CONTRIBUTION TO ANALYSIS OF CONSERVATIVITY, DISSIPATIVITY AND SYSTEM STABILITY

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Abstract: - The contribution is mainly concerned with presentation of a new fundamental approach to structural properties of linear and non-linear causal system representations. It has been proven that complete analysis of instability, conservativity, dissipativity, anti-dissipativity, stability, asymptotic stability and chaoticity reduces to two independent tests: the test of state energy monotonicity and that of complete state observability.

Key-Words: - Dissipativity, conservativity, state energy-metric, stability, chaoticity

1 Introduction

Almost in any field of science and technology some sort of stability problem can appear. Instability is certainly the most important phenomena, which should be investigated before any other aspect of reality will be attacked. Two typical situations should be distinguished in dynamical systems theory, if an in-stability problem has to be solved. The first one arises if an energy function *E[x(t)]* of a given system is known in a mathematical form. In such situations the time evolution of internal energy along any system motion can be described, and an energy monotonicity test, (see for instance [1], [2]), can be used:

$$E[\boldsymbol{x}(t)] > 0, \quad \frac{dE(\boldsymbol{x})}{dt} \le 0 \qquad (1)$$

On the other hand, there are certainly even more real world situations in which causality and energy conservation are known to play a crucial role, but any mathematical expression for the system energy evolution is not available.

In many cases, a sort of approximation seems to be adequate way to avoid this difficulty. If effects of parameter changes are neglected and a technique of approximate linearization is used, then a form of algebraic stability test based on the explicit knowledge of the solution of differential equations may help to simplify the solution. A set of necessary and sufficient conditions for roots:

$$\operatorname{Re} s_i < 0 \qquad (2)$$

or for *coefficients* a_i of the *system characteristic polynomials* $P(s)$, such as the well known *Hurwitz criterion*, for instance, have been frequently used. For the so-called *non-critical cases,* A. M. Lyapunov has legitimated the *approximate linearization approach* by his *first method*, also called *Indirect,* in the year 1892.

Unfortunately, it is *more an exception as a rule* that a *real world system* can be *a'priory* considered as *non-critical*.

In fact, more appreciated became the *second method of Lyapunov,* which instead of the physical energy E works with a *set of axiomatically defined scalar functions* V of the state $\mathbf{x}(t)$, called *Lyapunov functions*. Fundamental drawback if lack of a reliable technique of *Lyapunov functions generation.*

The main goal of the paper is to present an alternative method for stability analysis. Instead of Lyapunov functions a *state space metric* has been introduced as *an abstract measure of the total energy accumulated in the system state* [1, 2]. The essence of the new approach is demonstrated by variety of examples.

2 Internal and external stability

Recall that from general point of view any collection of trajectories constitutes a dynamical system, which, in principle, can be described either by its *external behavior*, or by an *internal structure*. Intuitively, the essence of *an external stability property* can formally be expressed as follows

$$\{\textit{(linear) system is stable}\} \Leftrightarrow \{|u(t)| < \delta \Rightarrow |y(t)| < \varepsilon\} \qquad (3)$$

In the present paper, mainly concepts concerning the internal stability will be examined. In such a case of the *initial state-to-future state framework*, only *an internal causality structure,* reflecting a *time orientation property* of the *causality relation,* describing a collection of all *state trajectories*, is appropriate:

$$\dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t)], \quad \mathbf{x}(t_0) \in X \subset R^n \qquad (4)$$

Definition 1: (Internal stability of an equilibrium state) The *equilibrium state* $\boldsymbol{x}^*$ of the internal system representation (4), defined by

$$f(\mathbf{x}^*)=0 \quad (5)$$

is: **Stable** (in the sense of Lyapunov) if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$:

$$\left\|\mathbf{x}(t_0) - \mathbf{x}^*\right\| < \delta \Rightarrow \left\|\mathbf{x}(t) - \mathbf{x}^*\right\| < \varepsilon, \forall t \geq t_0 \quad (6)$$

Asymptotically stable if it is table (Eq. (6)), and δ can be chosen such that:

$$\left\|\mathbf{x}(t_0) - \mathbf{x}^*\right\| < \delta \Rightarrow lim_{t\to\infty}\mathbf{x}(t) = \mathbf{x}^* \quad (7)$$

