Self Tuning Based Control of Mechanical Systems with Friction

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Abstract: - This paper presents a self tuning control algorithm for mechanical systems with friction. Mainly it applies the results of the identification procedures based on distributions theory to continuous time systems with friction. The tuning algorithm performs two actions: compensates the friction force and adapts the parameters of a linear controller. There are defined the so called generalized friction dynamic systems (GFDS) as a closed loop structure around a smooth system with discontinuous feedback loops representing friction reaction vectors. Only GFDS with static friction models (SFM) are c considered in self tuning process. The proposed method is a batch on-line identification and tuning method because identification results are obtained during the system evolution after some time intervals but not in any time moment. The advantages of representing information by distributions are pointed out when special evolutions as sliding mode, or limit cycle can appear. Some experimental results are presented to illuminate its advantages and practical use.

Keywords: - Identification; Distribution theory; Friction.

1 Introduction

There are many mechanical systems, from machine tool positioning to tracking in navigation, where the so-called friction forces influence the motion. Friction models contain some specific nonlinearity such as stiction, hysteretic, Stribeck effect, stick-slip, depending on velocity [1], [2], [3], [4]. The values of model parameters can change during the system evolution or are influenced by some other causes as external temperature, quality of materials etc. A large variety of friction models, as Coulomb friction model, Dahl model [5], [6], exponential model [7], bristle model [8], state variable model [9], there are accepted in literature [1].

Ignoring friction in controlling such systems can lead to tracking errors, limit cycles, undesired stick-slip motion [2]. To avoid these difficulties, adaptive control strategies, named model-based friction compensation techniques [2], are recommended. Such adaptive strategies involve identification procedures of the controlled system, including identification of the model friction parameters.

Unfortunately friction models are nonlinear, involving a discontinuous dependence with respect to velocity. Because of this, many techniques, as identification based on time-discretized models fail to offer good results.

A survey of models, analysis tools and compensation methods for the control of machines with friction is presented in [11]. A distribution-based approach of mechanical systems with friction identification is developed in [12]. This extends the results on continuous time system identification based on distribution theory, reported in [13], for linear systems, or in [14], for nonlinear systems. To perform continuous time domain identification the system differential equations is transformed to an algebraic system that reveals the unknown parameters, [15]. This can be done also by using some modulating functions to generate functionals to avoid the direct computation of the input-output data derivatives [16], [17], [18].

This paper presents a self-tuning control structure for both the friction compensation as an additional input correction signal and the controller parameters. Through this, is intended to demonstrate the effectiveness of the distribution based identification technique from [12], for friction mechanical systems. As the identification results are obtained during the system evolution after some time intervals but not in any time moment, proposed self-tuning method is a batch on-line.

The paper is organized as follows: After introduction in the first section, Section 2 presents the structure of the proposed self-tuning control system. Section 3 is dedicated to the generalized friction dynamic systems GFDS, as presented in [12]. Section 4, presents the main steps the of continuous time system identification based on distributions. In section 5 the tuning strategy is presented. Section 6 illustrates applications of the identification methods for different types of systems with frictions and results of the proposed self tuning control structure. Conclusions are resumed in Section 7.

2 Self-tuning control system structure

Figure 1 presents the structure of the proposed self tuning control system for friction mechanical system.

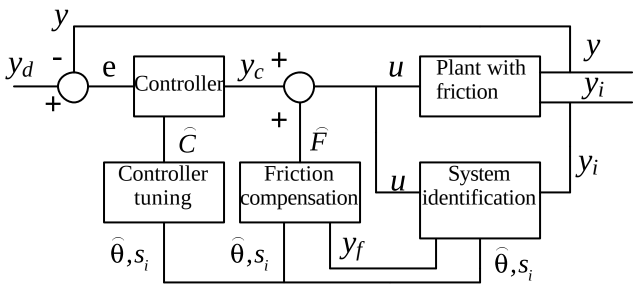

Figure 1, Self tuning control system structure.

The mechanical system with friction, as a controlled plant, called "Plant with friction" (PF), has a manipulated variable u and y as feedback variable for the closed loop system. Some other variables, denoted y_i, are utilized for identification purposes. The set point is y_d and the command variable, delivered by the "Controller" (CO), is y_c. A block, "System identification" (SI), receives the pair u, y_i, as measured signals, and realizes the identified parameters $\hat{\theta}$, the identification status s_i and additional variables, y_f necessary for the "Friction compensation" (FC) block. The output of FC, the estimated friction forces $\hat{F}$, is applied to PF as correction signals. The block "Controller tuning" (CT) adjusts some of the CO parameters, depending on the pair $\hat{\theta}, s_i$. All blocks SI, FC, CT incorporate also complex monitoring and decision functions.

