

$$\forall t \equiv k, k \in \{0,1,2,\ldots\} : \left\langle \Delta\mathbf{x}_\mathbf{k}, \boldsymbol{\eta}_\mathbf{k}^{\ T} \right\rangle = 0 \tag{7}$$

and in the continuous-time case it holds

$$\forall t : \left\langle \dot{\mathbf{x}}(t),\ \mathbf{x}^T(t) \right\rangle = 0 \tag{8}$$

A *geometric interpretation* of the *generalized Tellegen's principle* is visualized at the Fig. 2 with continuous-time version as a limit of the discrete-time case (Fig. 1).

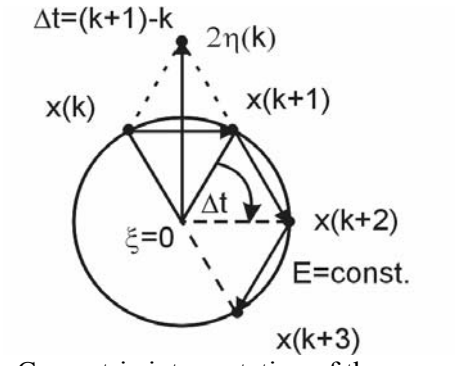

Fig. 1. Geometric interpretation of the generalized discrete Tellegen's principle (for n=2).

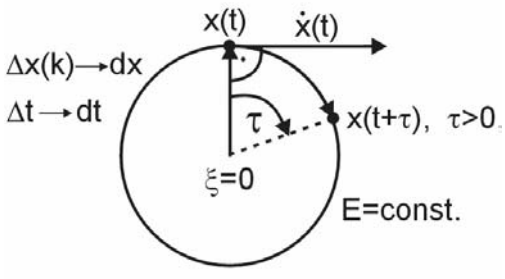

Fig. 2. Geometric interpretation of the generalized continuous Tellegen's principle (for n=2).

4 Dissipativity and conservativity

Recall that according to Liouville's theorem of vector analysis, *dissipative systems* have the important property that any volume of the state space strictly decreases under the action of the system flow [11 - 12]. For *continuous systems* with the *state velocity* given by the *nonlinear vector field* $\mathbf{f}$, the property of *dissipativity* is defined by using the *operation of divergence* :

$$\mathrm{div}\,\mathbf{f}(\mathbf{x}) \ = \ \sum_{i=1}^{n} \frac{\partial f_i(\mathbf{x})}{\partial x_i} < 0 \tag{9}$$

Thus a linear system is dissipative if and only if its *matrix* $\mathbf{A}$ *has negative trace*

$$Trace\ \mathbf{A} < 0 \tag{10}$$

Nonlinear systems having a dissipative approximate linearization are *locally dissipative*, but need not to be *globally dissipative. Vector fields* for which

$$\mathrm{div}\,\mathbf{f}(\mathbf{x}) = 0,\ \mathbf{f}(\mathbf{x}) = \mathbf{A}(.)\mathbf{x} \ \Leftrightarrow\ Trace\,\mathbf{A}(.) = 0 \tag{11}$$

preserve volume along state trajectories, and are referred to as *conservative.*

5 Causality and energy conservation: Some structural aspects

As an alternative to the well-known physical energy motivated method of Lyapunov functions a new conceptually different approach to stability problems has recently been proposed in [2 - 4] and called the Signal Energy-Metric approach. The *crucial idea* is that, in fact, *it is not the (physical or abstract) energy by itself*, but only a *measure of distance from the system equilibrium to the actual state* $\mathbf{x}(t)$, what is needed for stability/in-stability analysis. Thus a *state space metric* $\rho[\boldsymbol{x}(t), \boldsymbol{x}^*]$, where $\boldsymbol{x}^*$ denotes the *equilibrium state,* can be defined, and the basic idea of a new *state energy-metric approach* is formally expressed by:

$$E(\mathbf{x}) \ = \ \frac{1}{2}\rho^2\left[\mathbf{x}(t),\, \mathbf{x}^*\right] \tag{12}$$

