Some Aspects of Modeling and Robust Control of a Robotic Manipulator

EUGEN BOBAŞU, DAN POPESCU, MONICA ROMAN
Department of Automatic Control
University of Craiova
A.I. Cuza Str. No. 13, RO-200585 Craiova
ROMANIA
http://www.ace.ucv.ro

Abstract: - The problem of exact linearization via feedback consists in transforming a nonlinear system into a linear one using a state feedback. In the multivariable case, the nonlinear control law achieves also decoupling. The use of feedback linearization requires the complete knowledge of the nonlinear system. In practice, there are many processes whose dynamics is very complex, highly nonlinear and usually incompletely known. It is possible that the controlled system become unstable in the presence of significant model uncertainties. To improve robustness, it may be necessary to modify the exact linearization controller. In this paper, some robustification techniques for the exact linearization method are presented and applied for some multivariable models of a robotic manipulator. Numerical simulations are included to demonstrate the behavior and the performances of these controllers.

Key-Words: - Robotic arm, Modelling, Nonlinear control, Linearizing control, Robust control

1 Introduction

In this paper, by using the feedback linearizing techniques, a multivariable nonlinear control law is obtained for a robotic manipulator [1].

The model of a robot is obtained from the basic physical laws governing its movement. There are many methods to obtain the dynamical model (see [4], [7], [11]): Lagrange method, Euler method, d'Alembert method, Kane method etc. Here is used the Lagrange method to obtain the dynamical model for a robot, which works in cylindrical coordinates.

If we consider some approximations on the robot dynamical model we can do a linear analysis of the manipulator control problem. Without these approximations we have a nonlinear model.

In the last years, significant advances have been made in the development of ideas such as feedback linearizing and input-output decoupling techniques ([2], [3], [5]). The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system into a linear one using a state feedback and a coordinate transformation of the state.

Practical implementation of such controllers requires consideration of various sources of uncertainties such as: modelling errors, computation errors, unknown payloads, measurement noise, etc. It is possible that the controlled system become unstable in the presence of significant model uncertainties.

To improve robustness, it may be necessary to modify the exact linearization controller to guarantee its robustness.

Several techniques from linear and nonlinear control theory have been applied to the problem of robust feedback linearization: Lyapunov redesign method, sliding modes, the H_∞ approach, etc. Here we present two techniques that can be applied to obtain a robust controller for the feedback linearization. First, Glover-McFarlane H_∞ design is presented with the goal of increasing robustness of existing controllers without significantly compromising performance. The second approach is the two-degree of freedom controller design. In these methodologies, it is possible to separate the designing task of meeting performance specifications and robustness into two modular steps.

The paper is organized as follows: in Section 2, some basics of the exact linearization theory are presented. In Section 3, the Glover-McFarlane and the two-degree of freedom controller design methods are presented. The mathematical models of a robotic manipulator are analyzed in Section 4. A working example using both nonlinear control laws and the Clover-McFarlane method for robustification of exact linearization design is presented in Section 5 including some computer simulation. Finally, Section 6 collects the conclusions.

2 The Statement of the Exact Linearization Problem

A multivariable nonlinear system can be described in state space by equations of the following kind:

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i$$
$$y_j = h_j(x) \qquad j = 1...m \tag{1}$$

in which $f(x)$, $g_1(x)$, $g_2(x)$,...., $g_m(x)$ are smooth vector fields.

The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system (1) into a linear one using a state feedback and a coordinate transformation of the systems state. The exact feedback linearization theory is widely presented in [7]. Next, some basic results of this theory are presented. These results are applied in Section 4, where nonlinear control laws are developed for robotic manipulators.

Consider the Lie derivative of a function $h(x): R^n \to R$ along a vector field $f(x)$:

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \tag{2}$$

Definition. A multivariable nonlinear system of the form (1) has a relative degree $\{r_1, ..., r_m\}$ at a point x^0 if:

1) $L_{g_j} L_f^k h_i(x) = 0$ (3)

for all $1 \le j \le m$, for all $1 \le i \le m$ for all $k < r_i - 1$, and for x in a neighborhood of x^0,

2) the $m \times m$ matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_2(x) & . \quad . & L_{g_2} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & . \quad . & L_{g_2} L_f^{r_2-1} h_2(x) \\ . & . \quad . & . \\ L_{g_1} L_f^{r_m-1} h_m(x) & . \quad . & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{4}$$

is nonsingular at $x = x^0$.

