Wastewater Biodegradation Process Identification; a Multi Layer Approach via Distributions

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Abstract: - The paper extends the procedures of wastewater biodegradation process (WBP) identification, as have been first presented by the authors in [11], [12]. These procedures allow identification of all process parameters in both cases they are time constant or time variant. The identification problem is formulated as a condition to vanish the existence relation of the system. This relation is represented by functionals using techniques from distribution theory based on testing function from a finite dimensional fundamental space. As the WBP expresses rational dependences between parameters and some measurable variables, the main idea of these procedures is to use a hierarchical multi layer structure of identification, which allows obtaining string of linear algebraic systems of equations in the unknown parameters. The coefficients of these algebraic systems are functionals depending on the input and output variables evaluated through some testing functions from distribution theory. According to the proposed procedure, in the firs layer, only some state equations are evaluated throughout testing functions to obtain a set of linear equations in some parameters. The results of this first layer of identification are utilized for expressing other parameters by linear equations in the next layer. This process is repeated until all parameters are identified. The time variant laws are expressed as finite degree time polynomials whose parameters are included in the set of parameters to be identified. Applications for parameter identification of waste water biodegradation processes are presented. By examples, the potential of the method is revealed.

Key-Words: - Identification; Bioprocesses, Wastewater biodegradation, Distribution theory; Functionals.

1 Introduction

As presented in [11], progresses have been made in the area of continuous-time system identification. Many discussions, methods and results on continuous-time identification are presented in, [2]; [4], [8], [9]; [14], [15], [16].

A novel approach for continuous-time system identification is that based on distribution theory, using deterministic distributions [10] or random distributions [13]. Identification of the non-linear continuous-time systems is far away more complicated. The traditional procedures are based on the Volterra functional series, expressed in time domain [3] or frequency domain [6]. The parameter identification of deterministic nonlinear continuous-time systems (NCTS), modelled by polynomial type differential equation, has been considered by numerous authors, [15], [16]. In [11], it is presented a method for identification of nonlinear continuous time systems (NCTS) considering that the unknown parameters can appear in rational relations with measured variables. Using techniques utilized in distribution approach [7], [8], [9], the measurable functions and their derivatives are represented by functionals on a fundamental space of testing functions. Such systems are common in biotechnology [1], [17], [18].

The main idea from [11] is to use a hierarchical multi layer structure of identification. First, some state equations are utilized to obtain a set of linear equations in some parameters. The results of this first stage of identification are utilized for expressing other parameters by linear equations. This process is repeated until all parameters are identified. The above idea of hierarchical multi layer structure identification has been extended to time variant systems [12]. Variable parameters are modelled by finite degree time polynomials whose unknown coefficients are included in the set of parameters to be identified. To transform a differential time variant system of equations to an algebraic system of functionals, the so-called weighted distributions are considered. Weighted distributions are nothing else rather the product between time functions and distributions.

The paper is organized as follows: The mathematical model of wastewater biodegradation process is given in Section 2. Section 3, presents some aspects regarding distribution approach of identification. The hierarchical structure of identification and estimation equations takes the space of Section 4. Some experimental results are presented in Section 5, and conclusions in Section 6.

2 Mathematical model of wastewater biodegradation process

We consider a biomethanation process - wastewater biodegradation with production of methane gas that takes place inside a Continuous Stirred Tank Bioreactor whose reduced model is presented in [18]. It is a two phases process. In the first phase, the glucose from the wastewater is decomposed in fat volatile acids (acetates, propionic acid), hydrogen and inorganic carbon under action of the acidogenic bacteria. In the second phase, the ionised hydrogen decomposes the propionic acid CH3CH2COOH in acetates, H2 and carbon dioxide CO2. In the first methanogenic phase, the acetate is transformed into methane and CO2, and finally in the second methanogenic phase, the methane gas CH4 is obtained from H2 and CO2, [1], [17]. The following simplified reaction scheme is considered,

$$\begin{cases} S_1 \xrightarrow{\phi_1} X_1 + S_2 \\ S_2 \xrightarrow{\phi_2} X_2 + P_1 \end{cases} \tag{1}$$

where: S_1 represents the glucose substrate, S_2 the acetate substrate, X_1 is the acidogenic bacteria, X_2 the acetoclastic methanogenic bacteria and P_1 represents the product, i.e. the methane gas. The reaction rates are denoted by ϕ_1, ϕ_2.