3 State energy-metric and dissipation normal form

As an alternative to the well-known physical energy motivated method of Lyapunov functions, a new conceptually different approach to stability problems has recently been proposed in [3-10] and called the signal energy-metric approach. The *crucial idea* is that, in fact, *it is not the energy by itself*, but only a *measure of distance from the system equilibrium to the actual state* $\mathbf{x}(t)$, what is needed for stability/in-stability analysis. Thus a *state space metric* $\rho\left[\mathbf{x}(t), \mathbf{x}^*\right]$, where $\mathbf{x}^*$denotes the *equilibrium state,* has been defined, and the basic idea of a new *state energy-metric approach* is then formally expressed by:

$$E(\mathbf{x}) = \frac{1}{2}\rho^2\left[\mathbf{x}(t), \mathbf{x}^*\right] \quad (8)$$

To avoid confusion the concepts of the *signal power* and that of *state energy* for *system representations* $\Re\{S\}$ with $\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x}$ are *defined* first:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}[\mathbf{x}(t)] + \mathbf{B}\,\mathbf{u}(t),\ \mathbf{x}(t_0) = \mathbf{x}^0, \\ \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t) \end{aligned} \quad (9)$$

Let the *immediate value* of the *output signal power* and corresponding *value* of the system *energy,* accumulated in a *time instance t* in the *state* $\mathbf{x}(t)$ be defined:

$$P(t) = \left\|\mathbf{y}(t)\right\|^2,\ E(t) = \delta\left\|\mathbf{x}(t)\right\|^2, \frac{\mathrm{d}E(\mathbf{x})}{\mathrm{d}t} = -P(t),\ \delta > 0 \quad (10)$$

Putting $\mathbf{u}(t) = \mathbf{0}, \forall t \geq t_0$ and computing the *derivative of the energy* $E[\mathbf{x}(t)]$ along the equivalent representation of the given system we get the *signal power balance relation* in the form

$$\frac{\mathrm{d}E(\xi)}{\mathrm{d}t} = \delta\mathbf{x}^T(t)[\mathbf{A}(.) + \mathbf{A}^T(.)]\mathbf{x}(t) = -\left\|\mathbf{y}(t)\right\|^2 \quad (11)$$

and, by integration, the energy conservation principle for a proper chosen equivalent state space representation follows. Hence, in case of zero input $\mathbf{u}(t) = \mathbf{0}$*,* $t \geq t_0$, the *total energy accumulated in the system* in time t_0 must be equal to an *amount of the energy dissipated* on the interval $[t_0 , \infty)$ *by the output*:

$$E(t_0) = \int_{t_0}^{\infty}\left\|\mathbf{y}(t)\right\|^2 dt \quad (12)$$

It follows that a special form of a *structurally dissipative* state equivalent system representation called *dissipation normal form exists* and is given by matrices:

$$\mathbf{A} = \begin{pmatrix} -\alpha_1 & \alpha_2 & 0 & 0 & \cdots & 0 & 0 \\ -\alpha_2 & 0 & \alpha_3 & 0 & \cdots & 0 & 0 \\ 0 & -\alpha_3 & 0 & \alpha_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & -\alpha_{n-1} & 0 & \alpha_n \\ 0 & 0 & 0 & 0 & \cdot\cdot 0 & -\alpha_n & 0 \end{pmatrix}, \mathbf{C}^T = \begin{bmatrix} \gamma \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_{n-1} \\ \beta_n \end{bmatrix} \quad (13)$$

Structure of this representation is shown in Fig. 1.

