Fair NURBS Curve Generation using a Hand-drawn Sketch for Computer Aided Aesthetic Design

AKIRA YAMAGUCHI AND TETSUZO KURAGANO
Graduate School of Information Science, Meisei University
2-590 Nagabuchi, Ome-City, Tokyo, 198-8655, JAPAN

Abstract: - Image processing techniques are used to detect the edges of a hand-drawn sketch. Generally, various types of gradation are expressed in the background of a hand-drawn sketch. Therefore, the original image is transformed into a logarithmic image. The entire image has to be binarized for edge detection. To smooth the edge of the binary image, feature based erosion and dilation is applied. A Laplacian operation is applied to the binary image to detect the edge in the binary image. This edge position corresponds to the position of the line in the hand-drawn sketch image. Line segments which represent the detected edges are generated. Using the positions and gradients on the line segments, a NURBS curve is generated. The shape of the NURBS curve extracted from the sketch image is examined by the designer visually. The shape of this NURBS curve generally coincides with the intention of the designer, but does not coincide precisely. Therefore, a curve shape modification method based on the specified radius of curvature is applied. The sum of the squared differences between the radius of curvature of a curve and the specified radius of curvature of an entire curve is linearized by Taylor's theorem, then minimized. The specification of radius of curvature is performed by selecting a predefined fair NURBS curve using the similarity of radius of curvature distribution. Examples of NURBS curve generation based on a hand-drawn sketch are given according to the generation process.

Key-Words: - hand-drawn sketch, image processing, regression line, NURBS curve, curvature, radius of curvature, curve shape modification, correlation coefficient

1 Introduction

Conventional design procedures can easily produce simple shapes. However, currently, industrial designers are drawn to aesthetically pleasing freeform shapes because they have great consumer appeal, especially in a highly competitive and technically well-developed market, such as that for automobiles and electrical appliances. It takes a long time to design aesthetically pleasing products using conventional procedures.

A solution to this problem is to establish a method to shorten the product design period, especially the period from the first idea generation as it is narrowed down to the final design. Therefore, the objective of developing this method is to shorten the Computer Aided Aesthetic Design (CAAD) [1] period.

When designers begin a product design, designers create their ideas and expand them. Normally this process is performed on paper, and designers' hand-drawn lines are called sketches. If the designers' rough idea on the sketch can be realized as a real fair curve, it will be effective in shortening the design period. There are two canons for drawing techniques. One is perspective projection. The other is orthogonal projection. Generally, it is not easy to estimate the perspective transformation matrix from a sketch drawn by perspective projection. Therefore, the sketch in this study is limited to orthogonal projections such as front view, side view and top view.

First, hand-drawn sketches are put into a computer by using a scanner. Then, image processing techniques such as light intensity transformation, feature based erosion and dilation to smooth the edge of the binary sketch image, and edge detection of the binary sketch image are examined.

In addition to these image processing techniques, principal component analysis is introduced to generate regression line segments using the detected edges. Using the line segments determined, a quintic NURBS curve is generated. A NURBS curve, which is commonly used in the area of CAD・CAM and Computer Graphics, is used as an expression of a freeform curve. A quadratic NURBS curve is used as an expression of a quadratic curve using its weights.

In this study, a quadratic curve is not used to express the shape of a curve. Therefore, the weights of the NURBS curve are not used. A cubic NURBS curve is widely used also, but in this study, radius of curvature over the multi segments of the NURBS curve is modified based on the specified radius of curvature. So, a smooth radius of curvature continuity is needed. Accordingly, a quintic NURBS curve is used in this study.

The position of the midpoints and the gradient of the line segments are given to the NURBS curve equation and first derivative equation of the NURBS curve respectively. Then, a NURBS curve is generated. Afterwards, if necessary, the shape of this NURBS curve is modified according to the specified radius of curvature distribution.

The radius of curvature is specified by linear, quadratic, cubic, quartic, quintic, and six degrees function, while the shape of the curve generated from the sketch is modified according to the specified radius of curvature distribution using the shape modification algorithm.

Then, by introducing correlation matching, the similarities of the generated curve from the sketch to these six curves are examined. Among these predefined curves, the highest similarity curve to the curve generated from the sketch is selected and determined as a fair NURBS curve generated from the sketch.

There are many related works for generating curves based on a sketch. Curve generation by tracing a hand-drawn sketch is available as a function of a commercial computer aided aesthetic design system [2]. Curve generation using a sketch, design language and characteristic lines [3], curve generation based on a hand-drawn sketch [4], curves for a character such as a stuffed animal design based on a hand-drawn sketch [5], 3D shape reconstruction using hand-drawn lines [6], and simple polygonal shape reconstruction based on a hand-drawn sketch [7] have been published. Curve generation using a hand-drawn sketch and it's view points [8], remeshing based mesh smoothing by a sketch [9], shape generation using a volumetric modeling technique [10], and shape generation using 3D scenes [11] have also been published. In addition to these, mechanical parts such as a piston using a hand-drawn sketch [12], simple parts generated by constructed solid geometry [13], and simple mechanical parts design using digital clay [14] have been published. There are many related works for generating fair curves. Fair curve generation algorithms related to energy functions have been published. These find the unfair portion of a curve using energy function [15], and apply a low-pass filter to energy function [16]. Fair curve generation algorithms related to curvature control have been published. These make monotone curvature [17], use a clothoidal curve for specifying the curvature [18], and modify the curve based on the specified curvature [19]. In addition to these, fair curve generation by using the second derivative values [20], using an argument of Bézier control edges [21], minimizing positional, the first, the second, and the third derivative values [22] have been published.