The corresponding dynamical model is

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ S_1 \\ X_2 \\ S_2 \\ P_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -k_1 & 0 \\ 0 & 1 \\ k_2 & -k_3 \\ 0 & k_4 \end{bmatrix}\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} - D\begin{bmatrix} X_1 \\ S_1 \\ X_2 \\ S_2 \\ P_1 \end{bmatrix} + \begin{bmatrix} 0 \\ DS_{in} \\ 0 \\ 0 \\ -Q_1 \end{bmatrix} \tag{2}$$

where the state vector of the model is

$$\xi = [X_1 \ S_1 \ X_2 \ S_1 \ P_1]^T = [\xi_1 \ \xi_2 \ \xi_3 \ \xi_4 \ \xi_5]^T \tag{3}$$

whose components are concentrations in (g/l).

The reaction rates are nonlinear functions of the state components, expressed as

$$\phi = \phi(\xi) = [\phi_1(\xi) \ \phi_2(\xi)]^T . \tag{4}$$

The vector of feed rates and of rates of removal of components is denoted

$$F = [0 \ \ D \cdot S_{in} \ \ 0 \ \ 0 \ \ -Q_1]^T \tag{5}$$

where, D is the dilution rate, a scalar in this particular case, S_{in} represents the concentration of the externally influent substrate–glucose, Q_1 is the methane gas outflow rate.

The dynamical model (2) can be compactly written

$$d\xi / dt = K \cdot \phi(\xi) - D \cdot \xi + F . \tag{6}$$

In fact, this model describes the behavior of an entire class of biotechnological processes. It referees as the general dynamical state-space model of this class of bioprocesses [1]. In (6), K is the so-called matrix of the yield coefficients k_{ij}

$$K = \begin{bmatrix} 1 & -k_1 & 0 & k_2 & 0 \\ 0 & 0 & 1 & -k_3 & k_4 \end{bmatrix}^T \tag{7}$$

The reaction rates for this process are given by the Monod law

$$\phi_1(\xi) = \mu_1 \cdot \frac{S_1 \cdot X_1}{K_{M_1} + S_1}, \tag{8}$$

and the Haldane kinetic model

$$\phi_2(\xi) = \mu_2 \cdot \frac{S_2 \cdot X_2}{K_{M_2} + S_2 + S_2^2 / K_i}, \tag{9}$$

where K_{M_1}, K_{M_2} are Michaelis-Menten constants; μ_1, μ_2 represent specific growth rates coefficients and K_i is the inhibition constant. For simplicity, shall we denote the plant parameters by the vector

$$\theta = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5 \ \theta_6 \ \theta_7 \ \theta_8 \ \theta_9]^T \tag{10}$$

where

$$\theta_1 = k_1 ; \theta_2 = k_2 ; \theta_3 = k_3 ; \theta_4 = k_4 \tag{11}$$

$$\theta_5 = \mu_1 ; \theta_6 = \mu_2 \tag{12}$$

$$\theta_7 = K_{M_1} ; \ \theta_8 = K_{M_2} ; \ \theta_9 = K_i \tag{13}$$

Because the dilution rate D can be externally modified, it will be considered the third component of the input vector $u = [u_1 \ u_2 \ u_3]^T$. The other two components of u are the concentration S_{in} and the methane gas outflow rate Q_1 so,

$$u_1 = S_{in} ; \ u_2 = Q_1 ; \ u_3 = D; \tag{14}$$

Usually Q_1 depends on state variables, $Q_1 = \Psi(\xi)$, determining a feedback to the input u_2. Written explicitly by components, the state equations (2) or (6), within the above notations, takes the form,

$$\dot{\xi}_1 = \phi_1 - u_3 \cdot \xi_1 \tag{15}$$

$$\phi_1 = \theta_5 \cdot \frac{\xi_1 \cdot \xi_2}{\theta_7 + \xi_2} \tag{16}$$

$$\dot{\xi}_2 = -\theta_1 \cdot \phi_1 - u_3 \cdot \xi_2 + u_1 \cdot u_3 \tag{17}$$

$$\dot{\xi}_3 = \phi_2 - u_3 \cdot \xi_3 \tag{18}$$

$$\phi_2 = \theta_6 \cdot \frac{\xi_3 \cdot \xi_4}{\theta_8 + \xi_4 + \theta_9' \cdot \xi_4^2} \ , \ \theta_9' = \frac{1}{\theta_9} \tag{19}$$

$$\dot{\xi}_4 = \theta_2 \cdot \phi_1 - \theta_3 \cdot \phi_2 - u_3 \cdot \xi_4 \tag{20}$$

