# **On Adaptive Control of an Anaerobic Digestion Bioprocess**

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*Abstract:* - This paper presents the design and the analysis of some adaptive nonlinear control strategies for a class of depollution fermentation processes that are carried out in a continuous stirred tank bioreactor. The controllers design is based on the input-output linearization technique. The resulted control methods are applied in depollution control problem in the case of the anaerobic digestion bioprocess for which dynamical kinetics are strongly nonlinear and not exactly known, and for which not all the state variables are measurable. It must be noted that for the controlling of this very complex bioprocess it is necessary to reduce his model order. This is realized by using the singular perturbation method. Numerical simulations are included to illustrate the performances of the proposed controllers.

Key-Words: - Nonlinear systems, Adaptive control, On-line estimation, Anaerobic digestion bioprocess.

# **1** Introduction

During the last years, the control of biotechnological processes has been an important problem attracting wide attention. The main engineering motivation in applying control methods to such living processes is to improve operational stability and production efficiency. But the use of modern control for these bioprocesses is still low. Two factors make biotechnological processes control particularly difficult. First, these processes exhibit large nonlinearities, strongly coupled variables and often poorly understood dynamics. Second, the real-time monitoring and on-line measurements of biological process variables, for example, biomass and/or product concentrations, which are essential for control design, is hampered by the lack of cheap and reliable on-line sensors. Due to the two above characteristic factors, bioprocesses constitute a natural field of application for adaptive techniques [1], [6], [7], [8]. So, the difficulties encountered in the measurement of the state variables of the bioprocesses impose the use of the so-called "software sensors" [1]. Note that these software sensors are used not only for the estimation of the concentrations but also for the estimation of the kinetic parameters [1], [6].

This paper presents the design and the analysis of some nonlinear and adaptive control strategies capable to deal with the model uncertainties in an adaptive way for a complex anaerobic digestion bioprocess used as a depollution process. The controllers are obtained via the input-output linearization technique [3], [10]. The only information required about the process are the measurements of the state variables and its relative degree. It must be noted that if for the analyzed process there are no accessible state variables, these will be estimated by using an appropriately state observer. Numerical simulations performed under identical circumstances are included to demonstrate the performances of the designed controllers.

The rest of this paper is organized as follows. Section 2 is devoted to description and modelling of an anaerobic digestion bioprocess. Some nonlinear and adaptive control strategies are proposed in Section 3. Simulations results presented in Section 4 illustrate the performances of the proposed control algorithms and, finally, Section 5 concludes the paper.

# 2 Process description and modelling

Anaerobic digestion is a biological wastewater treatment process which produces methane. Four metabolic paths [1], [2] can be identified in this process: two for acidogenesis and two for methanisation (see Fig. 1).

In the first acidogenic path, glucose (or another complex substrate) is decomposed into volatile fatty acids (acetate, propionate), hydrogen and inorganic carbon by acidogenic bacteria. In the second acidogenic path, OHPA – Obligate Hydrogen Producing Acetogens decomposes propionate into acetate, hydrogen and inorganic carbon. In the first methanisation path, acetate is transformed into methane and inorganic carbon by acetoclastic methanogenic bacteria, while in the second methanisation path, hydrogen combines inorganic carbon to produce methane under the action of hydrogenophilic methanogenic bacteria.



Fig.1. A schematic view of anaerobic digestion process

The process can be described by the following reaction network [1], [2]:

$$S_1 \xrightarrow{\eta} \leftarrow X_1 + S_2 + S_3 + S_4 + S_5 \tag{1}$$

$$S_2 \xrightarrow{\gamma_2} X_2 + S_5 + P \tag{2}$$

$$S_3 \xrightarrow{r_5} X_3 + S_2 + S_4 + S_5 \tag{3}$$

$$S_4 + S_5 \xrightarrow{\gamma_4} X_4 + P \tag{4}$$

where  $X_1$ ,  $X_2$ ,  $X_3$ ,  $X_4$  are respectively acidogenic bacteria, acetoclastic methanogenic bacteria, OHPA, and hydrogenophilic methanogenic bacteria,  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ ,  $S_5$  are respectively glucose, acetate, propionate, hydrogen, inorganic carbon, and P is methane;  $r_1$  and  $r_2$  are respectively the rates of the first acidogenic reaction and methanisation reaction and  $r_3$  and  $r_4$  are respectively the rates of the second acidogenic reaction rate is a growth rate, and each growth reaction rate is a different microorganism population and may be written as a product of a specific growth rate by concentration of the biomass involved in the reaction [1], [2]:

$$r_i = \mu_i X_i , \ i = \overline{1, 4} \tag{5}$$

where  $\mu_i$ , *i* = 1,4 are the specific growth rates of reactions (1)-(4).

Let us consider that the anaerobic digestion process is operated in a Continuous Stirred Tank Reactor, and the only inlet substrate is the organic matter  $S_1$ . The dynamics of the anaerobic digestion process are described by the following equations:

$$\dot{X}_1 = r_1 - DX_1$$
 (6)

$$\dot{S}_1 = -k_{21}r_1 - DS_1 + DS_{in} \tag{7}$$

$$X_2 = r_2 - DX_2 \tag{8}$$

$$\dot{S}_2 = k_{41}r_1 - k_{42}r_2 + k_{43}r_3 - DS_2 \tag{9}$$

$$\dot{X}_3 = r_3 - DX_3$$
 (10)

$$\dot{S}_3 = k_{61}r_1 - k_{63}r_3 - DS_3 \tag{11}$$

$$\dot{X}_4 = r_4 - DX_4 \tag{12}$$

$$\dot{S}_4 = k_{81}r_1 + k_{83}r_3 - k_{84}r_4 - DS_4 - Q_{H_2}$$
(13)

$$\dot{S}_{5} = k_{01}r_{1} + k_{02}r_{2} + k_{02}r_{2} - k_{04}r_{4} - DS_{5} - O_{CO}$$
 (14)

$$\dot{P} = k_{02}r_2 + k_{04}r_4 - DP - O_P \tag{15}$$

where  $Q_P$ ,  $Q_{CO_2}$  and  $Q_{H_2}$  represent respectively gaseous outflow rates of  $CH_4$ ,  $CO_2$  and  $H_2$ ,  $S_{in}$  is the influent substrate concentration,  $k_{ij}$  (i = 0, 1, ..., 9; j = 1, 2, 3, 4) are the yield coefficients, and D is the dilution rate.

Since the anaerobic digestion is a very complex bioprocess, his dynamical model being described by ten differential equations, from an engineering point of view to control this bioprocess can be used an appropriately *reduced order model*. One possible systematic approach to achieve model simplification is to use the singular perturbation method, which is a technique that allows to transform a set of n+mdifferential equation into a set *n* differential equations and a set of *m* algebraic equations [2],[6].

To reduce the model order of bioprocesses it can be applied the following rule [2]: If in process there are some products  $\xi_j$  with low solubility, then in the dynamics corresponding to  $\xi_j$  given by

$$\dot{\xi}_j = K_j r(\xi) - D\xi_j + F_j - Q_j \tag{16}$$

where  $K_j$  is the line of *K* corresponding to  $\xi_j$ , the simplification is achieved by setting  $\xi_i$  and  $\xi_i$  to zero obtaining thus the following algebraic equation:

$$K_{j}\varphi(\xi) = Q_{j} - F_{j} \tag{17}$$

This rule is also applied for substrates involved in fast (slow) reactions [2].