Section 2 of this paper describes some techniques for image processing such as light intensity transformation, binarization of the sketch image, feature based erosion and dilation to smooth the edge of the binary image, and edge detection of the binary image. Section 3 describes the line segment generation based on the detected edges by introducing the principal component analysis. Section 4 of this paper describes a quintic NURBS curve, the first derivative of a quintic NURBS curve, curvature vector, curvature, and radius of curvature. Section 5 describes NURBS curve generation using the positions and the gradients on the line segments. Section 6 describes NURBS curve shape modification based on the specified radius of curvature. Section 7 describes a method to specify the radius of curvature distribution by selecting a predefined fair NURBS curve using the similarity of the radius of curvature distribution. In section 8, examples of NURBS curve generation based on a hand-drawn sketch according to the described generation process are given.

2 Image Processing

In this section, the image processing techniques used to detect the edges of a hand-drawn sketch are described. First, light intensity transformation to overcome the gradation given by the designer, binarization, feature based erosion and dilation to smooth the edge of the binary image, and edge detection are described.

2.1 Light intensity transformation

Generally, various types of gradation are expressed in the background of a hand-drawn sketch [23] as shown in Fig.1. In such cases, the human eye is able to recognize lines which are drawn in a dark area as well as lines which are drawn in a bright area. However, a computer can not recognize lines in a dark area, because of the small difference in light intensity

between the lines and the background gradation, which is dark. Therefore, the original image is transformed into a logarithmic image to detect edges. In the logarithmic image, the light intensity difference of adjacent image elements in the dark area is very close to the differences in the bright area. In other words, the light intensity of the dark area of the original image is amplified, while that of the bright area of the original image is reduced. The logarithmic image is shown in Fig.2.
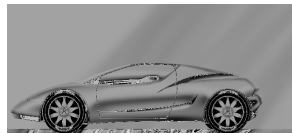
Fig.1 Hand-drawn sketch image   Fig.2 Logarithmic image

2.2   Binarization of sketch image

Binarization is performed according to the threshold temporarily decided. The candidate points for edges are displayed. This determination process should be done interactively by the designers. A single threshold is not enough in some cases to binarize an entire image. Therefore, the entire image is divided into multi regions, and thresholds for each region are established independently. In this case, the entire image is divided into 64 regions. The 64 thresholds for each region are determined and the entire image is then binarized.

2.3   Feature based erosion and dilation to smooth the edge of the binary image

"Erosion and dilation" is a well-known method to correct defects in a binary image. They are applied to the binary image as a combination of erosion-dilation or dilation-erosion. Erosion removes granules, and isolated lines and points, dilation fills holes and gaps in the binary image. "Feature based erosion and dilation" is used for the purpose of smoothing the edge of the binary image. The binary image is traditionally set to one for black, and to zero for white. The sum of the values of the 8 pixels adjacent to the target pixel is called the neighbor coefficient [24]. It is used to determine the target pixel value, which is zero or one. A neighbor coefficient varies from zero to eight. It is known by our experiments that a neighbor coefficient of over five does not work. In addition to this, in case the neighbor coefficient is zero, this process is exactly the same as "pixel based erosion and dilation". Therefore, a neighbor coefficient is determined interactively using the value from one to four, according to the visual inspection of the binary image.

For the erosion process, if the sum of the 8 neighbors is bigger than the neighbor coefficient, the target pixel value is set to zero, and if the sum of the 8 neighbors is smaller than the neighbor coefficient, the target pixel value is set to one.

For the dilation process, if the sum of the 8 neighbors is bigger than the neighbor coefficient, the target pixel value is set to one, and if the sum of the 8 neighbors is smaller than the neighbor coefficient, the target pixel value is set to zero.

As mentioned above, for the process of erosion and dilation, a method to decide the target pixel value, according to the sum of the 8 neighbors based on the neighbor coefficient is called "feature based erosion and dilation" [24] in this paper.

2.4   Edge detection

For Laplacian operators, there exist 3×3, 5×5, 7×7, 13×13 and so on. Our purpose of using a Laplacian operator is to detect the edge of the binary image. For this reason, a Laplacian operator is decided as 3×3 which is 8 neighbors. If a Laplacian operator is applied to the binary image, the Laplacian image has a positive and negative value, between these two values, the Laplacian value becomes zero. This is called zero crossing. The position for zero crossing is detected as the edge of the binary image.

Assuming $f(i,j)$ is the pixel value of position i,j of binary image, the pixel value $g(i,j)$ of the Laplacian image corresponding to $f(i,j)$ of the binary image is expressed by Eq.(1).

$$g(i,j) = f(i-1, j-1) + f(i-1, j) + f(i-1, j+1)$$
$$+ f(i, j-1) + f(i, j+1) + f(i+1, j-1) \qquad (1)$$
$$+ f(i+1, j) + f(i+1, j+1) - 8f(i,j)$$

3   Line Segment Generation using the Detected Edges

If lines are drawn that roughly follow the 160 detected edge data points, an image emerges as shown in Fig.3.
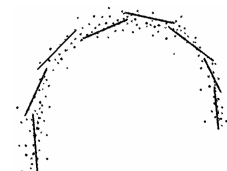
Fig.3 Detected edge data (160 points)
and their representative lines

For the principal component analysis, we have to place the data points into the desirable number of groups containing the desirable number of data points. But

we have no rule for grouping. Therefore, although the principal component analysis is not the statistical analysis, we follow the principle of Sturges rule [25]. The data points are then placed into 8 groups of 20 points each in this case. The principal component analysis is then applied to each group. A regression line segment representing the edge data for each group is then obtained. In this manner, the number of groups and the number of detected edge data in all groups are determined by following the principle of Sturges rule. Then, regression line segments for all groups are generated using the detected edges from the hand-drawn sketch.