To avoid confusion the concepts of the *signal power* and that of *signal energy* for the *continuous- time system representations* $\Re\{S\}$ are *defined* first:

$$\begin{aligned} \Re\{S\}:\ \dot{\mathbf{x}}(t) &= \mathbf{f}[\mathbf{x}(t)] + \mathbf{B}\mathbf{u}(t), \quad \mathbf{x}(t_0) = \mathbf{x}^0, \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{13}$$

Let the *immediate value of the output signal power and corresponding value of the system energy, accumulated in the state x(t)* be defined by:

$$P(t) = \left\|\mathbf{y}(t)\right\|^2,\ E(t) = \delta\left\|\mathbf{x}(t)\right\|^2,\ \frac{\mathrm{d}\,E(\mathbf{x})}{\mathrm{d}\,t} = -P(t),\ \delta > 0 \tag{14}$$

Putting $\mathbf{u}(t)=0$, $\forall t \geq t_0$ and computing the *derivative of the energy function E(t)* along the equivalent representation of the given system we get the *signal power balance relation* (for $\mathbf{f}(\mathbf{x})=\mathbf{A}(\mathbf{x})\,\mathbf{x}$) in the form

$$\frac{\mathrm{d}E(\mathbf{x})}{\mathrm{d}t} = \delta\,\mathbf{x}^T(t)[\mathbf{A}(.) + \mathbf{A}^T(.)]\,\mathbf{x}(t) = -\left\|\mathbf{y}(t)\right\|^2 \tag{15}$$

and, by integration, the *energy conservation principle for a proper chosen equivalent state space representation* follows. Hence, in case of zero input $\mathbf{u}(t) = \mathbf{0}$, $t \geq t_0$, the *total energy accumulated in the system* in time t_0 must be equal to an *amount of the energy dissipated* on the interval $[t_0 , \infty)$ *by the output*:

$$E(t_0) = \int_{t_0}^{\infty} \|\mathbf{y}(t)\|^2 \, dt \tag{16}$$

It follows that a special form of a *structurally dissipative state equivalent system representation* called *dissipation normal form exists* and is given by matrices:

$$\mathbf{A} = \begin{pmatrix} -\alpha_1 & \alpha_2 & 0 & 0 & \cdots & 0 & 0 \\ -\alpha_2 & 0 & \alpha_3 & 0 & \cdots & 0 & 0 \\ 0 & -\alpha_3 & 0 & \alpha_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & -\alpha_{n-1} & 0 & \alpha_n \\ 0 & 0 & 0 & 0 & \cdots 0 & -\alpha_n & 0 \end{pmatrix}, \quad \mathbf{C}^T = \begin{bmatrix} \gamma \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{n-1} \\ \beta_n \end{bmatrix} \tag{17}$$

6 Generalized Hessenberg structure

Let us consider the class of systems given by the *state space representation*

$$\Re\{S\}: \ \dot{\mathbf{x}}(t) = \mathbf{f}[t; \mathbf{x}(t), \mathbf{u}(t)] \tag{18}$$

$$\mathbf{y}(t) = \mathbf{h}[t; \mathbf{x}(t)] \tag{19}$$

with t as continuous *time* variable, $x_1, x_2, ..., x_n$ as *state* coordinates, $\dot{x}_1, \dot{x}_2, ..., \dot{x}_n$ as the *state velocity* coordinates, $u_1 , u_2, ... , u_r$ as the system *input signals*, and with $y_1 , y_2, ... , y_p$ as the observed system *output signals.*