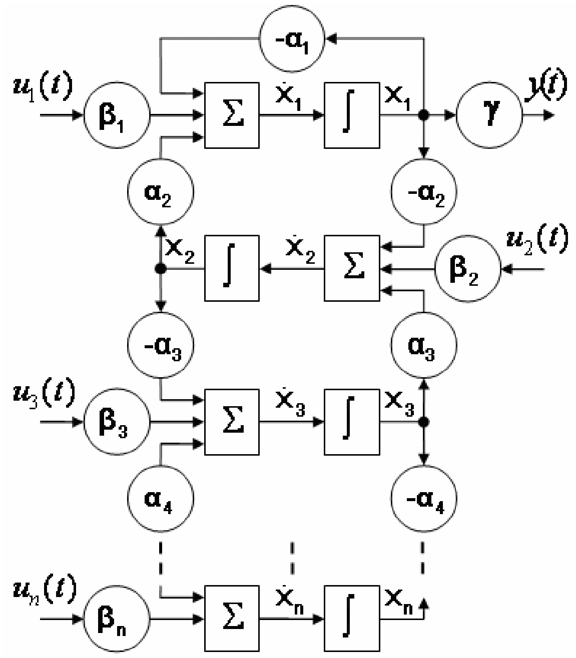

Fig. 1. Physically correct structure of the given representation

4 Dissipativity, conservativity and instability

Recall that according to Liouville's theorem of vector analysis, *dissipative systems* have the important property that any volume of the state space strictly decreases under the action of the system flow [11-20]. For nonlinear systems with *state velocity vector* given by a *vector field* **f**, the property of *dissipativity* is defined:

$$\operatorname{div}\mathbf{f}(\mathbf{x}) = \sum_{i=1}^{n}\frac{\partial f_i(\mathbf{x})}{\partial x_i} < 0 \quad (14)$$

Thus a system defined by a *matrix* **A**(.) is *dissipative* if **A** *has negative trace*

$$Trace\ \mathbf{A} = -\alpha_1 < 0 \quad (15)$$

Nonlinear systems having at an equilibrium state a dissipative approximate linearization are *locally dissipative* in a vicinity of this equilibrium state, but need not to be *globally dissipative.* Similarly systems with *vanishing divergence*

$$\operatorname{div} \mathbf{f}(\mathbf{x}) = 0 \quad \Rightarrow \quad \alpha_1 = 0 \qquad (16)$$

preserve volume along state trajectories and referred to as *conservative.* Hence, if a system is *neither dissipative, nor conservative, instability* appears.

5 Dissipativity, minimality and asymptotic stability

In this part mainly some *elementary consequences* of *dissipativity* and the *energy-metric approach* from stability analysis point of view will be discussed. We demonstrate that the set of *real parameters* α_i, γ, β_i satisfy the *conditions:*

1. $\alpha_1 < 0$ is *sufficient for instability*
2. $\alpha_1 = 0$, or $\alpha_1 > 0$ is *necessary and sufficient for stability (in the s. of L.)*
3. $\alpha_1 > 0$ is *necessary for asymptotic stability*
4. $\forall i, i \in \{2,3,...,n\} : 0 \neq \alpha_i, \gamma \neq 0, \exists i : \beta_i \neq 0 \Leftrightarrow$ *structural minimality*
5. $\forall i, i \in \{2,3,...,n\} : 0 \neq \alpha_i, \gamma \neq 0, \Leftrightarrow$ *structural observability*
6. $\forall i, i \in \{2,3,...,n\} : 0 \neq \alpha_i, \alpha_1 > 0, \Leftrightarrow$ *structural asymptotic stability*

<u>*Example 1*</u>. (*Stability/instability analysis* of an *n-th order* linear system)

Let us consider a system for $n = 6$, and for the input signal $u(t) = 0, t \geq t_0$, given by the linear differential equation with constant coefficients

$$y^{(6)} + a_1 y^{(5)} + ... + a_4 \ddot{y}(t) + a_5 \dot{y}(t) + a_6 y(t) = 0 \qquad (17)$$

and let the *internal structure* of the corresponding state space representation is given in the *dissipation normal form* with the characteristic polynomial $P_6(s)$ *given recursively by the relations*

$$\begin{aligned} &P_k(s) = s\, P_{k-1} + \alpha_k^2 P_{k-2}\,, \ k = 2,3,...,n \\ &\textit{with } P_0(s) = 1, \ \textit{and } \ P_1(s) = s + \alpha_1 \end{aligned} \qquad (18)$$

Now, let all the *parameters* a_1, a_2,…, a_n of $P_n(s)$ be considered as *unknown,* and let us specify the *region of asymptotic stability in the parameter space.* Recall that the *necessary and sufficient condition for existence of the unique equilibrium state* $\mathbf{x}^* = \mathbf{0}$ can explicitly be expressed as

$$\begin{aligned} &\det \mathbf{A} = a_6 = \alpha_2^2 \alpha_4^2 \alpha_6^2 \neq 0 \Leftrightarrow \\ &\exists \mathbf{A}^{-1} \Leftrightarrow \alpha_2 \neq 0, \alpha_4 \neq 0, \alpha_6 \neq 0 \end{aligned} \qquad (19)$$