It is known that for this bioprocess, the decomposition of propionate by OHPA and the composition of hydrogen and inorganic carbon are two reactions which may be characterized by fast dynamics. Then, using the above rule, the differential equations in (11) and (13) derive the following algebraic equations:

$$0 = k_{61}r_1 - k_{63}r_3; 0 = k_{81}r_1 + k_{83}r_3 - k_{84}r_4 - Q_{H_2}$$
(18)

If in (19) we neglect the outflow rate  $Q_{H_2}$  then the reaction rates  $r_1$  and  $r_3$  can be expressed as:

$$r_{3} = \frac{k_{61}}{k_{63}}r_{1}; r_{4} = \frac{k_{81}}{k_{84}}r_{1} + \frac{k_{83}}{k_{84}}r_{3} = \frac{k_{63}k_{81} + k_{61}k_{83}}{k_{63}k_{84}}r_{1} (19)$$

Using the above approximations, the anaerobic

digestion process can be described by the following reduced order dynamical model:

$$\dot{X}_1 = r_1 - DX_1 \tag{20}$$

$$\dot{S}_1 = -k_1 r_1 - DS_1 + DS_{in} \tag{21}$$

$$\dot{X}_2 = r_2 - DX_2 \tag{22}$$

$$\dot{S}_2 = k_3 r_1 - k_2 r_2 - DS_2 \tag{23}$$

$$\dot{S}_5 = k_4 r_1 + k_5 r_2 - DS_5 - Q_{CO_2} \tag{24}$$

$$\dot{P} = k_6 r_2 + k_7 r_1 - DP - Q_P \tag{25}$$

where:

$$\begin{split} k_1 &= k_{21} \, ; \ \ k_2 = k_{42} \, ; \ \ k_3 = k_{41} + \frac{k_{61}k_{43}}{k_{63}} \, ; \ \ k_5 = k_{92} \, ; \\ k_6 &= k_{02} \, ; \ \ k_4 = k_{91} + \frac{k_{61}k_{93}}{k_{63}} - k_{94} \frac{k_{63}k_{81} + k_{61}k_{83}}{k_{63}k_{84}} \, ; \\ k_7 &= k_{04} \frac{k_{63}k_{81} + k_{61}k_{83}}{k_{63}k_{84}} \, . \end{split}$$

## **3** Control Strategies

For the class of anaerobic bioprocesses described by dynamical model (16) we consider the problem of controlling the output pollution level by using an appropriately control input under the following assumptions: (i) the control input is the dilution rate; (ii) the reaction rates  $r_i$  are incompletely unknown; (iii) the matrix K is known; (iv) the vectors F and Q are known either by measurements or by user's choice.

To design the controllers we will use the simplified version of dynamical model given by (21)-(26) rewritten as:

$$\dot{\xi} = Kr(\xi) - D\xi + F - Q = KG(\xi)\alpha(\xi) - D\xi + F - Q$$
(26)

where  $r(\xi) = G(\xi)\alpha(\xi)$ ,  $\xi = [X_1 S_1 X_2 S_2 S_5 P]^T$ is the state vector,  $F = [0 \ DS_{in} \ 0 \ 0 \ 0 \ 0]^T$  is the vector of feed rates,  $Q = [0 \ 0 \ 0 \ 0 \ Q_{CO_2} Q_P]^T$  is the vector of gaseous outflow rates,  $r = [r_1 \ r_2]^T$  is the vector of reaction rates,  $\alpha = [\alpha_1 \ \alpha_2]^T$  is the vector of specific reaction rates, and

$$K = \begin{bmatrix} 1 & -k_1 & 0 & k_3 & k_4 & k_7 \\ 0 & 0 & 1 & -k_2 & k_5 & k_6 \end{bmatrix}^T, \ G = \begin{bmatrix} X_1 S_1 & 0 \\ 0 & X_2 S_2 \end{bmatrix}.$$

As controlled variable let us assume the output pollution level *y* defined as:

$$y = c_1 S_1 + c_2 S_2 \tag{27}$$

where  $c_1$  and  $c_2$  are known conversion constants, and as input control we chose the dilution rate, u = D. From (27) and (26) we obtain the following input-output process model whose relative degree is equal to 1:

$$\dot{y} = (c_2k_3 - c_1k_1)X_1S_1\alpha_1 - c_2k_2X_2S_2\alpha_2 - Dy + c_1u$$
(28)

For the anaerobic digestion process, the main *control objective* is to make the output pollution level y to maintain and to track a specified low level pollution denoted  $y^* \in \Re$  despite load variations and substrate concentration variations.

