Switched state-feedback controllers with multi-estimators for MIMO systems

LIBOR TŮMA, BOŘIVOJ HANUŠ AND JAROSLAV HLAVA
Department of Control Engineering
Technical University of Liberec, Faculty of Mechatronics
416 17 Liberec, Hálkova 6
CZECH REPUBLIC

Abstract: Switched controllers can often outperform traditional adaptive control. The paper starts with a brief overview of the development of the field of switched controllers. After that, it describes an original control architecture based on controller and estimator switching. This architecture uses state-feedback control with switched controllers to cope with controlled system dynamics changes. If conventional observer were used to estimate the unmeasurable state variables, the control performance would deteriorate because the estimation results could be corrupted by disturbances acting at various points of the controlled system. To overcome this problem, the proposed architecture includes also a bank of switched estimators. Each of these estimators is tuned for a specific input point of a disturbance. Switching logic is used to select the most suitable estimator that yields the most reliable estimate in any particular control situation. A considerable deal of attention is devoted to extensive testing of the proposed architecture using computer simulation. The testing results are promising.

Keywords: Switched systems, Hybrid systems, Switched controller, Switched-estimators

1. Introduction

In many practical situations, the controller design is complicated by considerable modelling uncertainties, system parameters changes, non-linearities and large unmeasurable disturbances. There are some classical solutions to this kind of problems. The aim of the robust control design methodology is to design controller that achieves a specified level of performance under any uncertainty from the assumed uncertainty set. However, if the uncertainty set is too large, the controller becomes very conservative with unacceptably sluggish nominal responses.

Another classical solution is adaptive control. In this approach, the controller is selected on the basis of the current estimate of the uncertain process and this selection is done over a continuously parameterized family of candidate controllers. Adaptive control can be used even if uncertainty is large and robust control design tools are inapplicable. However, this approach also has some well known inherent limitations. In particular, if unknown parameters enter the process model in a complicated way, it may be difficult to construct a continuously parametrized family of candidate controllers. Moreover, on-line identification of process model over a continuum may also be a difficult problem. Thus, the design of adaptive control algorithms may be fairly complicated and the final success will often depend on trial and error.

In this paper, we will focus on an alternative approach that seeks to overcome some of the above mentioned problems. Its main distinguishing feature is that controller selection is done using suitable switching logic not by continuous tuning.

2. Switching control

Unlike adaptive control, which can be considered a classical field of the control theory, the use of switched controllers is a relatively new idea. Although its origins can be traced back to the eighties (the pioneering work is by Mårtensson [6], another important early paper is [1]), most papers on switched controllers have been published during the last decade. Because of the hybrid (mixed continuous/logical) structure of the control system, switched controllers can be studied within a more general framework of hybrid control.

The algorithms proposed in the early papers used a very simple switching mechanism. They were based on a sequential or “pre-routed ”search among a set of candidate controllers. This blind search of an acceptable controller is time consuming and as a result of it, the control performance (in particular the transient behaviour) of such controllers is poor.

Considerably better results can be achieved with switching algorithms that evaluate online the potential performance of each candidate controller and use this to direct their search. These algorithms can further be divided into two main categories:

switching algorithms based on process estimation and algorithms based on a direct performance evaluation of each candidate controller.

The first category of algorithms can be represented e.g. by references [7], [8] and [5]. Its principle can briefly be explained in the following way. The switching algorithm (supervisor) continuously compares the behaviour of the process with the behaviour a several admissible process models in order to determine which model is more likely to describe the actual process. The decision is based on the estimation errors achieved with respective models. The model with smallest estimation error is regarded as an “estimate ”of the actual process and the supervisor places in the loop the candidate controller that is the most adequate one for this estimated model. Thus, the structure of an estimator-based supervisor consists of a multi-estimator responsible for evaluating which admissible model best matches the process and a decision logic that selects which candidate controller should be used.

Algorithms based on direct performance evaluation can be characterized by the fact that the supervisor attempts to assess directly the potential performance of every candidate controller, without trying to estimate the model of the process. To achieve this, the supervisor computes certain performance signals, that provide a measure of how well each of the candidate controllers C_n would perform in a conceptual experiment in which the actual control signal u would be generated by C_n as a response to the measured process output y. This type of supervision is inspired by the idea of controller falsification introduced in [10]. The structure of this kind of supervisor includes a performance monitor that generates the performance signals and a decision logic that selects which candidate controller should be used. This approach is the relatively less used one. An example of a paper using this approach is [9].