The parameters a_i of the characteristic polynomial $P_6(s)$ are given by the following *set of algebraic equations* with the *unique general solution:*

$$\begin{aligned} a_1 &= \alpha_1 \\ a_2 &= \alpha_2^2 + \alpha_3^2 + \alpha_4^2 + \alpha_5^2 + \alpha_6^2 \\ a_3 &= \alpha_1(\alpha_3^2 + \alpha_4^2 + \alpha_5^2 + \alpha_6^2) \\ a_4 &= \alpha_2^2(\alpha_4^2 + \alpha_5^2 + \alpha_6^2) + \alpha_3^2(\alpha_5^2 + \alpha_6^2) + \alpha_4^2\alpha_6^2 \\ a_5 &= \alpha_1\alpha_3^2(\alpha_5^2 + \alpha_6^2) + \alpha_1\alpha_4^2\alpha_6^2 \\ a_6 &= \alpha_2^2\alpha_4^2\alpha_6^2 \\ \alpha_1 &= a_1 = \Delta_1\,, \\ \alpha_2 &= \sqrt{\frac{a_1 a_2 - a_3}{a_1}} = \sqrt{\frac{\Delta_2}{\Delta_1}} \\ \alpha_3 &= \sqrt{\frac{a_1 a_2 a_3 - a_3^2 - a_1^2 a_4}{(a_1 a_2 - a_3) a_1}} = \sqrt{\frac{\Delta_3}{\Delta_2 \Delta_1}} \\ \alpha_k &= \sqrt{\frac{\Delta_k \Delta_{k-3}}{\Delta_{k-2}\Delta_{k-1}}} \ , \ k = 4, 5, 6, \ldots, n \end{aligned} \qquad (20)$$

It is easy to see that *for any finite order*, the *new parameters* Δ_k, k= 1, 2,…,n can be expressed as *diagonal minors* of the well known *Hurwitz determinant.* In order to use *energy monotonicity test* like the Eq. (1) we have to *compute the derivative of the output signal energy function* $E[\mathbf{x}(t)]$ *along the system representation* $\Re\{S\}$ given by the matrices (13) in the *dissipation normal form*:

$$\begin{aligned} \Re(S):\ \dot{x}_1(t) &= -\alpha_1 x_1(t) + \alpha_2 x_2(t) \\ \dot{x}_2(t) &= -\alpha_2 x_1(t) + \alpha_3 x_3(t) \\ \dot{x}_3(t) &= -\alpha_3 x_2(t) + \alpha_4 x_4(t) \\ \dot{x}_4(t) &= -\alpha_4 x_3(t) + \alpha_5 x_5(t) \\ \dot{x}_5(t) &= -\alpha_5 x_4(t) + \alpha_6 x_6(t) \\ \dot{x}_6(t) &= -\alpha_6 x_5(t) \end{aligned} \qquad (21)$$

$$\left. \frac{\mathrm{d}E(t)}{\mathrm{d}t} \right|_{\Re\{S\}} = -\alpha_1 x_1^2(t) = -\frac{\alpha_1}{\gamma^2}\, y^2(t) \qquad (22)$$

where γ is a *real output signal scaling parameter* $0<\gamma<\infty$. Thus, the *state energy* $E[\mathbf{x}(t)]$ *decreases monotonically* (for real $\gamma \neq 0$, and $y \neq 0$), if and only if:

$$P(t) = y^2(t) > 0 \ \Leftrightarrow \ \alpha_1 = \gamma^2 > 0 \qquad (23)$$

i.e. *real output dissipation power is positive,* and *vector field* $\mathbf{f}=\mathbf{A}\mathbf{x}$ *is dissipative.* It can be seen *directly* from the Eq. (22) that for $\alpha_1 = 0$ *the system is conservative* if and only if *its complete state is unobservable* by the output signal ($\gamma^2 = \alpha_1 = 0$). It means that the *equilibrium state can not be unstable* if the system representation is *dissipative, conservative, and/or totally unobservable.* On the other hand if $\alpha_1 < 0$, then we have

$$P(t) = y^2(t) < 0 \ \Leftrightarrow \ \alpha_1 = \gamma^2 < 0 \qquad (24)$$

Consequently, the output signal scaling parameter γ and (for real state) also the output signal itself have to be considered as imaginary. In analogy to non-active electrical power, it seems natural to conclude that: *if an active output power becomes negative, its positive value should be interpreted as non-active.*