$$\dot{\xi}_5 = -u_3 \cdot \xi_5 + \theta_4 \cdot \phi_2 - u_2 \tag{21}$$

3 Distribution approach of identification

Let us denote by Φ_n the fundamental space from distribution theory [5], of the real fundamental functions,

$$\varphi : \mathbb{R} \to \mathbb{R}, t \to \varphi(t), \quad (22)$$

with compact support T, having continuous derivatives at least up to the order n. A distribution is a linear, continuous (in the above topology) real functional on Φ_n, $F : \Phi_n \to \mathbb{R}, \varphi \to F(\varphi) \in \mathbb{R}$. Let

$$q : \mathbb{R} \to \mathbb{R}, t \to q(t) \quad (23)$$

be a function which admits a Riemann integral on any compact interval T from $\mathbb{R}$. Using this function, a unique distribution $F_q : \Phi_n \to \mathbb{R}, \varphi \to F_q(\varphi) \in \mathbb{R}$ can be build by the relation $F_q(\varphi) = \int_{\mathbb{R}} q(t) \cdot \varphi(t) \cdot dt, \forall \varphi \in \Phi_n$.

In distribution theory, the notion of distribution k-order derivative, $k = 0 : n$,[5], is,

$$F_q^{(k)}(\varphi) = (-1)^k \cdot F_q(\varphi^{(k)}), \forall \varphi \in \Phi_n$$
$$\varphi \to F_q^{(k)}(\varphi) = (-1)^k \cdot \int_{\mathbb{R}} q(t) \cdot \varphi^{(k)}(t) \cdot dt \in \mathbb{R} \quad (24)$$

Let now consider a dynamical continuous time system with n_u inputs, $u : \mathbb{R} \to \mathbb{R}^{n_u}, t \to u(t)$, $u \in \Omega$ and n_y outputs, $y : \mathbb{R} \to \mathbb{R}^{n_y}, t \to y(t)$, $y \in \Gamma$, where Ω represents the set of admissible inputs and Γ is the set of possible outputs. It can be expressed by a differential operator,

$$q_{\theta/(u,y)} = Q(u, y, \theta) = 0 \quad (25)$$

whose expression depends on a vector of parameters $\theta = [\theta_1 \ldots \theta_i \ldots \theta_p]^T$. The operator (25), whose class can be determined, represents a family of models with a given structure in constant parameters. A special case is the model (25) expressing a linear relation in the parameters

$$q_{\theta/(u,y)} = Q(u, y, \theta) = \sum_{i=1}^{p} w_i \cdot \theta - v = w^T \cdot \theta - v, \quad (26)$$

where w_i and v represent a sum of the derivatives of some known, possible nonlinear, functions ψ_i^j, ψ_0^j, with respect to the input and output variables,

$$w_i = \sum_{j=1}^{p_i} [\psi_i^j(u, y)]^{(n_i^j)}, i = 1 : p, \; v = \sum_{j=1}^{p_0} [\psi_0^j(u, y)]^{(n_0^j)} \quad (27)$$

Parameters p_i, n_i^j, p_0, n_0^j are given integer numbers. The identification problem, into condition (26), has a unique solution. An identification problem means to determine the parameter $\theta = \widehat{\theta}$, given the priori information on the model structure Q, (25), and the observed input-output pair (u_T, y_T), $\widehat{\theta} = \widehat{\theta}(u_T, y_T, Q)$, in a such a way that, $q_{\widehat{\theta}/(u_T, y_T)}(t) = 0, \forall t \in \mathbb{R}$.

Now let us consider known the set of continuous time scalar functions (27),

$$w_i(t) = \sum_{j=1}^{p_i} [\psi_i^j(u(t), y(t))]^{(n_i^j)} = \sum_{j=1}^{p_i} [\psi_i^j(t)]^{(n_i^j)}, i = 1 : p \quad (28)$$

$$v(t) = \sum_{j=1}^{p_0} [\psi_0^j(u(t), y(t))]^{(n_0^j)} = \sum_{j=1}^{p_0} [\psi_0^j(t)]^{(n_0^j)}. \quad (29)$$

Based on these functions, the regular distributions F_{w_i} , $i = 1 : p$, are generated by relations,

$$F_{w_i} = F_{\psi_i}^{(n_i)} = F_{\psi^{(n_i)}} : \Phi_n \to \mathbb{R}, \varphi \to F_{w_i}(\varphi) \quad (30)$$

$$F_{w_i}(\varphi) = \sum_{j=1}^{p_i} \int_{\mathbb{R}} [\psi_i^j(t)]^{(n_i^j)} \cdot \varphi(t) \cdot dt = \sum_{j=1}^{p_i} (-1)^{n_i^j} \int_{\mathbb{R}} [\psi_i^j(t)] \cdot \varphi^{(n_i^j)}(t) \cdot dt$$