In the sequel, we will basically follow the approach based on process estimation. However, the standard algorithm that makes the selection among different controllers in response to the controlled system dynamics changes will be augmented with another feature. If the controller uses state feedback, it is necessary not only to estimate which of the process models is the most appropriate one but it is also necessary to estimate the unmeasurable state variables. If conventional observer were used to estimate these unmeasurable state variables, the control performance would deteriorate because the estimation results can be corrupted by disturbances acting at various points of the controlled system (disturbance at system input or output, disturbance acting at some other point of the controlled system). To overcome this problem, the proposed architecture includes also a bank of switched estimators. Each of these estimators is tuned for a specific input point of a disturbance. Switching logic is extended so that it could select the most suitable estimator that yields the most reliable estimate in any particular control situation.

3. Control system architecture

The basic structure of the control system is shown in Fig. 1. A very simple set of candidate controllers is used for the sake of explanation. It includes only two controllers for two nominal models of the controlled system. However, extension to a larger set is straightforward. A bank of estimators tuned for different kinds of disturbances is associated with each controller. The following disturbances are considered: $\boldsymbol{d}_{IN}$ disturbance acting at system input, $\boldsymbol{d}_{OUT}$ disturbance at system output, $\boldsymbol{d}_{ARX}$, $\boldsymbol{d}_{ARMAX}$ general disturbances acting anywhere in the system described by ARX or ARMAX model.

The discrete integrators ($1/(1-z^{-1})$) are added to the basic state feedback structure in order to prevent steady state errors. The state feedback controllers are tuned for corresponding nominal models according to quadratic performance criterion

$$J = x^T \boldsymbol{Q}_x \, x + u^T \boldsymbol{Q}_u \, u \qquad (1)$$

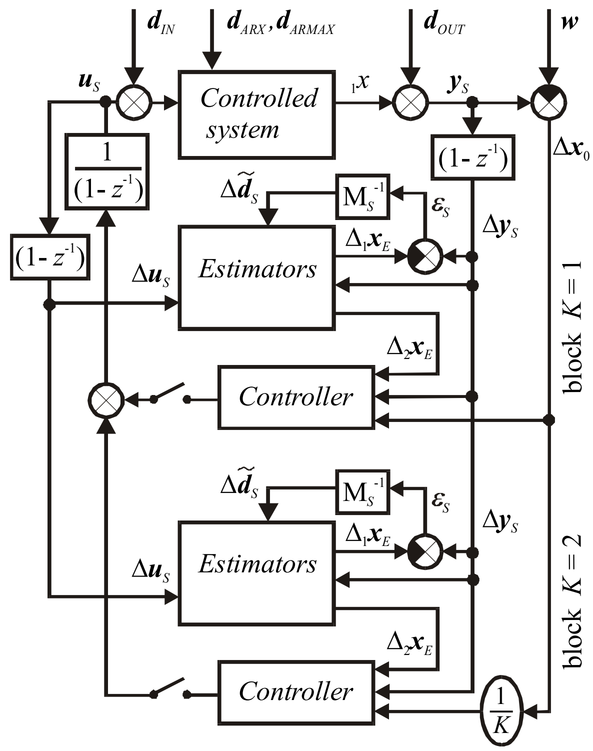

Fig. 1. The structure of the control system with switched controllers and switched incremental estimators.

Controlled system is described by

$$\begin{bmatrix} {}_1\boldsymbol{x} \\ {}_2\boldsymbol{x} \end{bmatrix} = z^{-1}\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}\begin{bmatrix} {}_1\boldsymbol{x} \\ {}_2\boldsymbol{x} \end{bmatrix} + z^{-1}\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}\boldsymbol{u} + z^{-1}\begin{bmatrix} \mathbf{B}_{S1} \\ \mathbf{B}_{S2} \end{bmatrix}\boldsymbol{d}_S \quad (2)$$

where ${}_1\boldsymbol{x}$ is vector of measurable state variables, ${}_2\boldsymbol{x}$ is vector of not measurable state variables, $\boldsymbol{d}_S$ is a disturbance ($\boldsymbol{d}_S = \boldsymbol{d}_{IN}$, $\boldsymbol{d}_{ARX}$, $\boldsymbol{d}_{ARMAX,}$ $\boldsymbol{d}_{OUT}$, otherwise $\boldsymbol{d}_S = 0$). Estimators are described by