#### 3.1 Exactly feedback linearizing control

Consider the ideal case where maximum prior knowledge concerning the process is available, that is in (26) the specific reaction rates  $\alpha_1$  and  $\alpha_2$  are assumed completely known, while all the state variables and all the inflow and outflow rates are available for on-line measurements. Then the following control law known as the exactly feedback linearizing control law:

$$u = \frac{1}{c_1 S_{in} - y} (\dot{y}^* + \lambda_1 (y^* - y) - (c_2 k_3 - c_1 k_1) X_1 S_1 \alpha_1 + c_2 k_2 X_2 S_2 \alpha_2)$$
(29)

determines a dynamical behaviour of closed-loop system described by the following fist order linear stable differential equation:

$$\frac{d}{dt}(y^* - y) + \lambda_1(y^* - y) = 0, \ \lambda_1 > 0$$
(30)

Since the performances of the closed-loop system by using the exactly control law (29) are the best, this case will be used as benchmark to compare with other situations obtained when the process model is incompletely known or there are state variables which are not measured.

The control law (29) leads to a linear error model  $\dot{e} = -\Lambda e$ , where  $e = y^* - y$  represents the tracking error. It is clear that for  $\lambda_1 > 0$ , the error model has an exponential stable point at e = 0.

#### **3.2** Adaptive control strategies

Since the prior knowledge concerning the process assumed in the previous subsection is not realistic, in this subsection we analyze some more realistic cases, where the process dynamics are incompletely known and time varying and some state variables are not accessible.

Let's assume that the only measurements available on-line are the output pollution level y, the acetate concentration  $S_2$  and methane gas outflow rate  $Q_{P}$ . Assume also that the specific reaction rates  $\alpha_1$  and  $\alpha_2$  are completely unknown. Since the control law (29) containes the state variables  $X_1, X_2, S_1$  and  $S_2$  from which only  $S_2$  can be on-line measured, one concludes that all other unmeasured variables must be estimated (even in the situation when the specific reaction rates  $\alpha_1$  and  $\alpha_2$  are completely unknown). For this, a solution is the using of an appropriately asymptotic state observer [4] that can be designed as follows.

Since in (26) rang(K) = 2, let's consider the following state partition:

$$\xi_a = [X_1 X_2]^T, \ \xi_b = [S_1 S_2 S_5 P]^T$$
(31)

wich induces on K, F and Q from (26) the following partitions:

$$K_{a} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \qquad F_{a} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \qquad Q_{a} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$K_{b} = \begin{bmatrix} -k_{1} & 0 \\ k_{3} & -k_{2} \\ k_{4} & k_{5} \\ k_{7} & k_{6} \end{bmatrix}; \qquad F_{b} = \begin{bmatrix} DS_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix}; \qquad Q_{b} = \begin{bmatrix} 0 \\ 0 \\ Q_{CO_{2}} \\ Q_{P} \end{bmatrix}$$

Let's define an auxiliary vector z as:

$$z = C\xi_a + \xi_b \tag{32}$$

where *C* is the uniquely solution of the equation  $CK_a + K_b = 0$ . We obtain:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = -\begin{bmatrix} -k_1 & 0 \\ k_3 & -k_2 \\ k_4 & k_5 \\ k_7 & k_6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} S_1 \\ S_2 \\ S_5 \\ P \end{bmatrix}$$
(33)

Assume that for our popose the  $CO_2$  concentration does not interesting; so that from the four variables, it must to be estimated only three state variables. Then from (34) we shall retain only three auxiliary variables  $z_1$ ,  $z_2$  and  $z_4$  given by:

$$z_{1} = k_{1}X_{1} + S_{1}; z_{2} = -k_{3}X_{1} + k_{2}X_{2} + S_{2}$$
  
$$z_{4} = -k_{7}X_{1} - k_{6}X_{2}$$
 (34)