4 NURBS Curve Expression

A quintic NURBS curve is used in this study. The objective of freeform curve design is to design the framework of surface patches. Surface patches are defined as tensor products, which are bi-variate and normally defined by u and v. In other words, one knot sequence in u direction, and another knot sequence in v direction are defined despite the complexity of the surface patches. Therefore, knot spacing is fixed in this study.

A quintic NURBS curve consists of $n-5$ segments $(n \geq 6)$, is composed of n control points such as $q_0, q_1, \cdots, q_{n-1}$ and n weights such as $\omega_0, \omega_1, \cdots, \omega_{n-1}$ as in Eq.(2).

$$R(t) = \frac{\sum_{i=0}^{n-1} N_{i,6}(t) \cdot \omega_i \cdot q_i}{\sum_{i=0}^{n-1} N_{i,6}(t) \cdot \omega_i} \qquad (2)$$

where $N_{i,6}(t)$ $(i = 0,1,\cdots,n-1)$ are NURBS basis functions.

These functions are recursively defined by knot sequence $t_0, t_1, \cdots, t_{n+5}$ as in Eq.(3).

$$\left. \begin{array}{l} N_{i,1}(t) = \begin{cases} 1 & (t_i \leq t < t_{i+1}) \\ 0 & \text{otherwise} \end{cases} \\[2ex] N_{i,M}(t) = \dfrac{t - t_i}{t_{i+M-1} - t_i} N_{i,M-1}(t) + \dfrac{t_{i+M} - t}{t_{i+M} - t_{i+1}} N_{i+1,M-1}(t) \end{array} \right\} \qquad (3)$$

where $i = 0,1,\cdots,n-1$ and $M = 2,3,\cdots,6$.

The basis functions are defined by the de Boor-Cox [26] recursion formulas. If the knot vector contains a sufficient number of repeated knot values, then a division of the form $N_{i,M-1}(t) / (t_{i+M-1} - t_i) = 0/0$ (for some i) may be encountered during the execution of the recursion. Whenever this occurs, it is assumed that $0/0 = 0$ [27]. A quintic NURBS curve with knot vector

$\{-5,-4,-3,-2,-1, 0, 1, 2, 3, 4, 5, 6\}$ is expressed as in Eq.(4).

$$\begin{aligned} R(t) = \frac{1}{120} \Big\{ & (1-t)^5 q_0 \\ & + (5t^5 - 20t^4 + 20t^3 + 20t^2 - 50t + 26)q_1 \\ & + (-10t^5 + 30t^4 - 60t^2 + 66)q_2 \\ & + (10t^5 - 20t^4 - 20t^3 + 20t^2 + 50t + 26)q_3 \\ & + (-5t^5 + 5t^4 + 10t^3 + 10t^2 + 5t + 1)q_4 \\ & + t^5 q_5 \Big\} \end{aligned} \qquad (4)$$

The first derivative of a quintic NURBS curve is expressed as in Eq.(5).

$$\begin{aligned} \frac{dR(t)}{dt} = \frac{1}{120} \Big\{ & -5(1-t)^4 q_0 \\ & + (25t^4 - 80t^3 + 60t^2 + 40t - 50)q_1 \\ & + (-50t^4 + 120t^3 - 120t)q_2 \\ & + (50t^4 - 80t^3 - 60t^2 + 40t + 50)q_3 \\ & + (-25t^4 + 20t^3 + 30t^2 + 20t + 5)q_4 \\ & + 5t^4 q_5 \Big\} \end{aligned} \qquad (5)$$

Curvature vector is expressed by Eq.(6).

$$\kappa(t) = \frac{\left(\dot{R}(t) \times \ddot{R}(t) \right) \times \dot{R}(t)}{\left(\dot{R}(t) \right)^4} \qquad (6)$$

where $\dot{R}(t)$ is the first derivative of a NURBS curve, and $\ddot{R}(t)$ is the second derivative of a NURBS curve.

Curvature is the magnitude of the curvature vector, therefore curvature is expressed as in Eq.(7).

$$\kappa(t) = |\kappa(t)| \qquad (7)$$

By definition, the curvature of a plane curve is nonnegative. However, in many cases it is useful to ascribe a sign to the curve [28]. The choosing of the sign is commonly connected with the tangent rotation (in moving along the curve in the direction of the increasing parameter): The curvature of the curve is positive when its tangent rotates counter-clockwise, the curvature of the curve is negative when its tangent rotates clockwise.

Radius of curvature is the reciprocal number of curvature, therefore, radius of curvature is expressed as in Eq.(8).

$$\rho(t) = \frac{1}{\kappa(t)} \qquad (8)$$

5 Generation of a NURBS Curve using the Line Segments

The concept of generation of a NURBS curve based on the regression line segments determined using the detected edges is illustrated in Fig.4.
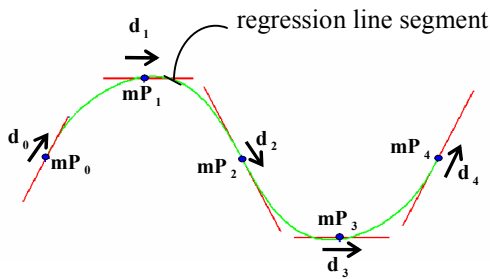
Fig.4 Concept of generation of a NURBS curve
which approximates the regression line segments

Where $mP_0, mP_1, \cdots, mP_4$ are the midpoints of the corresponding regression line segments.