3 Generalized Friction Dynamic Systems

As presented in [12], a generalized dynamic friction system (GFDS) is a system characterized by the state equation of the form

$$\dot{x} = f(x, u, r_1, .., r_i, .., r_p) \quad (1)$$

where x, is the state vector and u is the input vector. The vectors r_i are called friction reaction vectors. They depend on x and u through a specific operator $\Psi_i\{\}$, called friction operator,

$$r_i = \Psi_i\{x, u\}, i = 1 : p \quad (2)$$

There are two categories of friction models: static friction models (SFM) and dynamic friction models.(DFM). For SFM, we deal with only in this paper, the operator (2) is a non-dynamic mapping with a specific structure as follows. For any $i = 1 : p$,there are two functions $v_i = \nu_i(x, u)$, which determines the so called generalized velocity vector v_i, and $a_i = \alpha_i(x, u)$,expressing the so called active component of the velocity vector v_i. In SFM, the non dynamic mapping (2) can be expressed as a function of v_i, and a_i only, as depicted in Figure 2, $r_i = F_i\{x, u\}$.

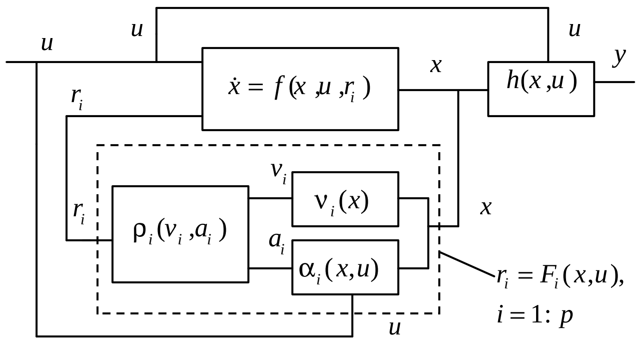

Fig. 2, The feedback structure of a GDFS with SFM.

Inspired from mechanics [11], the function ρ_i is explicitly defined for $v_i = 0$ and for $v_i \neq 0$. As a result, two components of the friction reaction vectors $r_i, i = 1 : p$ can be defined: static friction reaction r_i^s and cinematic friction reaction r_i^c, where, particularly, $r_i = r_i^s + r_i^c$,

$$r_i^s = \rho_i^s(v_i, a_i), v_i = 0;\ r_i^s = 0, v_i \neq 0; \quad (3)$$

$$r_i^c = \rho_i^c(v_i, a_i), v_i \neq 0;\ r_i^c = 0, v_i = 0 \quad (4)$$

The adjective static and dynamic, for the friction reaction vectors $r_i, i = 1:p$, must be understood with respect to the velocity vector v_i only. Also, for a vector $v_i \in \mathbb{R}^{m_i}, i = 1:p$, it is defined the function $sgn(v_i)$ as $sgn(v_i) = v_i / \|v_i\|$, where $\|v_i\|$ is the Euclidian norm of $\mathbb{R}^{m_i}$. In this norm the function $sgn(v_i)$ is a discontinuous function in the point $v_i = 0$. It is observed that

$$\| sgn(v_i) \| = sgn(\|v_i\|) = 1,\ v_i \neq 0,\ sgn(\|0\|) = 0 \quad (5)$$

If $m_i = 1$, v_i is a scalar variable, then (12) can be presented by using inequalities. Because of (3) and (4), the system state vector evolution $x(t)$ is characterized by a status of two values, related to each friction reaction vectors $r_i, i = 1:p$,

1. Evolution inside a surface characterized by zero value of the velocity vector v_i, $x(t) \in S_i$, where $S_i = \{x \in \mathbf{X}, v_i = 0\} = \{x \in \mathbf{X}, \nu_i(x) = 0\}$.
2. Evolution with nonzero value of the velocity vector v_i, that means outside the surface S_i, $x(t) \notin S_i$.