Theorem. Let be the nonlinear system of the form (1). Suppose that the matrix $g(x^0)$ has rank m. Then, the State Space Exact Linearization Problem is solvable if and only if:

1) for each $0 \le i \le n-1$, the distribution G_i has constant dimension near x^0;

2) the distribution G_{n-1} has dimension n;

3) for each $0 \le i \le n-2$, the distribution G_i is involutive.

If $r_1 + r_2 + ... + r_m = n$, the closed loop system it is composed from m chains of r_i integrators and it is described by a transfer matrix of the form (see Fig. 1):

$$H(s) = \begin{bmatrix} \frac{1}{s^{r_1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{s^{r_{21}}} & \cdots & 0 \\ . & . & . & . \\ 0 & 0 & 0 & \frac{1}{s^{r_m}} \end{bmatrix} \tag{5}$$

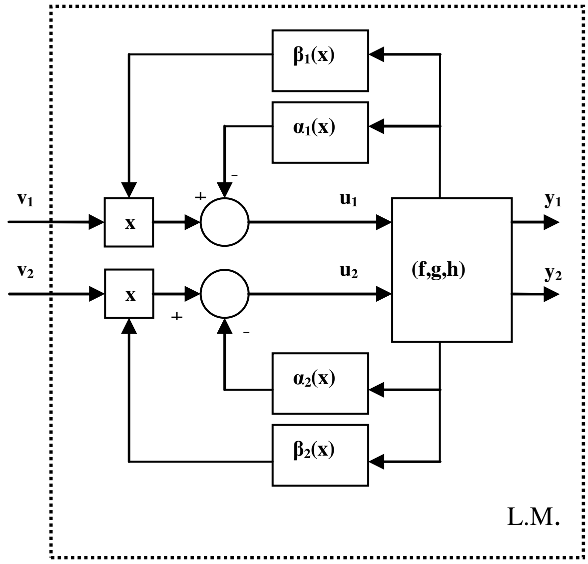

Fig. 1 The linearized model (L. M.)

Imposing for this system the feedback of the following form

$$v^i = c_0^i \left(y_i^{ref} - y_i - c_i^1 L_f h_i(x) - \ldots - c_i^{r_{i-1}} L_f^{r_1-1} h_i(x) \right),$$
$$1 \le i \le m \tag{6}$$

one obtains a linear input-output behavior characterized by a diagonal transfer matrix:

$$H(s) = \begin{bmatrix} \frac{1}{d_1(s)} & 0 & \cdots & 0 \\ 0 & \frac{1}{d_2(s)} & \cdots & 0 \\ . & . & . & . \\ 0 & 0 & 0 & \frac{1}{d_m(s)} \end{bmatrix} \tag{7}$$

where

$$d_i(s) = c_i^0 + c_i^1 s + ... + c_i^{r_i-1} s^{r_i-1} + s^{r_i} \tag{8}$$

3 Robust Control Design

3.1 Glover-McFarlane Control Design

We consider the structure of the control loop, for each decoupled channel, shown in Fig. 2, where is implemented the control law (6) and K_r is the robustifying controller (G_s is the nominal shaped plant).

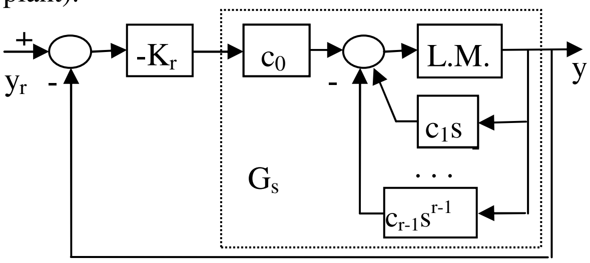

Fig. 2 The control loop

In this design, the model uncertainties are included as perturbations to the nominal model, and robustness is guaranteed by ensuring that the stability specifications are satisfied for the *worst-case* uncertainty.