Now we expres this reduced vector z in term of measurable  $\zeta_1$  and unmeasurable  $\zeta_2$  states as:

$$z = C_1 \zeta_1 + C_2 \zeta_2 \tag{35}$$

where  $\zeta_1 = S_2$  and  $\zeta_2 = [X_1 S_1 X_2]^T$ , and  $C_1$  and  $C_2$  are matrices with appropriately dimensions. From (33) and (35) one obtains:

$$C_{1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \quad C_{2} = \begin{bmatrix} k_{1} & 1 & 0 \\ -k_{3} & 0 & k_{2} \\ -k_{7} & 0 & -k_{6} \end{bmatrix}$$
(36)

Since  $det(C_2) = -(k_2k_7 + k_3k_6)$  that yields  $rang(C_2) = 3$  and  $C_2$  is a quadratic matrix, then the

unmeasurable state  $\zeta_2$  is given by  $\zeta_2 = C_2^{-1}(z - C_1\zeta_1)$ . To estimate these variables we use an asymptotic sate observer [6], described as:

$$\dot{\hat{z}} = -D\hat{z} + C(F_a - Q_a) + (F_b - Q_b)$$
(37)

$$\hat{\zeta}_2 = C_2^{-1}(\hat{z} - C_1\zeta_1) \tag{38}$$

whose equations here are particularized as:

$$\frac{d}{dt} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_4 \end{bmatrix} = -D \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_4 \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ -k_3 & k_2 \\ -k_7 & -k_6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} DS_{in} \\ 0 \\ -Q_P \end{bmatrix} (39)$$
$$\begin{bmatrix} \hat{X}_1 \\ \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} = (k_3k_6 + k_2k_7)^{-1} \begin{bmatrix} 0 & -k_6 & -k_2 \\ k_3k_6 + k_2k_7 & k_1k_6 & k_1k_2 \\ 0 & k_7 & -k_3 \end{bmatrix} \times \\ \times \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 & -S_2 \\ \hat{z}_4 \end{bmatrix} \qquad (40)$$

where  $S_{in}$ ,  $Q_P$  and  $S_2$  are on-line measurable and D is known.

Since the output y is on-line measurable, from (28) the variable  $S_1$  will be:

$$S_1 = (y - c_2 S_2) / c_1 \tag{41}$$

and thus  $S_1$  does not be estimated. As a conclusion, by using the asymptotic state observer (40), (41) it must be estimated only the variables  $X_1$  and  $X_2$ . From (40), (41) a first version of state observer is given by:

$$\dot{\hat{z}}_2 = -D\hat{z}_2; \quad \dot{\hat{z}}_4 = -D\hat{z}_4 - Q_P$$
(42)

$$X_{1} = (k_{3}k_{6} + k_{2}k_{7})^{-1}(k_{6}(S_{2} - \hat{z}_{2}) - k_{2}\hat{z}_{4})$$
(43)

$$\hat{X}_2 = (k_3k_6 + k_2k_7)^{-1} (k_7(\hat{z}_2 - S_2) - k_3\hat{z}_4)$$
(44)

Using the expression of  $z_1$  from (34), and his dynamics from (39), a second version of state observer is given by:

$$\dot{\hat{z}}_1 = -D\hat{z}_1 + DS_{in}; \quad \dot{\hat{z}}_4 = -D\hat{z}_4 - Q_P \tag{45}$$

$$\hat{X}_1 = \frac{1}{k_1} \left( \hat{z}_1 - \frac{y - c_2 S_2}{c_1} \right)$$
(46)

$$\hat{X}_2 = (k_3 k_6 + k_2 k_7)^{-1} (k_7 (\hat{z}_2 - S_2) - k_3 \hat{z}_4)$$
(47)