Let us now define a *constituent set* of *non-interacting elementary subsystems*

$$\begin{aligned} &\dot{x}_1 = f_1(t, x_1, u_1), \quad y_1 = h_1(t, x_1) \\ &\dot{x}_2 = f_2(t, x_2, u_2), \quad y_1 = h_2(t, x_2) \\ &\cdots \qquad\qquad \cdots \qquad\qquad \cdots \\ &\dot{x}_1 = f_n(t, x_n, u_n), \quad y_n = h_n(t, x_n) \end{aligned} \tag{20}$$

It follows that the constituent set (20) is dissipative if *at least one* of the *elementary subsystems is dissipative.*

Let us now investigate a *minimality property* of the *state velocity space.*

Definition 2: (Hessenberg structure of a matrix)

We say that a n-th order square *matrix* $\mathbf{A}$ has the *Hessenberg structure* if :

$$1^o \qquad a_{i,j} = 0, \quad j > i+1 \tag{21}$$

$$2^o \qquad a_{i,i+1} \neq 0, \ \text{and} \quad sign(a_{i,i+1}) = 1 \tag{22}$$

Definition 3: (Hessenberg structure of a vector field)

A *vector field* $\mathbf{f}$ has the *Hessenberg structure* if it holds

$$1^o \qquad \frac{\partial f_i}{\partial x_j} = 0, \quad j > i+1 \tag{23}$$

$$2^o \qquad \frac{\partial f_i}{\partial x_{i+1}} \neq 0, \quad sign\left(\frac{\partial f_i}{\partial x_{i+1}}\right) = 1 \tag{24}$$

Definition 4: (Generalized Hessenberg structure of a system representation)

We say that a *representation* has the *Generalized Hessenberg structure* if the

vector field $\mathbf{f}$ has the Hessenberg structure and in addition if it holds

$$3^o \qquad c_1 = \frac{\partial h_1}{\partial x_1} \neq 0, \quad sign\left(\frac{\partial h_1}{\partial x_1}\right) = 1 \tag{25}$$

$$4^o \qquad b_n = \frac{\partial f_n}{\partial u_n} \neq 0, \quad sign\left(\frac{\partial f_n}{\partial u_n}\right) = 1 \tag{26}$$

Remark 1: It is worthwhile to notice that each of the *Jacobian matrices* $\mathbf{J}_x(\mathbf{f})$, $\mathbf{J}_u(\mathbf{f})$, $\mathbf{J}_x(\mathbf{h})$ of the *velocity vector field* $\mathbf{f}$ as well as that one of the *observation map* h have a properly defined structure motivated by the system structure corresponding to the *cascade connection of the elementary subsystems* according to the Fig. 3. For the internal structure of subsystems S_k see Fig. 4.

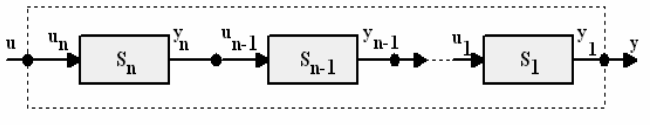

Fig. 3. Generalized Hessenberg structure.

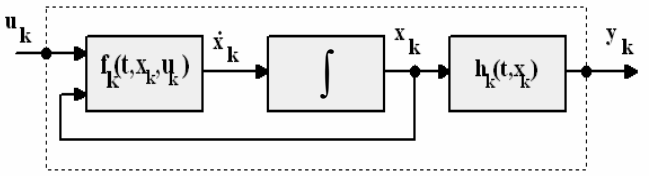

Fig. 4. Internal structure of the elementary subsystem S_k

The system representation in Generalized Hessenberg structure is obviously *always controllable and observable*, i.e. *structurally minimal* and is defined by

$$\begin{aligned} &\dot{x}_1 = f_1(t, x_1, x_2) \\ &\dot{x}_2 = f_2(t, x_2, x_3) \\ &\dot{x}_3 = f_3(t, x_3, x_4) \\ &\cdots \quad \cdots\cdots \\ &\dot{x}_{n-1} = f_{n-1}(t, x_{n-1}, x_n) \\ &\dot{x}_n = f_n(t, x_n) + u \\ &\boldsymbol{y}(t) = \boldsymbol{h}[t, \boldsymbol{x}(t)] = x_1(t) \end{aligned} \tag{27}$$

where the *set of external interactions* is given by

$$u(t) = u_n(t), \quad y(t) = x_1(t) \tag{28}$$

and the *set of internal interactions* is expressed by

$$u_i = x_{i+1} , \ i=1,2,..., n\text{-}1 \quad y_i = x_i , \ i = 1, 2,..., \tag{29}$$