They constitute the row vector,

$$F_w^T(\varphi) = [F_{w_1}(\varphi), \ldots, F_{w_i}(\varphi), \ldots, F_{w_p}(\varphi)] \in \mathbb{R}^p. \quad (31)$$

Also, the regular distribution F_v, is

$$F_v = F_{\psi_0}^{(n_0)} = F_{\psi^{(n_0)}} : \Phi_n \to \mathbb{R}, \varphi \to F_v(\varphi) \quad (32)$$

$$F_v(\varphi) = \sum_{j=1}^{p_0} \int_{\mathbb{R}} [\psi_0^j(t)]^{(n_0^j)} \cdot \varphi(t) \cdot dt = \sum_{j=1}^{p_0} (-1)^{n_0} \int_{\mathbb{R}} [\psi_0^j(t)] \cdot \varphi^{(n_0^j)}(t) \cdot dt$$

Into this conditions, any input-output pair (u, y) observed from the system (25) is described by a pair of regular distribution (F_w, F_v) for any $\varphi \in \Phi_n$.

In such a way, the problem of identification regarding the parameters of the real system (25) can be represented by distributions. For example, the regular distribution generated by the continuous function $q_{\theta/(u,y)}$ from (25), into the specific case of (26) is related to the parameter vector θ as

$$F_{q_\theta}(\varphi) = \sum_{i=1}^{p} F_{w_i}(\varphi) \cdot \theta_i - F_v(\varphi) = F_w^T(\varphi) \cdot \theta - F_y(\varphi), \varphi \in \Phi_n$$

If a triple (u^*, y^*, θ^*) is a realization of the model (25), then the identity (34) takes place,

$$F_{q_{\theta^*}}(\varphi) = F_{q_{\theta^*}/(u^*, y^*)}(\varphi) = 0, \forall \varphi \in \Phi_n \quad (34)$$

and vice versa, if an input-output pair (u^*, y^*) of the family of models (25), with unknown parameter θ, generates a distribution

$$F_{q_\theta}(\varphi) = F_{q_\theta/(u^*, y^*)}(\varphi) = \sum_{i=1}^{p} F_{w_i}(\varphi) \cdot \theta_i - F_v(\varphi) \quad (35)$$

which satisfies $F_{q_\theta}(\varphi) = F_{q_\theta/(u^*, y^*)}(\varphi) = 0, \forall \varphi \in \Phi_n$, then $\theta = \theta^*$. As θ has p components it is enough a chose (utilize) a finite number $N \geq p$ of fundamental function $\varphi_i, i = 1 : N$ and to build an algebraic equation,

$$\mathbf{F}_w \cdot \theta = \mathbf{F}_v \quad (36)$$

where $\mathbf{F_w}$ is an ($N \times p$) matrix of real numbers

$$\mathbf{F}_w = [F_w^T(\varphi_1); \ldots; F_w^T(\varphi_i); \ldots; F_w^T(\varphi_N)]^T \quad (37)$$

where i-th row $F_w^T(\varphi_i)$ is given by (31). The symbol $\mathbf{F}_v$ denotes an N-column real vector built from (32),

$$\mathbf{F}_v = [F_v(\varphi_1),...,F_v(\varphi_i),...,F_v(\varphi_N)]^T . \quad (38)$$

When only the restriction (u_T, y_T) of the pair (u, y) on the time interval T, is available, then one must chose φ_i such that $\text{supp}(\varphi_i) \subset T \subset \mathbb{R},\ i = 1:N$. If $r = rank(\mathbf{F}_w) = p$, then a unique solution is obtained.

$$\hat{\theta} = (\mathbf{F}_w^T \cdot \mathbf{F}_w)^{-1} \cdot \mathbf{F}_w^T \cdot \mathbf{F}_v = \theta^* \quad (39)$$

4 The hierarchical structure of identification and estimation equations

Consider all state variables accessible for measurements so $y = \xi$. The dynamical system $(15) \div (21)$ contains rational dependences between parameters and measured variables. To obtain linear equations in unknown parameters, the identification problem is split in several simpler interlinked identification problems called identification layers.
Based on the specific structure of this system, it is possible to group the state equations, in such way to determine five interconnected identification problems of the type (39), labelled Layer_*, *=a, b, c, d, e. They are organized in a hierarchical structure. First, in Layer_a, some state equations are utilized to obtain a set of linear equations in some parameters. The results of this first stage of identification are utilized for expressing other parameters by linear equations in Layer_b. This process is repeated in the other layers until all parameters are identified. For each identification layer, the same type of procedures and numerical algorithms are applied.