$d_0, d_1, \cdots, d_4$ are directional unit vectors of the corresponding line segments.

A NURBS curve which passes through the midpoints and has first derivatives which are proportional to the directional unit vectors is generated. Eq.(9) is applied to the midpoints of the line segments by setting the parameter of Eq.(4) to zero.

$$R_i = \frac{1}{120}(q_i + 26q_{i+1} + 66q_{i+2} + 26q_{i+3} + q_{i+4}) \qquad (9)$$

$(i = 0, 1, \cdots, m-1)$

where m is the number of line segments.

Eq.(10) is applied to the directional unit vectors of the line segments by setting the parameter of Eq.(5) to zero while considering the magnitude of the first derivatives.

$$\frac{dR_i}{dt} = \frac{1}{24}(-q_i - 10q_{i+1} + 10q_{i+3} + q_{i+4}) \qquad (10)$$

$(i = 0, 1, \cdots, m-1)$

where m is the number of line segments.

A NURBS curve is generated by solving Eq.(9) and Eq.(10) simultaneously. If the number of line segments is 4, the number of NURBS curve equations (Eq.(9)) is 4 and the number of first derivative equations (Eq.(10)) is 4. As a linear system, the total number of equations is 8, whereas the total number of control points of a NURBS curve is 8. Therefore, this linear system is determined. That is, the rank of a coefficient matrix of a linear system is equal to the number of unknowns. The solution to this linear system is exact.

However, in case the number of line segments is 3, the number of equations (Eq.(9)) which pass through the midpoints is 3, and the number of equations of the first derivative (Eq.(10)) is 3. In this case, as a linear system, the number of equations is 6, whereas the number of control points of the NURBS curve is 7. That is, the number of equations is less than the

number of unknowns. Therefore, this linear system is underdetermined [29].

For an underdetermined system, while setting auxiliary function, the linear system is solved under the constraint condition by selecting one solution from infinite number of exact solutions using the Lagrange's method of indeterminate multipliers.

In case the number of line segments is 5, the number of equations (Eq.(9)) is 5, and the number of equations of the first derivative (Eq.(10)) is 5. In this case, as a linear system, the number of equations is 10, whereas the number of control points of the NURBS curve is 9. That is, the number of equations exceeds the number of unknowns. Therefore, this linear system is overdetermined [30]. For an overdetermined system, the differences of right and left side of all the equations of the system are minimized. The control points calculated are approximations.

For a system where the number of line segments is more than 5, the linear system is overdetermined. For these systems, in accordance with the increment of the difference between the number of equations and the number of unknowns, the status of approximation solution becomes worse.

6 Curve Shape Modification based on the Specified Radius of Curvature

In this section, a method to modify a NURBS curve shape according to the specified radius of curvature distribution to realize an aesthetically pleasing freeform curve is described.

The concept of radius of curvature specification and NURBS curve shape modification based on the specified radius of curvature distribution is shown in Fig.5. A NURBS curve and its radius of curvature plots are shown in Fig.5(a). A method to modify the shape of the NURBS curve shown in Fig.5(a) to the curve shown in Fig.5(b) is examined.

Considering the parameter of the NURBS curve is different from the perimeter of the curve, the perimeter of a NURBS curve as a straight line is set to the horizontal axis, and the radius of curvature is set to the vertical axis as shown in Fig.5(c). Then, the radius of curvature distribution to the perimeter is drawn. After this, specified radius of curvature is superimposed on the current radius of curvature distribution. Linear, quadratic, cubic, quartic, quintic, and six degree algebraic function is applied as specified radius of curvature to the current radius of curvature distribution to modify the shape of the NURBS curve.

To be more in detail, coefficients of the algebraic function are calculated by introducing the least-squares method using the current radius of curvature distribution. Then, the radius of curvature is specified by the determined algebraic function.

current NURBS curve

knot position

shape modified NURBS curve

knot position

radius of curvature

radius of curvature

(a) current NURBS curve and its radius of curvature plots

(b) shape modified NURBS curve and its radius of curvature plots

O perimeter 1

O perimeter 1

radius of curvature

radius of curvature

ρ_i

$\hat{\rho}_i$

δ_i

specified radius of curvature

specified radius of curvature

radius of curvature distribution of current NURBS curve

radius of curvature distribution of modified NURBS curve

(c) difference between current radius of curvature and specified radius of curvature

(d) radius of curvature of shape modified NURBS curve and specified radius of curvature (same as in (c))
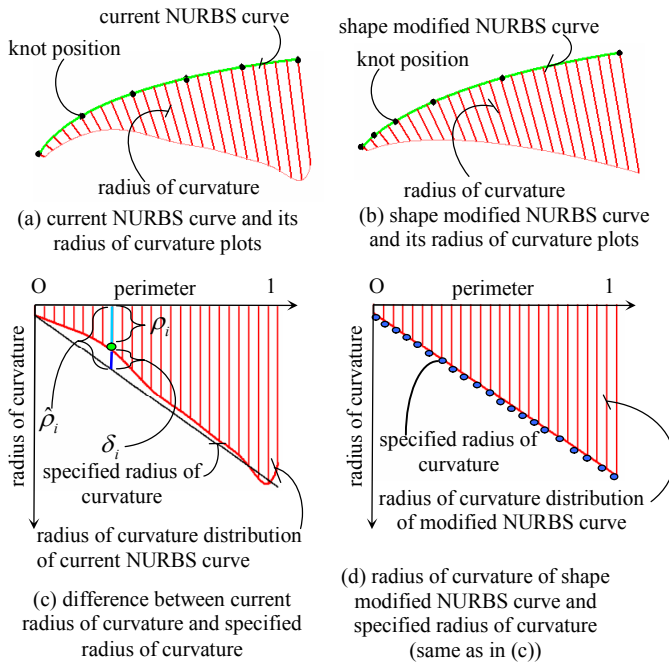
Fig.5 Concept of radius of curvature specification and NURBS curve shape modification based on the specified radius of curvature distribution