7 Bi-orthonormal basis of the state velocity space

In order to specify the *physically correct internal system structure* in the sense of energy conservation principle, we introduce a *structural representation*

$$\Re^*\{S\}: \quad \mathbf{Q}\,\dot{\mathbf{x}}(t) = \mathbf{A}^*\mathbf{x}(t) + \mathbf{B}^*\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}^*\mathbf{x}(t) \tag{30}$$

where the columns $\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_n$ of the matrix $\mathbf{Q}$ form a *biorthonormal basis* in the *state velocity space* given by the *backwards recursion:*

$$\mathbf{q}_k + \mathbf{q}_{k+1} = \mathbf{e}_k, \; k = 1,2,\ldots,n\text{-}1, \; \mathbf{q}_n = \mathbf{e}_n \tag{31}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & & 0 & 0 \\ -1 & 1 & & 0 & 0 \\ 1 & -1 & & . & . \\ -1 & 1 & \ldots & . & . \\ . & -1 & . & 0 & 0 \\ . & . & . & 1 & 0 \\ . & . & . & -1 & 1 \end{bmatrix}, \mathbf{Q}^{-1} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 1 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & \ldots & 1 & 0 \\ 0 & 0 & . & \ldots & 1 & 1 \end{bmatrix} \tag{32}$$

$$\mathbf{B}^* = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{bmatrix}, \quad (\mathbf{C}^*)^T = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

8 Dissipativity and minimality

If *each* elementary subsystem S_k of the constituent set is *dissipative*, then it holds

$$\forall i: \quad \frac{\partial f_i}{\partial x_i} < 0, \quad i = 1,2,.....n \tag{33}$$

Hence the *simpliest dissipative* Hessenberg form *of the structural matrix* A^* and the resulting *generic minimal and dissipative structure of the matrix A(.)* follow:

$$\mathbf{A}^* = \begin{bmatrix} -1 & 1 & 0 & \ldots & 0 & 0 & \\ 0 & -1 & 1 & \ldots & 0 & 0 & 0 \\ 0 & 0 & -1 & \ldots & 0 & 0 & 0 \\ : & : & : & ::: & : & : & : \\ 0 & 0 & 0 & \ldots & 0 & -1 & 1 \\ 0 & 0 & 0 & \ldots & 0 & 0 & -1 \end{bmatrix} \tag{34}$$

$$\mathbf{A} = \mathbf{Q}^{-1}\mathbf{A}^* = \begin{bmatrix} -1 & 1 & 0 & 0 & \ldots & 0 & 0 \\ -1 & 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & -1 & 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & -1 & 0 & \ldots & 0 & 0 \\ : & : & : & : & ::: & : & : \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & 0 & 0 & \ldots & -1 & 0 \end{bmatrix}$$

and thus the internal structure of the dissipation normal form has been justified.