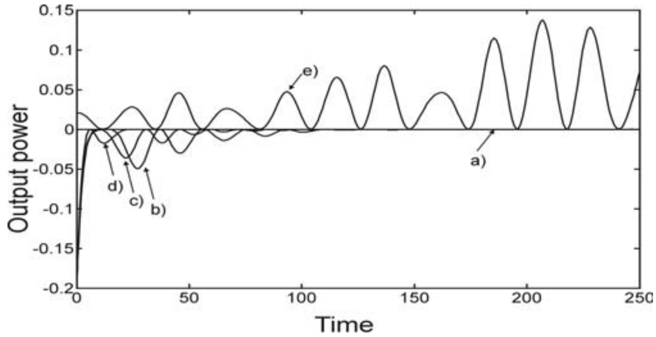

Fig. 2. Evolution of output power $P(t)$
a) conservative case $\alpha_1 = 0$, *b) stability* $\alpha_1>0$, $\alpha_3=0$, *c) stability* $\alpha_1 >0$, $\alpha_5 =0$, *d) asymptotic stability* $\alpha_1 >0$, $\alpha_k \neq 0$, $k = 2,3,\ldots,n$, *e)* instability $\alpha_1< 0$.

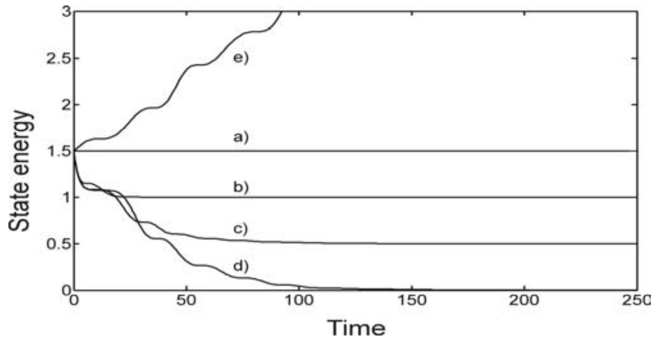

Fig. 3. Evolution of state energy $E[\mathbf{x}(t)]$
a) conservative case $\alpha_1 = 0$, *b) stability* $\alpha_1>0$, $\alpha_3=0$, *c) stability* $\alpha_1 >0$, $\alpha_5 =0$, *d) asymptotic stability* $\alpha_1 >0$, $\alpha_k \neq 0$, $k = 2,3,\ldots,n$, *e)* instability $\alpha_1< 0$.

It is easy to deduce from the Eq.(22) that for $\alpha_1 < 0$, the *velocity vector field is anti-dissipative,* and *instability* of the zero equilibrium state results. From the *existence of a unique equilibrium state* point of view Eq. (19) it follows that *not only* the *dissipation parameter* α_1, but also the *interaction parameters* α_3, α_5 can be chosen *arbitrary*. Notice that if we put $\alpha_5=0$, then the state variables x_i, $i=5,6$ become *unobservable by the output y*; thus *only the first isolated subsystem* with the state variables x_i, $i=1,2,3,4$, which is *observable, will be asymptotic stable*, while the *second one will oscillate on the constant energy level*, *(see Fig. 2. for course of the signal power and Fig. 3. for the state energy evolution),* corresponding to its initial state energy with the *frequency* given by the *natural frequency parameter* α_6. As a result the *whole system* is *stable in the sense of Lyapunov, but not asymptotically*. In such a case the polynomial $P_6(s)$ becomes:

$$P(s) = (s^2 + \alpha_6^2)\prod_{i=1}^{4}(s - s_i) \qquad (25)$$

and we get the set of *standard algebraic stability conditions*

$$\operatorname{Re} s_i<0, \text{ for } i = 1,2,3,4, \text{ and } \operatorname{Re} s_5 = 0, \operatorname{Re} s_6 = 0 \qquad (26)$$