As an example, the linear algebraic function as a specified radius of curvature specification is shown in Fig.5(c). The i th of radius of curvature distribution of a perimetrically represented NURBS curve is denoted as ρ_i, the specified radius of curvature at the same spot is denoted as $\hat{\rho}_i$, the difference δ_i is shown by Eq.(11) and is illustrated in Fig.5(c).

$$\delta_i = \rho_i(q_1^x, \cdots, q_{n-2}^x, q_1^y, \cdots, q_{n-2}^y) - \hat{\rho}_i \qquad (11)$$

Where $i = 0,1,2,\cdots,m-1$, m is the number of specified radius of curvature, and n is the number of NURBS curve segments plus 5, which is the degree of the curve.

$S(q_1^x, \cdots, q_{n-2}^x, q_1^y, \cdots, q_{n-2}^y)$ which is the sum of the squared differences for all specified radius of curvatures in Eq.(12) is minimized by introducing the least-squares method. The radius of curvature expression is non-linear. Therefore, by Taylor's theorem, Eq.(12) is linearlized as in Eq.(13).

$$S(q_1^x, \cdots, q_{n-2}^x, q_1^y, \cdots, q_{n-2}^y)$$
$$= \sum_{i=0}^{m-1} \left[\rho_i(q_1^x, \cdots, q_{n-2}^x, q_1^y, \cdots, q_{n-2}^y) - \hat{\rho}_i \right]^2 \qquad (12)$$

$$S(q_1^x + \Delta q_1^x, \cdots, q_{n-2}^x + \Delta q_{n-2}^x, q_1^y + \Delta q_1^y, \cdots, q_{n-2}^y + \Delta q_{n-2}^y)$$
$$= \sum_{i=0}^{m-1} \left[\rho_i(q_1^x, \cdots, q_{n-2}^x, q_1^y, \cdots, q_{n-2}^y) + \frac{\partial \rho_i}{\partial q_1^x} \Delta q_1^x + \right.$$
$$\left. \cdots + \frac{\partial \rho_i}{\partial q_{n-2}^x} \Delta q_{n-2}^x + \frac{\partial \rho_i}{\partial q_1^y} \Delta q_1^y + \cdots + \frac{\partial \rho_i}{\partial q_{n-2}^y} \Delta q_{n-2}^y - \hat{\rho}_i \right]^2 \qquad (13)$$

Eq.(13) is minimized by equating to zero all the partial derivatives of $S(q_1^x + \Delta q_1^x, \cdots, q_{n-2}^x + \Delta q_{n-2}^x, q_1^y + \Delta q_1^y, \cdots, q_{n-2}^y + \Delta q_{n-2}^y)$ with respect to Δq_r^x and Δq_r^y ($r = 1, 2, \cdots, n-2$) as in Eq.(14).

$$\frac{\partial S}{\partial \Delta q_r^x} = 0 \quad (r = 1,2,\cdots,n-2)$$
$$\frac{\partial S}{\partial \Delta q_r^y} = 0 \quad (r = 1,2,\cdots,n-2) \qquad (14)$$

Using these simultaneous linear equations, Δq_r^x and Δq_r^y ($r = 1,2,\cdots,n-2$) are calculated. Then, q_r^x and q_r^y are determined.

7 A Method to Specify the Radius of Curvature Distribution by Selecting a fair NURBS Curve using the Similarity of Radius of Curvature Distribution

In this section, a method to specify the radius of curvature distribution by selecting a predefined fair NURBS curve using the similarity of radius of curvature distribution is described.

The shape of a NURBS curve is defined by the number, the location of its control points, and the knot sequence of the knot vector. A curve with a monotone radius of curvature distribution is considered as a fair curve in the area of Computer Aided Aesthetic Design. As a measure of curve fairness evaluation, radius of curvature distribution is used as an alternative characteristic of a curve. The radius of curvature is specified by linear, quadratic, cubic, quartic, quintic, and six degree function, while curve shape is modified according to the specified radius of curvature distribution using the shape modification algorithm mentioned in the previous section.

Then, six curves, whose radius of curvature are linear, quadratic, cubic, quartic, quintic, and six degrees are generated based on the curve generated from the sketch. These six functions are used as the specified radius of curvature.

Moreover, these six curves can be considered as fair, since their radius of curvatures are monotone because their applied algebraic functions independent variables are monotone to the dependent variables.

The similarity of the curve generated from the sketch to these six curves is examined. The similarity is evaluated by correlation coefficient. The curve whose similarity is the highest among these curves is determined as the curve generated from the sketch. From now on, to make description simple, the curve generated from the sketch is called curve-S, the shape modified curve according to the radius of curvature distribution specified by a linear function is called *curve-1*, the shape modified curve according to the radius of curvature distribution specified by a quadratic function is called *curve-2*, the shape modified curve according to the radius of curvature distribution specified by a cubic function is called *curve-3*, the shape modified curve according to the radius of curvature distribution specified by a quartic function is called *curve-4*, the shape modified curve according to the radius of curvature distribution specified by a quintic function is called *curve-5*, and the shape modified curve according to the radius of curvature distribution specified by a six degrees function is called *curve-6*.