Let $G_S = N / M$ be the normalized coprime factorization of the nominal shaped plant.

The normalized coprime factor uncertainty characterization is given by

$$\left\{ \frac{N+\Delta_N}{M+\Delta_M} : \left\| \left[\Delta_N \Delta_M \right] \right\| \le \varepsilon \right\} \tag{9}$$

The following steps yield the optimal controller that assumes a state-space (A, B, C) available for the transfer function G_S:

1) Obtain Z by solving the algebraic Riccati equation (ARE)

$$AZ + ZA - ZC^T CZ + BB^T = 0 \tag{10}$$

2) Obtain X by solving the ARE

$$AX + XA - XBB^T X + C^T C = 0 \tag{11}$$

3) Compute the maximum possible ε for the given nominal shaped plant

$$\varepsilon_{\max} = (1 + \rho(XZ)^{-1/2}) \tag{12}$$

where ρ denotes the spectral radius. Hence, in this design scheme there is no need for an explicit characterization of uncertainty. The method detects and solves for the worst-case scenario.

4) The robustness margin ε is chosen to be slightly less than $\varepsilon_{\max}$. Let $\gamma = 1/\varepsilon$.

5) The state-space realization of the robustifying controller K_γ is given by

$$\begin{bmatrix} A + BF + \gamma^2 (L^T)^{-1} ZC^T C & \gamma^2 (L^T)^{-1} ZC^T \\ B^T X & 0 \end{bmatrix} \tag{13}$$

where $F = -B^T X$ and $L = (1-\gamma^2)I + XZ$.

An important feature of this algorithm [8] is that the loop transfer functions before and after robustification are not significantly different.

3.2 Two-degree of freedom controller

Now, we consider the overall control system represented by the configuration of Fig.3, with a two-parameter compensator (R,S,T). Our design objective is to specify the two-parameter compensator to achieve the following two aims:

a) The compensator can robustly stabilize nominal model $G_0(s)$ against the uncertainty ΔG by specifying R(s) and S(s).

b) The transfer function from r to y is as close to the desired model M(s) as possible via an adequately chosen T(s).

Here, the nominal model $G_0(s)$ can be chosen as the transfer function of the linearized model (5).

The algorithm for designing the controller parameters (R,S,T) can be found in [9].

Remark: In this method it is necessary to evaluate the norm of the uncertainty.

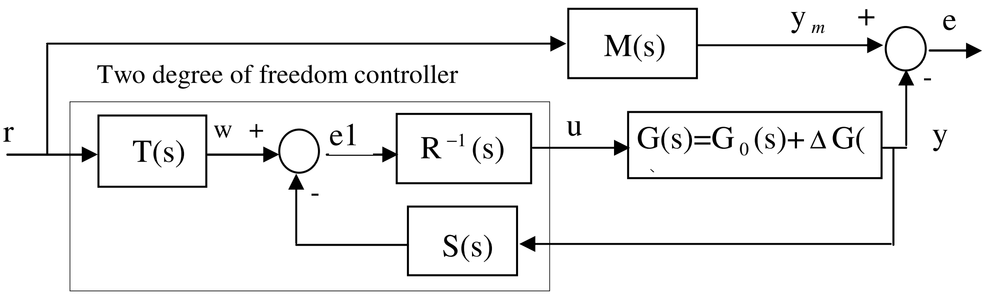

Fig. 3 The overall control system

4 Mathematical Model of Robotic Manipulators

In order to develop some adaptive control strategies for the robotic arms, it is necessary to obtain useful models of these plants. In this section a manipulator with three axes is analyzed and a multivariable model is developed.