Outside the surface S_i, r_i is a vector opposite to $v_i \neq 0$ but inside the surface S_i, r_i is a vector opposite to a_i. There is a closed subset $S_i^0(u) \subseteq S_i$, called sticky area (SA), which keeps the system state inside . This means

$$\tfrac{d}{dt} v_i(x(t)) = [\tfrac{d}{dx} v_i(x)]^T \cdot \dot{x}(t) = 0, \forall x \in S_i^0(u). \quad (6)$$

Inside the SA $r_i = -a_i$. Because the input u can change the SA position the state x can be forced to be out of $S_i^0(u)$, crossing its border. For any admissible u, the function $r_i = F_i(x,u)$ is continuous with respect to $\forall x \in S_i^0(u)$. Because of this, when the system state $x(t)$ arrives on or leaves out $S_i^0(u)$ the friction reaction $r_i(t)$ is a continuous time function. Condition c, is called the smooth sticky condition (SSC). However, when $x(t) \in S_i \setminus S_i^0(u)$, $r_i(t)$ has a discontinuity and $\frac{d}{dt} v_i(x(t)) \neq 0$. In this case $x(t)$ passes from one side to other of $S_i \setminus S_i^0(u)$, as a switching mode or as a sliding mode. For example, expressions as (7) and (8) of (3) and (4) respectively, satisfy the above conditions, where by a_i it must understand $a_i = a_i(x,u)$,

$$r_i^s = \rho_i^s(v_i, a_i) = -max\{Q_i, \|a_i\|\} \cdot sgn(a_i) \cdot [1 - sgn(\|v_i\|)] \quad (7)$$

$$r_i^c = \rho_i^c(v_i, a_i) = -[Q_i + K_{vi} \cdot \|v_i\| + B_i \cdot (e^{-\beta_i \|v_i\|} - 1)] \cdot sgn(v_i) \quad (8)$$

As it can be observed, the cinematic reaction r_i^c is a sum of three components, $r_i^{cc}, r_i^{cv}, r_i^{cs}$ expressing respectively Coulomb friction, viscous friction and the so called Stribeck effect, [4], [11],

$$r_i^c = r_i^{cc} + r_i^{cv} + r_i^{cs}. \quad (9)$$

For $m_i = 1$, all r_i, a_i, v_i are scalar variables so the static reaction (7), r_i^s, is illustrated in Fig. 3.a. and the cinematic reaction (8), r_i^c, in Fig. .b.

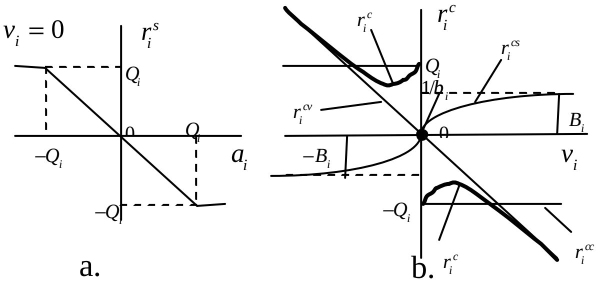

Fig.3, Static and cinematic components of a scalar friction reaction.

A friction reaction vector r_i, as above defined, has a sticky characteristic which means there is a subset $S_i^S(u) \subseteq S_i$, called sticky set (SS), such

$$\dot{v}_i(t) = d/dt\{v_i(x(t))\} = 0, \forall x(t) \in S_i^S(u(t)) \subseteq S_i \quad (10)$$

The position of SS depends on input vector u. When the system state $x(t)$ approaches $S_i^S(u)$, generated by a vector r_i, it remains inside of that SS till the input $u(t)$ changes the position of $S_i^S(u)$, forcing $x(t)$ to be outside of it. Substituting (4) into (1) and denoting

$$\mathbf{f}(x,u) = f(x,u,F_1(x,u),..,F_i(x,u),..,F_p(x,u)) \quad (11)$$

the GDFS takes the compact form

$$\dot{x} = \mathbf{f}(x,u), x(t_0) = x_0, t \geq t_0. \quad (12)$$

This is a differential system with a discontinuous function on right side so for its analytical description, special mathematical approaches are necessary. For example approaches describing the solution in the Charatheodory sense [4], using the Filippov approach , or differential inclusions and differential inequalities . However, for the identification it is supposed a solution exist for (28) and are available as measurements the input variable u and the output variable y where

$$y = h(x,u). \quad (13)$$

4 Identification Based on Distributions of a Friction Mechanical System

To combine the elements of GFS with the theory of identification based on distributions, as presented in [12], let us consider the simplest system with a single friction, as in Fig. 3. It is, represented by a mass m attached to a spring with stiffness K_p, moving on a horizontal surface. The end of the spring is a fixed point. A horizontal force u acts on the mass.