9 Non-linear phenomena: chaoticity and resonance

Let a simple 4th order nonlinear system in the dissipation normal form is given

$$\dot{x}_1 = -\alpha_1(x_1, x_2)x_1 + \alpha_2 x_2$$
$$\dot{x}_2 = -\alpha_2 x_1 + \alpha_3 x_3$$
$$\dot{x}_3 = -\alpha_3 x_2 + \alpha_4 x_4$$
$$\dot{x}_4 = -\alpha_4 x_3$$

where coefficients and initial conditions are given by:

$$\alpha_1(x_1,x_2) = -(\beta_0 - \beta_1 x_1^2 - \beta_2 x_2^2) \qquad x_1(0) = 0$$
$$\beta_0 = 1, \beta_1 = \text{var}, \beta_2 = 10 \qquad x_2(0) = 0$$
$$\alpha_2 = 1, \alpha_3 = 1 \qquad x_3(0) = 0.5$$
$$\alpha_4 = \text{frequency parameter} \qquad x_4(0) = 0$$

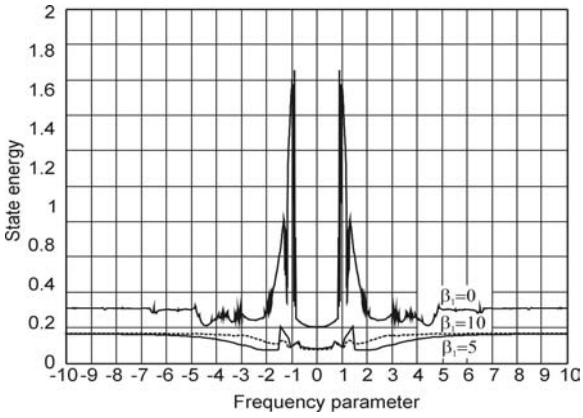

Fig. 5. Chaoticity and resonance

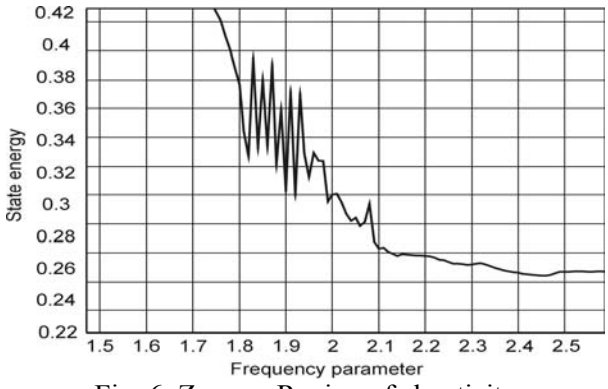

Fig. 6. Zoom – Region of chaoticity

At the Fig. 7.-10. some typical cases of the system behavior are displayed by means of the 3-D projections of the state space trajectories in dependence on the values of the frequency parameter α_4, chosen from the energy diagram Fig. 5., 6. for: α_4 =1 region of sub-

resonance behavior, α_4 =1.7 sub-chaotic behavior, α_4 =2 region of the chaotic behavior, α_4 =2.2 quasi-periodic behavior. Notice also the dependence of the energy plot in the Fig.5. on the value of β_1,[13 - 22].

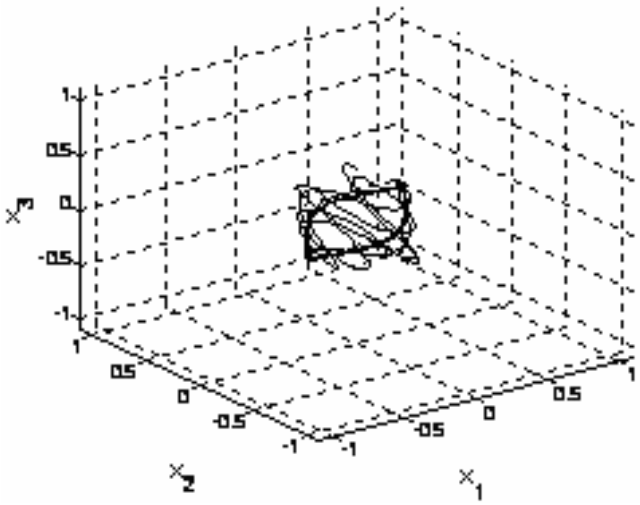

Fig. 7. 3-D Projection of the state α_4=1.