We consider the robot manipulator with three axes described in Fig. 4, which is driven by a d.c. motor controlled in current. For this robot arm, which works in cylindrical coordinates, the kinetic energy is:

$$K = \frac{1}{2}\left(I_1 + I_2 + I_3 + m_3 q_3^2\right)\dot{q}_1^2 + \\ + \frac{1}{2}\left(m_2 + m_3\right)\dot{q}_2^2 + \frac{1}{2} m_3 \dot{q}_3^2 \quad (14)$$

The potential energy is:

$$P = \left(m_2 + m_3\right) g \quad (15)$$

Lagrange's equations of motion for a conservative system are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = \tau \quad (16)$$

where q is an n-vector of generalized coordinates q_i, τ is an n-vector of generalized forces τ_i, and the Lagrangian (L) is the difference between the kinetic (K) and potential (P) energies.

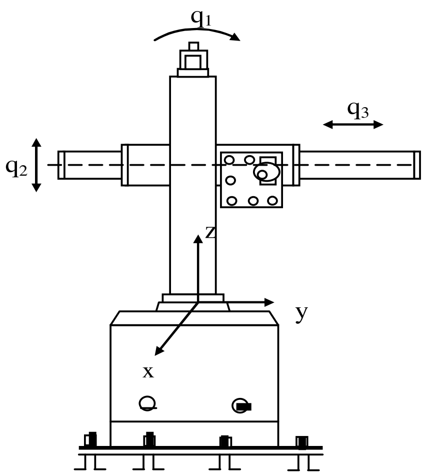

Fig. 4. Structure of a robotic manipulator

Now, we shall use Lagrange's equation to derive the general robot arm dynamics. The system is characterized by a set of three first order differential equations:

$$\begin{aligned} &\left(I_1 + I_2 + I_3 + m_3 q_3^2\right)\ddot{q}_1 + 2 m_3 q_3 \dot{q}_1 \dot{q}_3 = \tau_1 \\ &\left(m_2 + m_3\right)\ddot{q}_2 = \tau_2 - \left(m_2 + m_3\right) g \\ &m_3 \ddot{q}_3 - m_3 q_3 \dot{q}_1^2 = \tau_3 \end{aligned} \quad (17)$$

where I_1, I_2, I_3 *r*epresent the moments of inertia of the solids with respect to the axis z; m_2, m_3 are the solids' masses; τ_1, τ_2, τ_3 are the generalized forces.

(i). For the beginning we consider $q_2 = 0$ and we note $I = I_1 + I_2 + I_3$. The state equations are the following:

$$\dot{x} = f(x) + \sum_{i=1}^{2} g_i(x) u_i \quad (18)$$

where

$$f(x) = \begin{pmatrix} x_3 \\ x_4 \\ -\dfrac{2 m_3 x_2 x_3 x_4}{I + m_3 x_2^2} \\ x_2 x_3^2 \end{pmatrix}; \; g(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{I + m_3 x_2^2} & 0 \\ 0 & \dfrac{1}{m_3} \end{pmatrix} \quad (19)$$

$x^T = \left[q_1, q_3, \dot{q}_1, \dot{q}_3\right]$ and $u^T = \left[u_1, u_2\right] = \left[\tau_1, \tau_3\right]$

For the system (18), we consider as output variables the generalized coordinates q_1 and q_3:

$$\begin{aligned} y_1 &= h_1(x) = q_1(t) = x_1(t) \\ y_2 &= h_2(x) = q_3(t) = x_2(t) \end{aligned} \quad (20)$$

In this situation, the mathematical model is multivariable and it has two inputs and two outputs.

(ii). If $q_2 \neq 0$, the state equations are the following:

$$\begin{aligned} &\dot{x} = f(x) + \sum_{i=1}^{3} g_i(x) u_i \\ &y_i = h_i(x) = x_i, \; i = 1,2,3 \\ &x^T = \left[q_1, q_2, q_3, \dot{q}_1, \dot{q}_2, \dot{q}_3\right] \text{ and } u^T = \left[\tau_1, \tau_2, \tau_3\right] \end{aligned} \quad (21)$$

$$f(x) = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ \dfrac{-2 m_3 x_3 x_4 x_6}{I + m_3 x_3^2} \\ -g \\ x_3 x_4^2 \end{pmatrix} \quad (22)$$

$$g(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{1}{I + m_3 x_3^2} & 0 & 0 \\ 0 & \dfrac{1}{m_2 + m_3} & 0 \\ 0 & 0 & \dfrac{1}{m_3} \end{pmatrix}$$