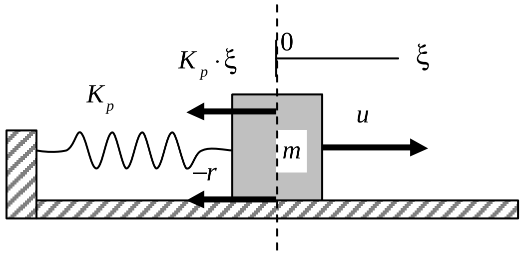

Fig.4, Principle diagram of the friction mechanical system.

Outside $S^0(u)$, the system can be described by

$$m\cdot\ddot{\xi}+K_v\cdot\dot{\xi}+K_p\cdot\xi+Q\cdot sgn(\dot{\xi})+B\cdot(e^{-\beta|\dot{\xi}|}-1)sgn(\dot{\xi})=u \quad (14)$$

except a set of points of a zero measure. A state equations can be determined considering $x=[x_1 \quad x_2]^T=[\xi \quad \dot{\xi}]^T$. Five parameters $\theta=[m;K_v;K_p;Q;B]$ are simultaneously identified. Their identified values are denoted $\hat{\theta}=[\hat{m};\hat{K}_v;\hat{K}_p;\hat{Q};\hat{B}]$ respectively. The distributions involved in identification [12], are expressed by $i=1:6$, integrals of the form

$$F_{w_i}(\varphi_k)=\int_{t_a^k}^{t_b^k}\psi_i^k(t)dt, \text{ for } k=1:N \quad (15)$$

on the testing intervals $T_k=[t_a^k,t_b^k]\subseteq T$, where,

$$\psi_1^k(t)=\xi(t)\cdot\ddot{\varphi}_k(t); \qquad \psi_2^k(t)=-\xi(t)\cdot\dot{\varphi}_k(t);$$
$$\psi_3^k(t)=\xi(t)\cdot\varphi_k(t); \qquad \psi_4^k(t)=sgn(\dot{\xi}(t))\cdot\varphi_k(t);$$
$$\psi_5^k(t)=[(-1+e^{\beta\cdot|\dot{\xi}(t)|})]sgn(\dot{\xi}(t))]\cdot\varphi_k(t); \quad (16)$$
$$\psi_6^k(t)=u(t)\cdot\varphi_k(t).$$

The testing functions $\varphi_k(t)$ are of the form

$$\varphi_k(t)=\alpha_k\cdot\beta_k(t_a^k,t_b^k)\cdot\Psi_k(t,t_a^k,t_b^k) \quad (17)$$

$$\Psi(t,t_a^k,t_b^k)=\begin{cases} sin^{n_k}[\pi\cdot(t-t_b^k)/(t_b^k-t_a^k)], \forall k,n_k\geq n \\ 0, \quad \forall t\in(-\infty,t_a^k]\cup[t_b^k,\infty) \end{cases}$$

where α_i is a scaling factor and β_i normalizes the area $\beta_k(t_a^k,t_b^k)=1/\int_{t_a^k}^{t_b^k}\Psi_k(t,t_a^k,t_b^k), \forall t_a^k<t_b^k$.

Based on these integrals, an algebraic equation,

$$\mathbf{F}_w\cdot\theta=\mathbf{F}_v$$

is built, where $\mathbf{F_w}$ is an $(N\times5)$ matrix of real numbers $\mathbf{F}_w=[F_w^T(\varphi_1);..;F_w^T(\varphi_k);..;F_w^T(\varphi_N)]^T$ where k-the row is $F_w^T(\varphi_k)=[F_{w_1}(\varphi_k),...,F_{w_i}(\varphi_k),...,F_{w_5}(\varphi_k)]$.

$\mathbf{F}_v$ is a vector

$$\mathbf{F}_v=[F_{w_6}(\varphi_1),...,F_{w_6}(\varphi_k),...,F_{w_6}(\varphi_N)]^T.$$

The identified parameter vector $\hat{\theta}$ is given by

$$\hat{\theta}=(\mathbf{F}_w^T\cdot\mathbf{F}_w)^{-1}\cdot\mathbf{F}_w^T\cdot\mathbf{F}_v. \quad (18)$$

5 Self Tuning Control System

The proposed control structure, depicted in Fig.1, considers the PF from Fig.4, and a PI controller with a transfer function

$$H_R(s)=K_R/T_i\cdot(T_i\cdot s+1).$$

As mentioned, blocks SI, FC and CT contain complex monitoring and decision functions, not revealed in this paper.