Curve-S is shown in Fig.6(a), *curve-1* is shown in Fig.6(b), *curve-2* is shown in Fig.6(c), *curve-3* is shown in Fig.6(d), *curve-4* is shown in Fig.6(e), *curve-5* is shown in Fig.6(f), and *curve-6* is shown in Fig.6(g) with their curvature and radius of curvature plots respectively. Point marks of these figures indicate knot position of a NURBS curve.

Radius of curvature distribution corresponding to Fig.6(a), (b), (c), (d), (e), (f), and (g) is shown in Fig.7(a), (b), (c), (d), (e), (f), and (g) respectively, while perimeters of curves are rescaled as 1.

The correlation coefficient of *curve-S* and *curve-1*, that of *curve-S* and *curve-2*, that of *curve-S* and *curve-3*, that of *curve-S* to *curve-4*, that of *curve-S* to *curve-5*, and that of *curve-S* to *curve-6* are calculated. These are summarized in Table 1. From Table 1, the correlation coefficient between *curve-S* and *curve-5* is recognized as the highest. That is, the radius of curvature distribution of *curve-S* is recognized as being very similar to that of *curve-5* according to the correlation coefficient. Therefore, *curve-5* is determined as the curve generated from the sketch. In this way, a fair NURBS curve is generated from a hand-drawn sketch.
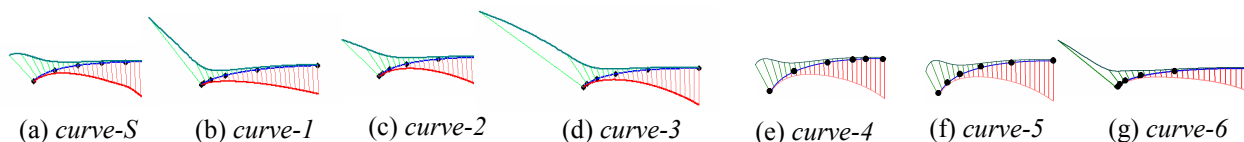

(a) *curve-S*   (b) *curve-1*   (c) *curve-2*   (d) *curve-3*   (e) *curve-4*   (f) *curve-5*   (g) *curve-6*

Fig.6 Curves with curvature and radius of curvature plots

(a) radius of curvature distribution of *curve-S*   (b) radius of curvature distribution of *curve-1*   (c) radius of curvature distribution of *curve-2*   (d) radius of curvature distribution of *curve-3*   (e) radius of curvature distribution of *curve-4*   (f) radius of curvature distribution of *curve-5*   (g) radius of curvature distribution of *curve-6*
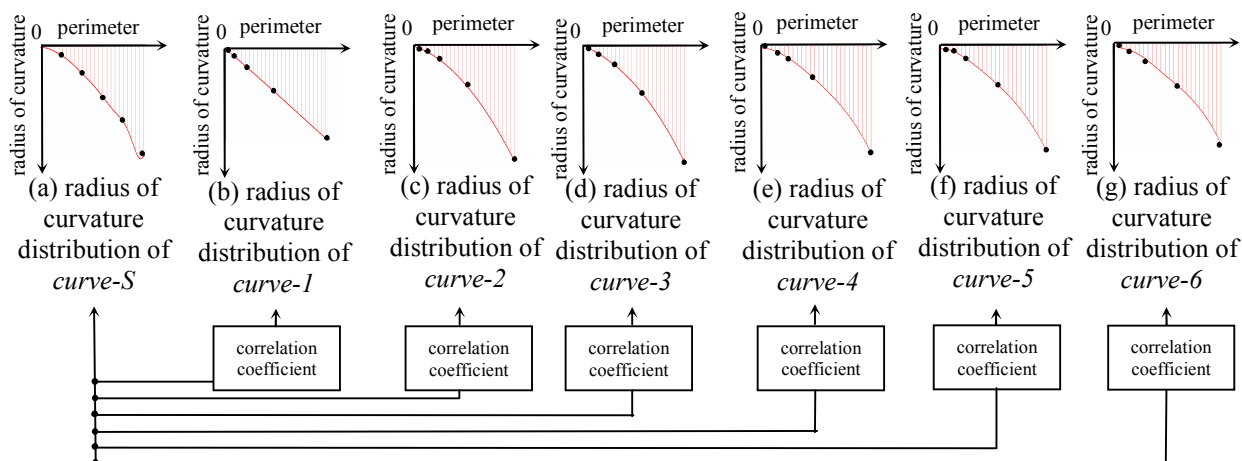
Fig.7 Radius of curvature distribution and correlation coefficient

Table 1 Correlation coefficient

	curve-1	curve-2	curve-3	curve-4	curve-5	curve-6
curve-S	0.966819	0.999420	0.999398	0.999618	0.999627	0.999570

8   Examples   of   NURBS   Curve Generation based on a Hand-drawn Sketch

Using the techniques mentioned in the previous sections, a fair NURBS curve generation process illustrated in Fig.8 is described according to the following steps.

The hand-drawn sketch image shown in Fig.8(a) is put into a computer by using a scanner set at 120 dots per inch.   A gradated background is shown in this hand-drawn sketch.   Therefore, transformation to a logarithmic image is necessary.   The logarithmic image is shown in Fig.8(b).   Then, the hand-drawn sketch is binarized.   The binary hand-drawn sketch image is shown in Fig.8(c).