Layer_a: Identification of θ_1.

Substituting expression ϕ_1 from (15) into (17) we obtain, the Layer_a model (25)

$$q^a_{\theta/(u,y)} = (\dot{\xi}_1 + u_3 \cdot \xi_1) \cdot \theta_1 - (-\dot{\xi}_2 - u_3 \cdot \xi_2 + u_1 \cdot u_3) \quad (40)$$

characterized by $\theta^a = [\theta^a_1] = [\theta_1]$, $p^a = 1$

$$F^a_{w_1}(\varphi) = \int_{\mathbb{R}} [-\xi_1(t)] \cdot \varphi^{(1)}(t) \cdot dt + \int_{\mathbb{R}} [u_3(t) \cdot \xi_1(t)] \cdot \varphi^{(0)}(t) \cdot dt$$

$F^{a,T}_w(\varphi) = [F^a_{w_1}(\varphi)]$. Also, $F^a_v(\varphi) = \int_{\mathbb{R}} [\xi_2(t)] \cdot \varphi^{(1)}(t) \cdot dt +$

$$+\int_{\mathbb{R}} [-u_3(t) \cdot \xi_2(t)] \cdot \varphi^{(0)}(t) \cdot dt + \int_{\mathbb{R}} [u_1(t) \cdot u_3(t)] \cdot \varphi^{(0)}(t) \cdot dt$$

Layer_b: Identification of θ_5, θ_7.

Considering known $\theta_1 = \hat{\theta}_1$ from the Layer_a, and substituting (16), equation (17) becomes,

$$\dot{\xi}_2 = -\hat{\theta}_1 \cdot \theta_5 \cdot \frac{\xi_1 \cdot \xi_2}{\theta_7 + \xi_2} - u_3 \cdot \xi_2 + u_1 \cdot u_3$$

The Layer_b model (25) is now,

$$q^b_{\theta/(u,y)} = (\xi_1 \cdot \xi_2 \cdot \hat{\theta}_1) \cdot \theta_5 + (\dot{\xi}_2 + u_3 \cdot \xi_2 - u_1 \cdot u_3) \cdot \theta_7 -$$
$$-(-\xi_2 \cdot \dot{\xi}_2 - u_3 \cdot \xi_2^2 + u_1 \cdot u_3 \cdot \xi_2) \quad (41)$$

characterized by $\theta^b = [\theta^b_1\ \ \theta^b_2] = [\theta_5\ \ \theta_7]$, $p^b = 2$

$$F^b_{w_1}(\varphi) = \int_{\mathbb{R}} [\xi_1(t) \cdot \xi_2(t) \cdot \hat{\theta}_1] \cdot \varphi^{(0)}(t) \cdot dt$$

$$F^b_{w_2}(\varphi) = \int_{\mathbb{R}} [-\xi_2(t)] \cdot \varphi^{(1)}(t) \cdot dt + \int_{\mathbb{R}} [u_3(t) \cdot \xi_2(t)] \cdot \varphi^{(0)}(t) \cdot dt +$$

$+\int_{\mathbb{R}} [-u_1(t) \cdot u_3(t)] \cdot \varphi^{(0)}(t) \cdot dt$; $F^{b,T}_w(\varphi) = [F^b_{w_1}(\varphi)\ \ F^b_{w_2}(\varphi)]$

Also, $F^b_v(\varphi) = \int_{\mathbb{R}} [\frac{1}{2} \cdot \xi_2^2(t)] \cdot \varphi^{(1)}(t) \cdot dt +$

$$+\int_{\mathbb{R}} [-u_3(t) \cdot \xi_2^2(t)] \cdot \varphi^{(0)}(t) \cdot dt + +\int_{\mathbb{R}} [u_1(t) \cdot u_3(t) \cdot \xi_2(t)] \cdot \varphi^{(0)}(t) \cdot dt$$

Layer_c: Identification of θ_2, θ_3

Considering known $\theta_5 = \hat{\theta}_5 ; \theta_7 = \hat{\theta}_7$ from the Layer_b the estimated expression $\hat{\phi}_1$, of the rational ϕ_1, is

$$\hat{\phi}_1 = \hat{\theta}_5 \cdot \frac{\xi_1 \cdot \xi_2}{\hat{\theta}_7 + \xi_2} \quad (42)$$

whose time expression is $\hat{\phi}_1(t) = \hat{\theta}_5 \cdot \frac{\xi_1(t) \cdot \xi_2(t)}{\hat{\theta}_7 + \xi_2(t)}$.