Similarly, if we put $\alpha_3 = 0$, then the state variables x_i, $i=3,4,5,6$ become *unobservable by the output y,* and only the first isolated subsystem

$$\begin{aligned} \dot{x}_1(t) &= -\alpha_1 x_1(t) + \alpha_2 x_2(t) \\ \dot{x}_2(t) &= -\alpha_2 x_2(t) \\ y(t) &= \gamma x_1(t) \end{aligned} \qquad (27)$$

which is *observable, will be asymptotic stable*, while the *second one will oscillate on the constant energy level*, *(see Fig. 2, 3),* corresponding to its initial state energy with frequencies given by the *natural frequency parameters* α_4, α_6, modified by the *interaction parameter* α_5. Again, the *whole system* is *stable in the sense of Lyapunov, but not asymptotically.* The characteristic polynomial is:

$$P(s) = (s^2 + \alpha_1 s + \alpha_2^2)\prod_{i=3}^{6}(s - s_i) \qquad (28)$$

and the set of *standard algebraic stability conditions* results:

$$\operatorname{Re} s_i<0, \text{ for } i=1,2 \text{ and } \operatorname{Re} s_i=0, \text{ for } i=3,4,5,6 \qquad (29)$$

<u>*Example 2*</u>. (*Asymptotic* stability *analysis* of an *n-th order* linear system)
It is easy to prove in general that the conditions mentioned above are *necessary but not sufficient* for *asymptotic stability.* If, in addition, the couple (**A, C**) has the well known *observability property,* then the resulting conditions become *necessary and sufficient for asymptotic stability, too.*

Let $n = 6$, and the matrices **A** and **C** are given by the Eq. (13); recall that the *observability matrix* $\mathbf{H}_0$ is defined by

$$\boldsymbol{H}_0 = [\boldsymbol{C}^T; \boldsymbol{A}^T\boldsymbol{C}^T; (\boldsymbol{A}^T)^2\boldsymbol{C}^T; \ldots; (\boldsymbol{A}^T)^{n-1}\boldsymbol{C}^T] \qquad (30)$$

and the set of *necessary and sufficient observability conditions* has the form:

$$\begin{aligned} \det\mathbf{H}_0 \neq 0 \Leftrightarrow \alpha_2 \neq 0, \alpha_3 \neq 0, \alpha_4 \neq 0, \\ \alpha_5 \neq 0, \alpha_6 \neq 0, \gamma \neq 0 \end{aligned} \qquad (31)$$

Thus the set of *necessary and sufficient conditions of asymptotic stability* reads:

$$\alpha_k \neq 0, \ k \in \{1,2,\ldots,6\}, \ \alpha_1 > 0, \qquad (32)$$

Using the expressions (18), it is very easy to prove that the resulting conditions (30) are *equivalent to the set of the well-known Hurwitz conditions*:

$$\Delta_k > 0, \quad k=1, 2, \ldots, n \qquad (33)$$

It has been clearly demonstrated that linear *algebraic methods* for stability analysis can be seen as a *special case* of methods based on the proposed *signal energy-*

metric approach. Moreover, the state energy interpretation makes it possible to gain a better insight into classical results.

6 Analysis of asymptotic stability of non-linear systems

Example 3. (Non-linear asymptotic stability analysis)
Let us consider a simple *non-linear system given* by the representation:

$$\ddot{y}(t)+\varepsilon\left[\alpha-\beta y^2(t)\right]\dot{y}(t)+a_2 y(t) = 0 \qquad (34)$$

Let the matrix **C** is defined by **C**=[γ, 0], and the *structure of the matrix* **A**(**x**) is

$$\mathbf{A}(x_1,x_2)=\begin{bmatrix}-\varepsilon\left[\alpha-\frac{1}{3}\beta x_1^2\right], & \sqrt{a_2}\\ -\sqrt{a_2}\,, & 0\end{bmatrix} \qquad (35)$$

The system representation is *locally observable if and only if* $\gamma\neq 0$, $a_2>0$, and

$$\left.\frac{\mathrm{d}E(t)}{\mathrm{d}t}\right|_{\Re(s)} = -P(t)\leq 0,\;\; P(t)=\varepsilon\left[\alpha-\frac{1}{3}\beta x_1^2\right]x_1^2 \quad (36)$$

i.e. the *state* $\mathbf{x}^*=\mathbf{0}$ is *asymptotically stable in the region* $D\subset X\subset R^2$.