There are two variants for FC: Direct friction compensation and Observer based compensation. In the first variant the friction force is reconstruct by an algebraic relation,

$$\hat{F}=\hat{Q}\cdot sgn(\dot{\xi})+\hat{B}\cdot(e^{-\beta|\dot{\xi}|}-1)sgn(\dot{\xi}) \quad (19)$$

considering the parameter β off-line determined.

The compensating force $\hat{F}$ depends on the identified parameters $\hat{Q};\hat{B}$ and the both position ξ and the speed $\dot{\xi}$.

In the second variant, the observer's parameters are tuned based on whole identified parameter $\hat{\theta}$. The linear term $K_v\cdot\dot{\xi}$ is not considered in compensation process. The controller parameters $K_R;T_i$ are tuned to assure a desired closed loop behavior of the friction compensated system.

When a tuning decision is taken the new values of parameters are changed by a recursive relation,

6 Experimental Results

Because of limited space in this paper, only one example is analyzed, based [12]. In the first example, the measured signals, as indicated in Fig.5., are generated by a step input $u(t)=2\cdot\mathbf{1}(t)$ with initial state and x(0)=[1 2] considering B=0 and m, Kv , Kp, Q as parameters for identification. Four testing functions φ_k on Tk , as (17), with nk=4 and T1=[0,3];T2=[3,6]; T3=[6,9]; T4=[9,12] are utilized.

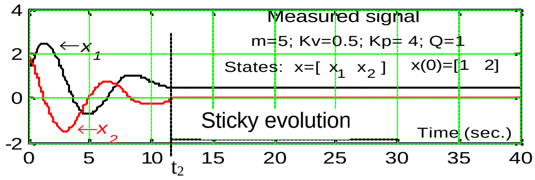

Fig. 5. Measured variables for the friction system without Stribeck effect and initial state x(0)=[1 2]

The real and identified parameter values are respectively

m: 5.00 4.99804 K$_p$: 4.00 3.99776
K$_v$: 0.50 0.50843 Q: 1.00 .99648,

and the conditioning number of Fw, **cond(Fw)=12.3548**. For the same input but with x(0)=[2 6] and T1=[0,5]; T2=[5,10]; T3=[10,15]; T4=[15,20]**;** **cond(Fw)=11.6314**, the identification results are **m: 5.00 4.99793 K$_p$: 4.00 3.99857 K$_v$: 0.50 0.499711 Q: 1.00 0.99918.**

The third example refers to the same conditions but considering errors in the measurement of both input and output. A zoom of these measurements containing error is shown in Fig,6.

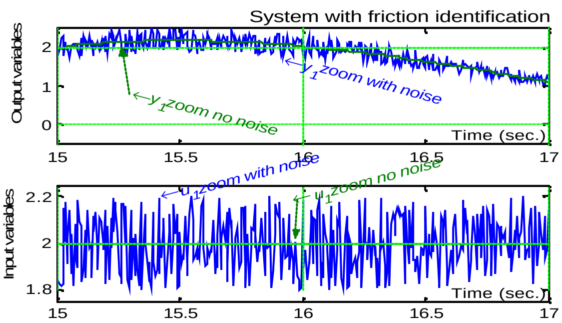

Fig. 6. A zoom representation of measurements containing error for the friction system without Stribeck effect and initial state x(0)=[2 6]

Using these noise contamined measurements, the matrix Fw is still well conditioned, cond(Fw)=11.7178., with the results

m: 5.00 4.99634 K$_p$: 4.00 4.00747
K$_v$: 0.50 0.51198 Q: 1.00 0.97137

If the Coulomb friction parameter Q=1 changed to Q=2, the proposed algorithm gives the new identification results

m: 5.00 5.00496 K$_p$: 4.00 4.00367
K$_v$: 0.50 0.49554 Q: 2.00 2.01137

having **cond(Fw)=48.7245**. In the last presented example, the Stribeck effect is considered, with $\beta=10$ and coefficient B, that has to be identified, together the other previous four parameters.