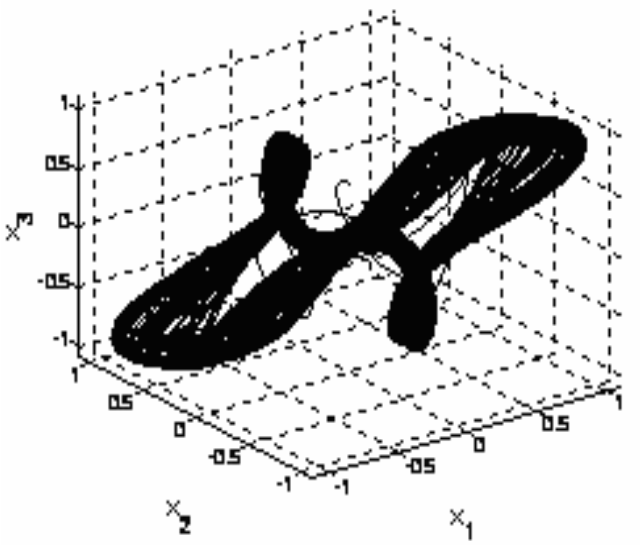

Fig. 8. 3-D Projection of the state α_4=1.7.

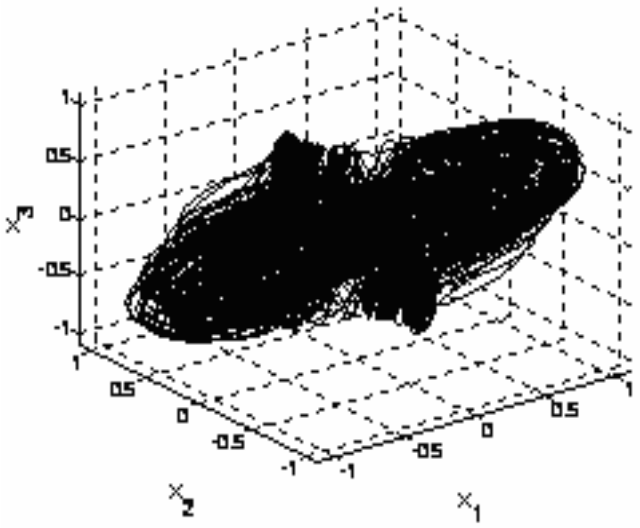

Fig. 9. 3-D Projection of the state α_4=2.0.

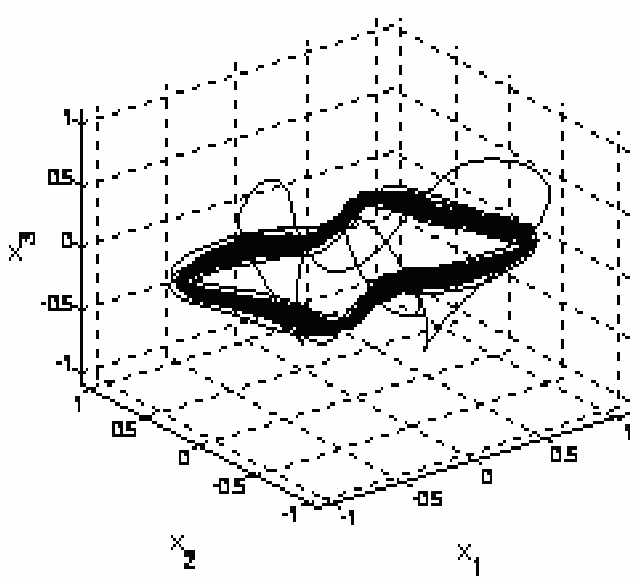

Fig. 10. 3-D Projection of the state α_4=2.2.

10 Conclusion

In the contribution a new unifying, systematic and constructive approach to non-linear phenomena, based on concepts of the system energy-metric of the state space, and on the dissipation normal form has been presented. The main features of the proposed method are illustrated by several typical examples.

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