Example 4. (Generation of Lyapunov functions using the state energy function)
Let us consider the same *non-linear system* but instead of the *structure of* **A**(**x**) the state **x**(t) is defined by the standard way: $x_1=y$, $x_2=dy/dt$. Then the *unique Lyapunov function* V(**x**) can be determined by *isometric transformations of the energy function E(.)* and for $\alpha=\beta=a_2=1$ we get:

$$V(\mathbf{x})=E(\mathbf{x})= \frac{1}{2}\left[\frac{1}{9}\varepsilon^2x_1^6-\frac{2}{3}\varepsilon^2x_1^4+(1+\varepsilon^2)x_1^2-\frac{2}{3}\varepsilon x_1^3x_2+2\varepsilon x_1x_2+x_2^2\right] \qquad (37)$$

and for the *linear conservative* special case (ε = 0) it reduces to

$$V(\mathbf{x})=E(\mathbf{x})=\frac{1}{2}\left(x_1^2+x_2^2\right)=\frac{1}{2}\rho_2^2\left[\mathbf{x},0\right] \qquad (38)$$

Example 8. (State energy and mechanism of chaoticity in a causal system) [21, 22].
Let a 4th order system with chaotic behavior state be given by:

$$\begin{aligned}\dot{x}_1&=-\alpha_1x_1+\alpha_2x_2\\ \dot{x}_2&=-\alpha_2x_1+\alpha_3x_3\\ \dot{x}_3&=-\alpha_3x_2+\alpha_4x_4\\ \dot{x}_4&=-\alpha_4x_3\end{aligned}\quad \alpha=\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\end{bmatrix}=\begin{bmatrix}-1+10x_2^2\\ 1\\ 1\\ 2.00\end{bmatrix},\; x(0)=\begin{bmatrix}0\\ 0\\ 0.5\\ 0\end{bmatrix} \quad (39)$$

Projection of chaotic state and state energy evolution of this example are shown in Fig. 4 and 5.

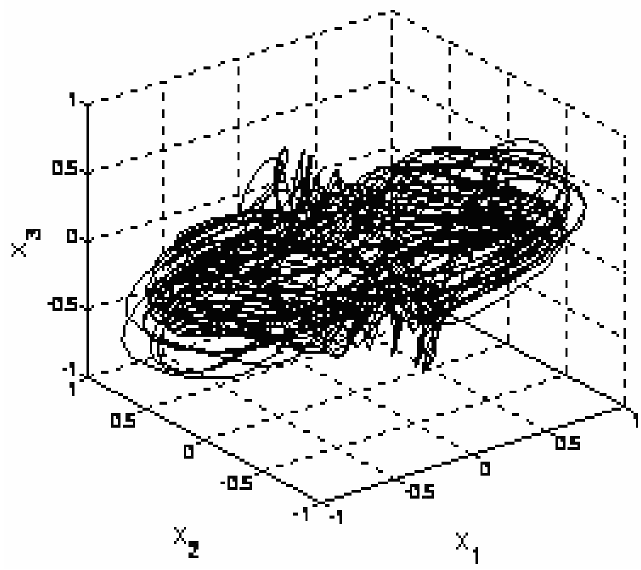

Fig. 4. 3D-Projection of chaotic state

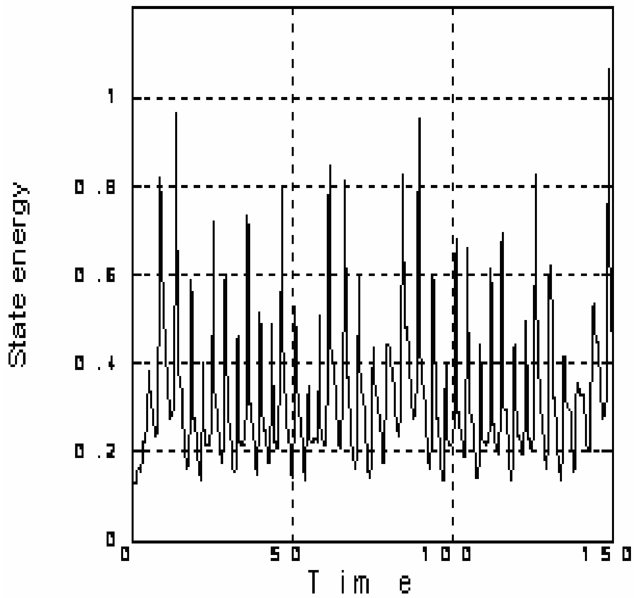

Fig. 5. State energy evolution

7 Conclusion

In the contribution a new unifying and constructive approach to linear and non-linear stability problems, based on a state-energy-metric of the system state space, has been presented.

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