Substituting expression ϕ_2 from (18) and (42) instead of ϕ_1 into (20) we obtain,

$$\dot{\xi}_4 = \theta_2 \cdot \hat{\phi}_1 - \theta_3 \cdot [\dot{\xi}_3 + u_3 \cdot \xi_3] - u_3 \cdot \xi_4$$

which determines the Layer_c model (25)

$$q^c_{\theta/(u,y)} = (\hat{\phi}_1) \cdot \theta_2 + (-\dot{\xi}_3 - u_3 \cdot \xi_3) \cdot \theta_3 - (\dot{\xi}_4 + u_3 \cdot \xi_4) \quad (43)$$

characterized by $\theta^c = [\theta^c_1\ \ \theta^c_2] = [\theta_2\ \ \theta_3]$, $p^c = 2$

$$F^c_{w_1}(\varphi) = \int_{\mathbb{R}} [\hat{\phi}_1(t)] \cdot \varphi^{(0)}(t) \cdot dt$$

$$F^c_{w_2}(\varphi) = \int_{\mathbb{R}} [\xi_3(t)] \cdot \varphi^{(1)}(t) \cdot dt + \int_{\mathbb{R}} [-u_3(t) \cdot \xi_3(t)] \cdot \varphi^{(0)}(t) \cdot dt$$

$$F^{c,T}_w(\varphi) = [F^c_{w_1}(\varphi)\ \ F^c_{w_2}(\varphi)]$$

Also, $F^c_v(\varphi) = \int_{\mathbb{R}} [-\xi_4(t)] \cdot \varphi^{(1)}(t) \cdot dt +$

$$+\int_{\mathbb{R}} [u_3(t) \cdot \xi_4(t)] \cdot \varphi^{(0)}(t) \cdot dt$$

Layer_d: Identification of $\theta_6, \theta_8, \theta_9^{/}$.

Considering known $\theta_2 = \hat{\theta}_2, \theta_3 = \hat{\theta}_3$ from the Layer_c, and substituting (19) in equation (20) where ϕ_1 is replaced by $\hat{\phi}_1$ we obtain,

$$\dot{\xi}_4 = \hat{\theta}_2 \cdot \hat{\phi}_1 - \hat{\theta}_3 \cdot \frac{\xi_3 \cdot \xi_4}{\theta_8 + \xi_4 + \theta_9^{'} \cdot \xi_4^2} - u_3 \cdot \xi_4$$

The Layer_d model (25) is now,

$$q^d_{\theta/(u,y)} = (\xi_3 \cdot \xi_4 \cdot \hat{\theta}_3) \cdot \theta_6 + (\dot{\xi}_4 + u_3 \cdot \xi_4 - \hat{\theta}_2 \cdot \hat{\phi}_1) \cdot \theta_8 +$$
$$+(\xi_4^2 \cdot \dot{\xi}_4 + u_3 \cdot \xi_4^3 - \hat{\theta}_2 \cdot \hat{\phi}_1 \cdot \xi_4^2) \cdot \theta_9^{/} -$$
$$-(-\xi_4 \cdot \dot{\xi}_4 - u_3 \cdot \xi_4^2 + \hat{\theta}_2 \cdot \hat{\phi}_1 \cdot \xi_4) \quad (44)$$

characterized by

$$\theta^d = [\theta^d_1\ \ \theta^d_2\ \ \theta^d_3] = [\theta_6\ \ \theta_8\ \ \theta_9^{/}],\ p^d = 3$$

$$F^d_{w_1}(\varphi) = \int_{\mathbb{R}} [\xi_3(t) \cdot \xi_4(t) \cdot \hat{\theta}_3] \cdot \varphi^{(0)}(t) \cdot dt$$