To obtain the on-line estimates  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  of the unknown specific rates  $\alpha_1$  and  $\alpha_2$  we will use a linear regressive parameter estimator [4] given by:

$$\dot{\Psi}^T = -\omega \Psi^T + KG(\xi) \tag{48}$$

$$\dot{\Psi}_0 = -\omega\Psi_0 + (\omega - D)\xi + F \tag{49}$$

$$\dot{\hat{\alpha}} = \Gamma \Psi \left( \xi - \Psi_0 - \Psi^T \hat{\rho} \right)$$
(50)

$$\dot{\Gamma} = -\Gamma \Psi \Psi^T \Gamma + \lambda \Gamma , \quad \Gamma(0) > 0 \tag{51}$$

where  $\Psi^T$  is the regressor matrix,  $\Gamma$  is a diagonal gain matrix of the updating law (50), and  $\omega > 0$  and  $\lambda$  (forgetting factor) are design parameters at the user's disposal to control the stability and the tracking properties of the estimator [4], [6], [9]. This estimator will be applied to a submodel of model (26), which contains the unknown rates  $\alpha_1$  and  $\alpha_2$ . So, taking in the acount only the dynamics of the substrates  $S_1$  and  $S_2$ , the algorithm (48)-(51) with the regressor matrix chosen of the form:

$$\Psi^{T} = \begin{bmatrix} -k_{1} & 0 \\ k_{3} & -k_{2} \end{bmatrix} \begin{bmatrix} \psi_{1} & 0 \\ 0 & \psi_{2} \end{bmatrix} = \begin{bmatrix} -k_{1}\psi_{1} & 0 \\ k_{3}\psi_{1} & -k_{2}\psi_{2} \end{bmatrix} \text{ and}$$

 $\Psi_0 = \begin{bmatrix} \Psi_{01} \\ \Psi_{02} \end{bmatrix}$ , takes the following particularly form:

$$\dot{\psi}_1 = -\omega\psi_1 + \hat{X}_1 S_1; \ \dot{\psi}_2 = -\omega\psi_2 + \hat{X}_2 S_2$$
 (52)

$$\psi_{01} = -\omega\psi_{01} + (\omega - D)S_1 + F_1$$
(53)  

$$\dot{\psi}_{02} = -\omega\psi_{02} + (\omega - D)S_2$$
(54)

$$\frac{d}{dt} \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix} = \Gamma \cdot \begin{bmatrix} -k_1 \psi_1 & k_3 \psi_1 \\ 0 & -k_2 \psi_2 \end{bmatrix} \times \\ \times \begin{bmatrix} S_1 - \psi_{01} + k_1 \psi_1 \hat{\alpha}_1 \\ S_2 - \psi_{02} - k_3 \psi_1 \hat{\alpha}_1 + k_2 \psi_2 \hat{\alpha}_2 \end{bmatrix}$$
(55)

$$\dot{\Gamma} = -\Gamma \Psi \Psi^T \Gamma + \lambda \Gamma , \ \Gamma(0) > 0$$
(56)

Finally, the full adaptive linearizing algorithm for controlling the fermentation bioprocess (6)-(15) is made up by combination of the equations (41), (42)-(44) or (45)-(47) and (52)-(56) with the control law (29) rewritten as:

$$u = \frac{1}{c_1 S_{in} - y} \left( \dot{y}^* + \lambda_1 (y^* - y) - (c_2 k_3 - c_1 k_1) \hat{X}_1 S_1 \hat{\alpha}_1 + c_2 k_2 \hat{X}_2 S_2 \hat{\alpha}_2 \right)$$
(57)

and is schematized in Fig. 2.

## **4** Simulation Results

The performances of the desiged controllers have been tested through extensive simulation experiments by using the process model (6)-(15) under realistic conditions. The values of yield coefficients used in simulations are [6]:  $k_{21} = 3.2$ ,  $k_{41}$ = 0.77,  $k_{42} = 16.7$ ,  $k_{43} = 0.53$ ,  $k_{61} = 0.75$ ,  $k_{63} = 1.5$ ,  $k_{81} = 0.6$ ,  $k_{83} = 0.1$ ,  $k_{84} = 1.15$ ,  $k_{91} = 1.15$ ,  $k_{92} = 1.5$ ,  $k_{93} = 0.2$ ,  $k_{94} = 0.1$ ,  $k_{02} = 3.0$ ,  $k_{04} = 0.2$ .