5 A Working Example

The mathematical model in the multivariable case (i) is of the form (18), but where the inputs are the generalized coordinates τ_1 and τ_3. In this situation, we consider as output variables the generalized coordinates q_1 and q_3:

$$\begin{aligned} y_1 &= h_1(x) = x_1(t) \\ y_2 &= h_2(x) = x_2(t) \end{aligned} \tag{23}$$

For this system we have decoupling matrix

$$A(x) = \begin{bmatrix} \dfrac{1}{I + m_3 x_2^2} & 0 \\ 0 & \dfrac{1}{m_3} \end{bmatrix} \tag{24}$$

and the nonlinearities canceling vector is

$$b(x) = \begin{bmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{bmatrix} = \begin{bmatrix} -\dfrac{2m_3 x_2 x_3 x_4}{I + m_3 x_2^2} \\ x_2 x_3^2 \end{bmatrix} \tag{25}$$

Using relations (24) and (25), the input-output system can be written in the form:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = b(x) + A(x) \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{26}$$

An easy calculus shows that the matrix for mathematical model of the robot is nonsingular and the (vector) relative degree is $\{r_1, r_2\} = \{2, 2\}$. Because the decoupling matrix (24) is not singular, it is possible to design a nonlinear input:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A^{-1}(x) \cdot \begin{bmatrix} -L_f^2 h_1(x) + v_1 \\ -L_f^2 h_2(x) + v_2 \end{bmatrix} \tag{27}$$

such that the obtained linear system has the transfer matrix:

$$H(s) = \begin{bmatrix} \dfrac{1}{s^2} & 0 \\ 0 & \dfrac{1}{s^2} \end{bmatrix} \tag{28}$$

Imposing on the linear system an additional feedback of the form:

$$\begin{aligned} v_1 &= c_1^0 \cdot \left(q_{1ref} - x_1\right) - c_1^1 \cdot x_3 \\ v_2 &= c_2^0 \cdot (q_{3ref} - x_2) - c_2^1 \cdot x_4 \end{aligned} \tag{29}$$

then, the obtained system has a linear input-output behavior, described by the following diagonal transfer function matrix

$$H(s) = \begin{bmatrix} \dfrac{c_1^0}{s^2 + c_1^1 s + c_1^0} & 0 \\ 0 & \dfrac{c_2^0}{s^2 + c_2^1 s + c_2^0} \end{bmatrix} \tag{30}$$

In the multivariable case (ii), for the system (17), we consider as output variables the generalized coordinates q_1, q_2 and q_3:

$$\begin{aligned} y_1 &= h_1(x) = x_1(t) \\ y_2 &= h_2(x) = x_2(t) \\ y_3 &= h_3(x) = x_3(t) \end{aligned} \tag{31}$$

For this system we have decoupling matrix

$$A(x) = \begin{bmatrix} \dfrac{1}{I + m_3 x_3^2} & 0 & 0 \\ 0 & \dfrac{1}{m_2 + m_3} & 0 \\ 0 & 0 & \dfrac{1}{m_3} \end{bmatrix} \tag{32}$$

and the nonlinearities canceling vector:

$$b(x) = \begin{bmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \\ L_f^2 h_3(x) \end{bmatrix} = \begin{bmatrix} \dfrac{-2m_3}{I + m_3 x_3^2} x_3 x_4 x_6 \\ -g \\ x_3 x_4^2 \end{bmatrix} \tag{33}$$

Using relations (32) and (33), the input-output system can be written in the form:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix} = b(x) + A(x) \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{34}$$

An easy calculus shows that the matrix for the mathematical model of the robot is nonsingular and the (vector) relative degree is $\{r_1, r_2, r_3\} = \{2, 2, 2\}$. Because the decoupling matrix (32) is not singular, it is possible to design a nonlinear input:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = A^{-1}(x) \cdot \begin{bmatrix} -L_f^2 h_1(x) + v_1 \\ -L_f^2 h_2(x) + v_2 \\ -L_f^2 h_3(x) + v_3 \end{bmatrix} \tag{35}$$

such that the obtained linear system has the transfer matrix:

$$H(s) = \begin{bmatrix} \dfrac{1}{s^2} & 0 & 0 \\ 0 & \dfrac{1}{s^2} & 0 \\ 0 & 0 & \dfrac{1}{s^2} \end{bmatrix} \tag{36}$$