$$F_{w_2}^d(\varphi)=\int_{\mathbb{R}}[-\xi_4(t)]\cdot\varphi^{(1)}(t)\cdot dt+\int_{\mathbb{R}}[u_3(t)\cdot\xi_4(t)]\cdot\varphi^{(0)}(t)\cdot dt+$$
$$+\int_{\mathbb{R}}[-\widehat{\theta}_2(t)\cdot\widehat{\phi}_1(t)]\cdot\varphi^{(0)}(t)\cdot dt\ ;\ F_w^{d,T}(\varphi)=[F_{w_1}^d(\varphi)F_{w_2}^d(\varphi)F_{w_3}^d(\varphi)]$$
$$F_v^d(\varphi)=\int_{\mathbb{R}}[\frac{1}{2}\cdot\xi_4^2(t)]\cdot\varphi^{(1)}(t)\cdot dt+\int_{\mathbb{R}}[-u_3(t)\cdot\xi_4^2(t)]\cdot\varphi^{(0)}(t)\cdot dt+$$
$$+\int_{\mathbb{R}}[\widehat{\theta}_2(t)\cdot\widehat{\phi}_1(t)\cdot\xi_4(t)]\cdot\varphi^{(0)}(t)\cdot dt$$

Layer_e: Identification of θ_4

Considering known $\theta_6=\widehat{\theta}_6;\theta_8=\widehat{\theta}_8;\theta_9^/=\widehat{\theta}_9^/$, from the Layer d identification, the estimated expression $\widehat{\phi}_2$, of the nonlinear function ϕ_2, is

$$\widehat{\phi}_2=\widehat{\theta}_6\cdot\frac{\xi_3\cdot\xi_4}{\theta_8+\xi_4+\widehat{\theta}_9^/\cdot\xi_4^2} \tag{45}$$

whose time expression is

$$\widehat{\phi}_2=\widehat{\theta}_6\cdot\frac{\xi_3(t)\cdot\xi_4(t)}{\theta_8+\xi_4(t)+\widehat{\theta}_9^/\cdot\xi_4^2(t)}\ .$$

Substituting expression (45) instead of ϕ_2 into (21),

$$\dot{\xi}_5=-u_3\cdot\xi_5+\theta_4\cdot\widehat{\phi}_2-u_2$$

which determines the Layer_e model (25)

$$q^e_{\theta/(u,y)}=(\widehat{\phi}_2)\cdot\theta_4-(\dot{\xi}_5+u_3\cdot\xi_5+u_2) \tag{46}$$

characterized by $\theta^e=[\theta_1^e]=[\theta_4]$, $p^e=1$

$F_{w_1}^e(\varphi)=\int_{\mathbb{R}}[\widehat{\phi}_2(t)]\cdot\varphi^{(0)}(t)\cdot dt$; $F_w^{e,T}(\varphi)=[F_{w_1}^e(\varphi)]$. Also,

$$F_v^e(\varphi)=\int_{\mathbb{R}}[-\xi_5(t)]\cdot\varphi^{(1)}(t)\cdot dt+$$
$$+\int_{\mathbb{R}}[u_3(t)\cdot\xi_5(t)]\cdot\varphi^{(0)}(t)\cdot dt+\int_{\mathbb{R}}[u_2(t)]\cdot\varphi^{(0)}(t)\cdot dt$$

7 Experimental results

The model given by (15) - (21) and the hierarchical identification procedure developed in this paper has been implemented in Matlab. Three types of experiments ware performed. 1. Noise free; 2. Constant parameters but output measurements are noise contamined; 3. Some process parameters have random variations around constant values. Twelve types of testing functions $\varphi(t)$, characterized by a bounded support $T=[t_a,t_b],t_a<t_b$ are considered. All of these accomplish the condition $\varphi(t)=0,t\notin(t_a,t_b)$,

The nonzero restriction, is of the form $\varphi(t)=\alpha\cdot\beta(t_a,t_b)\cdot\Psi(t,t_a,t_b)\in\Phi_n$, where, for $p\geq n+1$

$\Psi(t)=\Psi(t,t_a,t_b)\in C^n[(t_a,t_b)]$ is one of the four types,

1. Exponential: $\Psi(t)=\exp[|t_a\cdot t_b|/(t-t_a)\cdot(t-t_b)]$;
2. Sinusoidal: $\Psi(t)=\sin^p[\pi\cdot(t-t_b)/(t_b-t_a)]$,
3. Polynomial : $\Psi(t)=(t-t_a)^p\cdot(t-t_b)^p$,
4. Product : $\Psi(t)=f_a(t)\cdot f_b(t)$, where

$$f_a\in C^n[(t_a,t_b)], f_b\in C^n[(t_a,t_b)]\ ,\ p_a;p_b\geq n+1$$

$$f_a^{(k)}(t_a)=0,k=0:p_a\ ;\ f_b^{(k)}(t_b)=0,k=0:p_b\ .$$

For each of the four types, three variants can be implemented with respect to the coefficient $\beta=\beta(t_a,t_b)$. Here , α is a scaling factor.