The reaction rates  $r_1$  and  $r_3$  are described as:

 $r_i = \mu_i X_i = \alpha_i X_i S_i$ , i = 1,3, where  $\mu_i = \mu_i^* S_i / (K_{M_i} + S_i)$  - model Monod, with:  $\mu_1^* = 0.2 h^{-1}$ ,  $K_{M_1} = 0.5 g/l$  and  $\mu_3^* = 0.5 h^{-1}$ ,  $K_{M_5} = 0.4 g/l$ . The reaction rates  $r_2$  and  $r_4$  are described as:

$$r_k = \mu_k X_k = \alpha_k X_k S_k$$
 where



Fig.2. The structure of the adaptive system

 $\mu_{k} = \mu_{k}^{*}S_{k} / (K_{M_{k}} + S_{k} + S_{k}^{2} / K_{I_{k}}) - \text{model Haldane,}$ with:  $\mu_{2}^{*} = 0.35 h^{-1}$ ,  $K_{M_{2}} = 4 g/l$ ,  $K_{I_{2}} = 21 g/l$  and  $\mu_{4}^{*} = 0.5 h^{-1}$ ,  $K_{M_{4}} = 4 mM$ ,  $K_{I_{4}} = 3 mM$ . The value of yield coefficients of the reduced

The value of yield coefficients of the reduced model (16) are [6]:  $k_1 = 3.2$ ,  $k_2 = 16.7$ ,  $k_3 = 1.035$ ,  $k_4 = 1.1935$ ,  $k_5 = 1.5$ ,  $k_6 = 3$ ,  $k_7 = 0.113$ , and the values of the conversion coefficients  $c_1$  and  $c_2$  in (28) are:  $c_1 = 1.2$ ,  $c_2 = 0.75$ .

The system's behaviour was analyzed assuming that the pollutant concentration  $S_{in}$  acts as a perturbation of the form presented in Fig. 3

The behaviour of closed-loop system using the adaptive controller (57) by comparison to the exactly linearizing law (29) is presented in Fig. 3, where the evolutions of output pollution level y and of control input D are shown. Fig. 4 shows the evolution of some estimated states variables and of some specific reaction rates. Note that in these figures the graphics indexed by b correspond to the benchmark case. To verify the regulation properties of the controller for the reference variable, a piecewise constant variation was considered as:  $y^* = 1.5$  for 0 < t < 35h;  $y^* = 0.75$  for  $35 \le t < 70h$ ;  $y^* = 0.45$  for  $70 \le t < 100h$ .

The system evolves in open loop from the time 0 to  $t_1 = 2 h$ , after which the system is closed by using the control law (29) respectively (57).

For a proper comparison of the two control strategies, the simulations were carried out under identical conditions and the results were judged using the same set of criteria. The value of the gain parameter  $\lambda_1$  in (29) and (57) is  $\lambda_1 = 1.5$ .

From graphics in Figs. 4a-4d it can be seen that the behaviour of adaptive system, even if this used much less priority information, is good, being very close to the behaviour of closed loop system in the ideal case when the process model is completely known. Note also the regulation properties and ability of the controller to maintain the controlled output y close to his desired values (very low level for  $y^*$ ) despite the process uncertainties.



Fig.3. The behaviour of nonlinear adaptive system by comparison to behaviour of benchmark system



Fig.4. The evolution of some estimated states variables and of some specific reaction rates

One can observe a good behaviour both of state observer (42)-(44) or (45)-(47) and parameter estimator (52)-(56).

## 5 Conclusions

Some nonlinear and adaptive control strategies for controlling the pollution level for a class of nonlinear plants with incompletely known dynamics were presented and compared.

The performances of the proposed controllers were analyzed by simulations conducted in the case of an anaerobic fermentation process. It can be concluded that in most practical situations, when the process nonlinearities are not completely known and/or the process dynamics are time varying, the adaptive controllers are viable alternatives.

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