A step input $u(t)=2\cdot\mathbf{1}(t)$ is applied from initial state x(0)=[2 6] considering B=0.75 and five testing functions on T1=[0,5]; T2=[5,10]; T3=[10,15]; T4=[15,20]; T5=[20,25]; The results are

m: 5.00 4.99999 K$_p$: 0.50 0.50000
K$_v$: 0.50 0.50000 Q: 1.00 0.99997
B: 0.75 0.74997

To point out the Stribeck effect, the Fig. 7 shows the ramp response of this system.

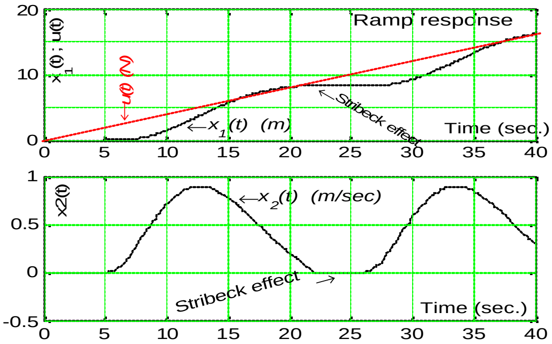

Fig. 7. The ramp response of system with Stribeck effect.

Figures 8,9,10 and 11 present the step response of the closed loop system with and without self tuning facility for different parameters.

m=5; Kp=0.5; Kv=5; Q=6; B=2; bet=10; KR=3; Ti=3

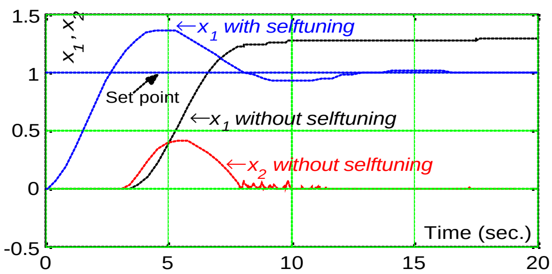

Fig. 8. The step response of the closed loop system with steady state error.

m=5; Kp=0.5; Kv=0.5; Q=1; B=0.75; bet=10; KR=0.025; Ti=5

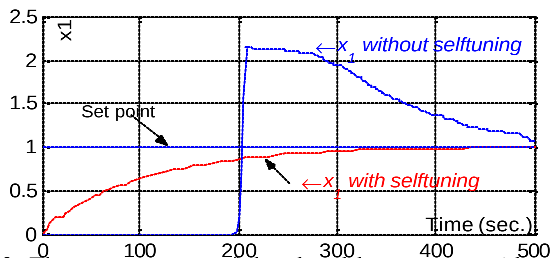

Fig. 9. The step response of the closed loop system with sticky regime.

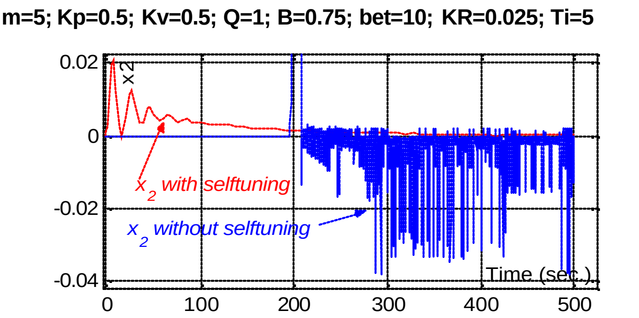

Fig. 10. The step response of the closed loop system with respect to the speed.

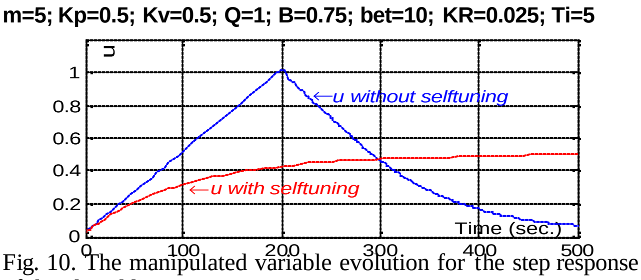

Fig. 10. The manipulated variable evolution for the step response of the closed loop system .

6 Conclusion

The above results illustrate the advantages of distribution-based identification for self-tuning friction systems control. Description by functionals allows enlarging the area of systems to which identification and control procedures can be applied.

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