a. Free amplitude: $\beta(t_a,t_b)=1,\ \forall t_a,t_b$

b. Normalized peak:

$$\beta(t_a,t_b)=1/\max_{t\in T}|\Psi(t,t_a,t_b)|,\ \forall t_a,t_b$$

c. Normalized area:

$$\beta(t_a,t_b)=1/\int_{t_a}^{t_b}\Psi(t,t_a,t_b),\ \forall t_a,t_b$$

Fig.1 shows the noise free system time response.

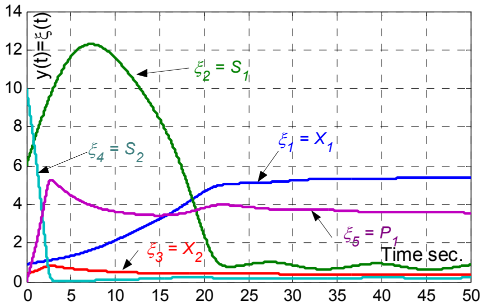

Fig.1 Time response for noise free system

Noise free identification results

Real values	Identified values
5.40000000000000	5.39999999968883
1.00000000000000	0.99999999988881
14.70000000000000	14.70000000094740
10.00000000000000	9.99999993145097
0.20000000000000	0.20000000002676
0.60000000000000	0.60000000028379
0.75000000000000	0.75000000078192
1.00000000000000	1.00000000065981
21.00000000000000	20.99999936217280

For this identification, 3 testing functions $\varphi_1,\varphi_2,\varphi_3$ of the type 2c (sinusoidal-normalized area), of the degree $p=4$, on the intervals $T_1=(0,5)$, $T_2=(5,10)$, $T_3=(10,15)$, has been utilized. Fig.2 shows the noise contamined measured variables utilized in identification.

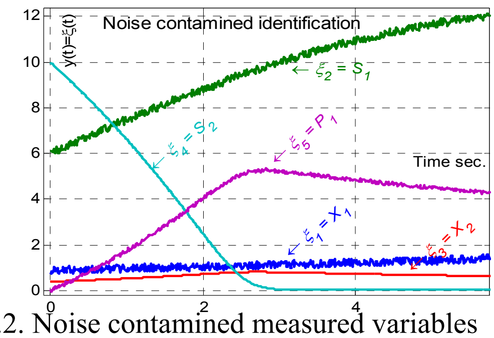

Fig.2. Noise contamined measured variables

Noise contamined identification

Real values	Identified values
5.40000000000000	5.38519339180525
1.00000000000000	1.00400800428003
14.70000000000000	14.70349693406084
10.00000000000000	9.99770002079475
0.20000000000000	0.19740257180333
0.60000000000000	0.59212639221193
0.75000000000000	0.63453032445845
1.00000000000000	0.98606989066886
21.00000000000000	22.94469606537222

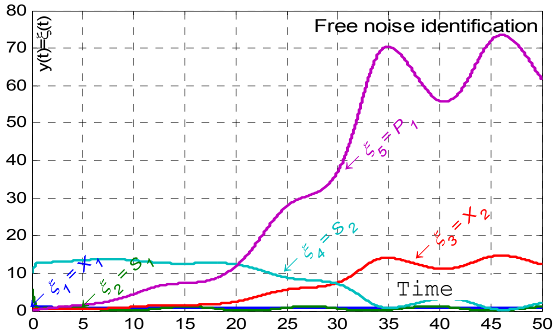

Fig.3. Time response for other unknown parameters

Free noise contamined identification

Real values	Identified values
40.00000000000000	40.00000000117525
20.00000000000000	19.99999999873951
1.00000000000000	0.99999999883225
5.00000000000000	5.00000000015483
1.00000000000000	1.00000001236970
2.00000000000000	1.99999586831905
5.00000000000000	5.00000007181871
30.00000000000000	29.99993050529158
2.00000000000000	2.00000458980052

8 Conclusion

Through this research has been proved that it is possible to identify all parameters of continuous time nonlinear systems even if they are related to measured variables by rational expressions. This is possible if the identification problem is formulated as a set of interconnected identification problems with linear dependences between parameters and measured variables. The problem of functionals based identification consistency has to be analyzed for a broader class of nonlinear systems.

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