The edge of the binary sketch is smoothed by applying feature based erosion and dilation.

The objective of this process is to smooth the outer edge of the binary image.   Therefore, internal image contours are eliminated.

The edge smoothed binary sketch image is shown in Fig.8(d).   Then, applying a Laplacian operation, the edges of the smoothed binary sketch image are detected.   The detected edges indicated by dots are shown with the sketch image as shown in Fig.8(e). Using the detected edges, regression line segments are determined.

These line segments are shown in Fig.8(f).   Using the determined line segments, a quintic NURBS curve is generated.

The generated NURBS curve with radius of curvature plots is shown in Fig.8(g).

Radius of curvature distribution and specified radius of curvature by quartic function according to the similarity of the radius of curvature distribution are shown in Fig.8(h).   The curve shape modification algorithm is then applied to this NURBS curve.

A shape modified NURBS curve is shown with its radius of curvature plots in Fig.8(i).

Radius of curvature distribution of a shape modified NURBS curve is shown with the specified radius of curvature distribution in Fig.8(j).

It is visually recognized that the radius of curvature distribution of the shape modified curve is in accordance with the specified radius of curvature.

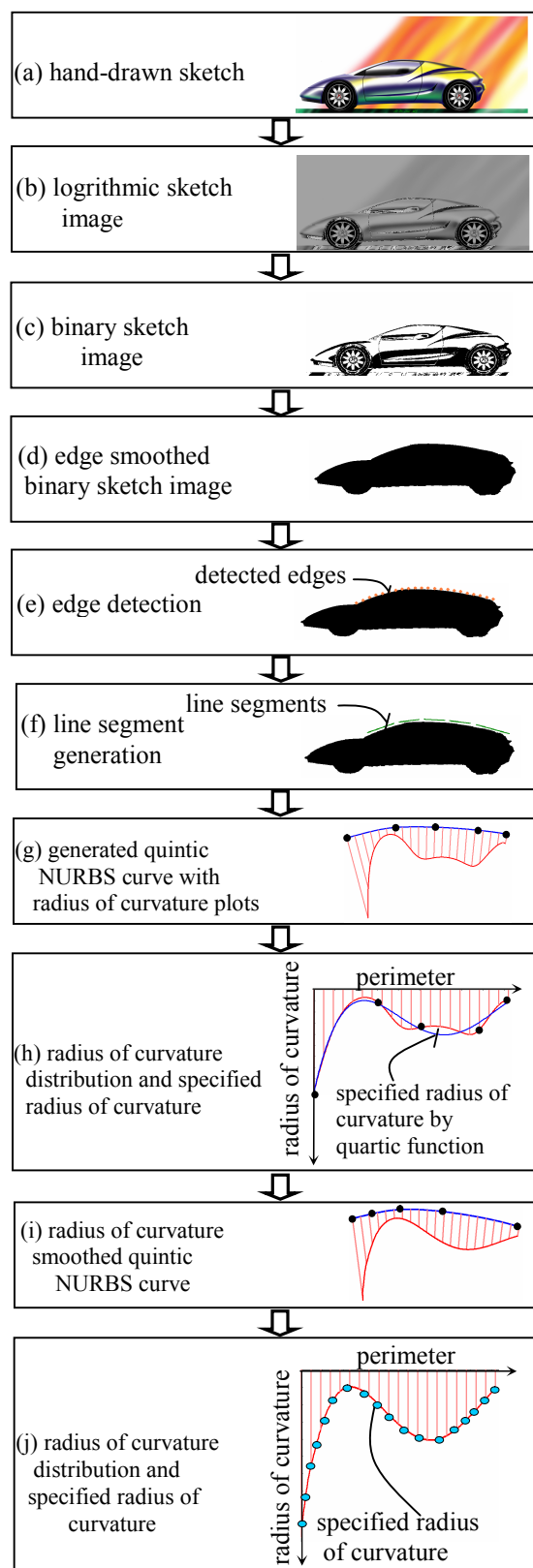Thus, following these steps, a fair NURBS curve can be generated.

Fig.8 Illustration for the steps from data input
to generation of shape modified NURBS curve

9   Concluding Remarks

The objective of this study is to shorten the product design period by extracting a freeform curve from a hand-drawn sketch. Edge smoothing of the binary sketch image by feature based erosion and dilation, and line segment generation using the detected edges are described. A freeform curve is expressed as a quintic NURBS curve, and NURBS curve generation based on the line segments are described. Then, a method to modify a NURBS curve shape based on the specified radius of curvature distribution is described. In addition to this, a method to specify the radius of curvature distribution by selecting a predefined fair NURBS curve using the similarity of radius of curvature distribution is described.

Examples of NURBS curve generation based on a hand-drawn sketch are given according to the generation steps.

We have proposed an edge smoothing method of the binary image by feature based erosion and dilation. We have also proposed a method to generate a NURBS curve using the given points and the gradient on the line segments simultaneously.

Moreover, we have also proposed a method to modify the shape of the NURBS curve according to the specified radius of curvature distribution.

In addition to this, we have also proposed a method to specify the radius of curvature distribution by selecting a predefined fair NURBS curve using the similarity of radius of curvature distribution.

A definition of a fair curve and specification of the radius of curvature are issues to be examined thoroughly in the future.

References:

[1] C. Werner Dankwort and Gerd Podehl, "A New Aesthetic Design Workflow-Results from the European Project FIORES", *CAD Tools and Algorithms for Product Design*, Springer-Verlag Berlin (2000), pp.16-30.