Imposing on the linear system an additional feedback of the form:

$$\begin{aligned} v_1 &= c_1^0 \cdot \left(q_{1ref} - x_1\right) - c_1^1 \cdot x_4 \\ v_2 &= c_2^0 \cdot (q_{2ref} - x_2) - c_2^1 \cdot x_5 \\ v_3 &= c_3^0 \cdot (q_{3ref} - x_3) - c_3^1 \cdot x_6 \end{aligned} \tag{37}$$

the obtained decoupled closed-loop system has a desired behavior. In the exact linearization case, the design parameters are computed using a pole-placement design technique. Then, applying Glover-McFarlane algorithm described in Section 3, the controller K_r was computed.

The implementation of the obtained nonlinear control laws is hampered if some of robot parameters are unknown or variable in time (slowly). Two simulation cases were considered in order to test the performances of the proposed nonlinear and robust controllers.

The simulation was done for the model equations (18), the nonlinear control law (27), (29). The performance of the controlled system is presented in Fig. 5–Fig.6.

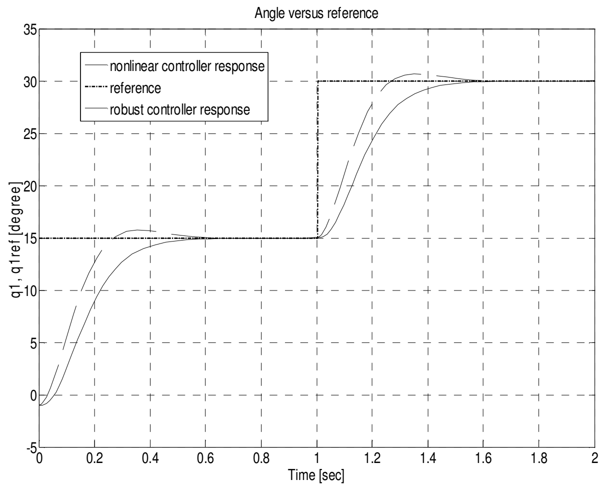

Fig. 5 Evolution of the arm angle

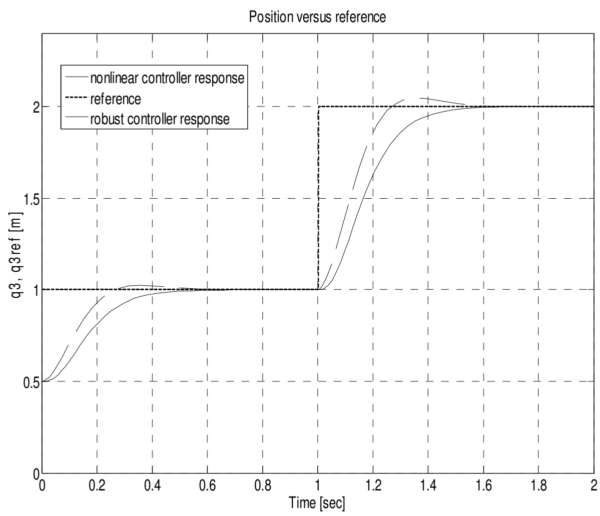

Fig. 6 Evolution of the arm position

These figures depict the behavior of the robot manipulator in this situation: nonlinear controller response (dashed line) versus robust controller response (dotted line).

6 Concluding Remarks

This paper deals with some control techniques for robotic manipulators. First, a nonlinear control technique based on the exact linearization via feedback is developed. This control law achieves also input-output decoupling of the multivariable robotic system. Second, a Glover-McFarlane controller design is applied in order to obtain some robust performances of the controlled system.

The numerical simulations performed for the robotic manipulator show a good behavior of the proposed control laws. However, the comparisons demonstrate that the robust controller ensures better setpoint tracking performance.

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