[2] Alias Learning Tools, "Learning Design with Alias StudioTools: A Hands-on Guide to Modeling and Visualization in 3D (Official Alias Training Guide)", Sybex; Pap/Dvdr edition (2006).

[3] H. Aoyama, Y. Urabe, M. Ohta, and T.Kusunoki "Aesthetic design system based on sketch, design language, and characteristic lines", *CIRP Journal of Manufacturing systems*, vol 32, (2003), pp. 101-106.

[4] S. Saga, "A freehand interface for computer aided drawing systems based on the fuzzy spline curve identifier", *IEEE*, (1995), pp. 2754-2759.

[5] T. Igarashi, S. Matsuoka, H. Tanaka, "Teddy: A sketching interface for 3D freeform design", *SIGGRAPH 99*, (1999), pp. 409-416.

[6] J. Mitani, H. Suzuki, F. Kimura, "3D Sketch: Sketch-based model reconstruction and rendering", *7th IFIP WG5.2 International Workshop on Geometric Modeling* (GEO-7), (2000), pp. 85-112.

[7] S. Sugishita, K. Kondo, H. Sato, S. Shimada, and F. Kimura, "Interactive freehand sketch interpreter for geometric modeling", Symbiosis of Human and Artifact, (1995), pp. 561-566.

[8] K. Matsuda, S. Sugishita, K. Kondo, H. Sato, and S. Shimada, "Freehand sketch system for 3D geometric modeling", *IEEE*, (1997), pp.55-62.

[9] C.C.L. Wang, Y. Wang, M.M.F. Yuen, "Remeshing based mesh smoothing by 2D sketches input. *Proceedings of DETC'02 ASME 2002 Design Engineering Technical Conference and Computer and Information in Engineering Conference*", (2002), pp.579-587.

[10] T.A. Galyean, J.F. Hughes, "Sculpting : An interactive volumetric modeling technique", *Proc. ACM SIGGRAPH '91*, Vol 25, Num 4, (1991), pp. 267-273.

[11] R.C. Zeleznik, K.P. Herndon, J.F. Hughes, "SKETCH : An interface for sketchnig 3D scenes" *Computer graphics Proceedings*, (1996), pp. 163-170.

[12] Y. Zeng Y, A. Pardasani, H. Antunes, Z.Li,J. Dickinson, V. Gupta, D. Baulier, "Representation and interpretation of sketches in mechanical design: experimental and theoretical approaches", *Proceedings of DETC'03 ASME 2003 Design Engineering Technical Conferences and Computers and Information in Engineering Conference,* (2003), pp. 613-624.

[13] A. Shesh, and B. Chen, "SMARTPAPER : An interactive and user friendly sketching system", *EUROGRAPHICS 2004*, vol 23, (2004), pp. 301-310.

[14] E. Schweikardt, M.D. Gross, "Digital clay : deriving digital models from free hand sketches", *Automation in Construction*, 9, (2000), pp. 107-115.

[15] C. Zhang, P. Zhang, F.(F). Cheng, "Fairing spline curves and surface by minimizing energy", *Computer Aided Design*, 33, (2001), pp.913-923.

[16] X. Yang, G. Wang, "Planar point set fairing and fitting by arc splines", *Computer Aided Design*, 33, (2001), pp. 35-43.

[17] W.H. Frey, D.A. Field, "Designing Bézier conic segments with monotone curvature", *Computer Aided Geometric Design*, 17, (2000), pp. 457-483.

[18] M. Kuroda, M. Higashi, T. Saitoh, Y. Watanabe, T. Kuragano, "Interpolating curve with B-spline curvature function", Mathematical Methods for Curves and Surfaces Ⅱ, Vanderbilt University Press, Nashville, TN, (1998), pp. 303-310.

[19] W. Li, S. Xu, J. Zheng, G. Zhao, "Target curvature driven fairing algorithm for planar cubic B-spline curves", *Computer Aided Geometric Design*, 21, (2004), pp. 499-513.

[20] H. Nowacki, D. Liu, X. Lu, "Fairing Bézier curves with constraints", *Computer Aided Geometric Design*, 7, (1990), pp. 43-55.

[21] Y. Mineur, T. Lichah, J.M. Castelain, H. Giaume, "A shape controled fitting method for Bézier curves", *Computer Aided Geometric Design*, 15, (1998), pp. 879-891.

[22] L. Fang, D.C. Gossard, "Multidimensional curve fitting to unorganized data points by nonlinear minimization", *Computer Aided Design*, 27,1, (1995), pp. 48-58.

[23] J. Unger, "Rendering in mixed media", Watson-Guptill Publications, (1985).

[24] J.C. Russ J.C, *The image processing handbook forth edition*, CRC Press, Florida, (2002), pp.123-124.

[25] Avallone E.A., Barry P., Bonn G. S., Bowman W. G., *McGraw-Hill Dictionary of Scientific and technical terms Sixth Edition*. McGraw-Hill, (2002), pp.2054.

[26] C. de Boor, "On calculating with B-spline", Japprox. Theory, 6(1), (1972), pp.50-62.

[27] D. Marsh, "Applied Geometry for Computer Graphics and CAD", Springer-Verlag, (2005), pp.188.

[28] Eugene V. Shikin.: Hand Book and Atlas of CURVES. CRC Press Boca Raton, Florida, (1995), pp.29.

[29] W. Boehm, and H. Prautzsch, "Geometric Concepts for Geometric Design", (1994), pp.26-27.

[30] G. Farin, and D. Hansford, "Practical Linear Algebra –A Geometry Toolbox–", (2005), pp.253-254.