$$\begin{bmatrix} \Delta_1\boldsymbol{x}_E \\ \Delta_2\boldsymbol{x}_E \end{bmatrix} = z^{-1}\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}\begin{bmatrix} \Delta\,\boldsymbol{y} \\ \Delta_2\boldsymbol{x}_E \end{bmatrix} + z^{-1}\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}\Delta\,\boldsymbol{u} + + z^{-1}\begin{bmatrix} \mathbf{0} \\ \mathbf{B}_{S2} \end{bmatrix}\Delta\,\tilde{\boldsymbol{d}}_S \quad (3)$$

where $\Delta_1\boldsymbol{x}_E$, $\Delta_2\boldsymbol{x}_E$ are the state variables of the estimator, corresponding to the time increments of the controlled system state variables, $\Delta\,\tilde{\boldsymbol{d}}_S$ - the input disturbance, which is estimated by means of the estimator ($\Delta\,\tilde{\boldsymbol{d}}_S = \mathbf{M}_S^{-1}\boldsymbol{\varepsilon}_s$, $\mathbf{M}_S = \mathbf{B}_{S1}$, for $\tilde{\boldsymbol{d}}_S = \boldsymbol{d}_{IN}$, $\boldsymbol{d}_{ARMAX}$ otherwise $\Delta\,\tilde{\boldsymbol{d}}_S = 0$)

The block with the incremental estimators of the type E_{ARX}, E_{ARMAX}, E_{IN} and E_{OUT}, which also model the controlled system in the nominal regimes, is connected to the output of the controlled system and forms input to the controller. The estimators are tuned for the types of the expected disturbances $\boldsymbol{d}_s$ inputs. The information about the step disturbance input is estimated by means of the matrix **M** (see Fig. 1) from the estimators error $\boldsymbol{\varepsilon}_s$ and it is transferred further into the estimator by means of the corrective feedback (some further results regarding the structure of these estimators were published by the authors in [2] and [3]).

The maximum number of disturbances entering the controlled system simultaneously in one group is equal to the number of the measurable state variables. The number of such groups is unlimited. The estimator error is also used to estimate, which nominal model tuned in the block of the estimators is closest to the current behaviour of the controlled system. If discrete controller is used, the estimator yields the estimate of all unmeasurable state variables. Every estimator of every nominal block is connected permanently to the measurable output of the controlled system and its error $\boldsymbol{\varepsilon}_s$ is evaluated. The estimator operates either in its nominal functions (e.g. E_{IN}, E_{ARMAX}, E_{OUT}) or it is switched over in the E_{ARX} function. It is switched into the control operation, if it is evaluated as the most suitable one (after the evaluation of its error $\boldsymbol{\varepsilon}_s$). The strategy of the estimators switching aims at reaching the smallest value of the estimators errors $\boldsymbol{\varepsilon}_s$ as fast as possible. The switching algorithm is realized by means of the logic functions.

4. Testing experiments

The following controlled system was used for testing

$$\begin{aligned} y_1 &= \frac{1}{K0.5s+1}u_1 - \frac{0.5}{K1.5s+1}u_2 \ , \\ y_2 &= -\frac{0.5}{Ks+1}u_1 + \frac{1}{K2s+1}u_2 \ , \end{aligned} \quad (4)$$

where K is integer coefficient that parametrizes nominal models. Now only two nominal regimes are considered for simplicity ($K = 1,2$).

The corresponding discrete time transfer functions with the sampling interval $\Delta T = 0.2s$ are

$$\begin{aligned} y_1 &= \frac{0.3297z^{-1}}{1-0.6703z^{-1}}u_1 - \frac{0.06241z^{-1}}{1-0.8752z^{-1}}u_2 \ , \\ y_2 &= \frac{-0.09063z^{-1}}{1-0.8187z^{-1}}u_1 + \frac{0.09516z^{-1}}{1-0.9048z^{-1}}u_2 \ , \end{aligned} \quad (5)$$

for $K = 1$ and

$$\begin{aligned} y_1 &= \frac{0.1813z^{-1}}{1-0.8187z^{-1}}u_1 - \frac{0.03225z^{-1}}{1-0.9355z^{-1}}u_2 \ , \\ y_2 &= \frac{-0.04758z^{-1}}{1-0.9048z^{-1}}u_1 + \frac{0.04877z^{-1}}{1-0.9512z^{-1}}u_2 \ , \end{aligned} \quad (6)$$

for $K = 2$.

The discrete integrators are combined with the description of the controlled system

$$\begin{bmatrix} {}_1\boldsymbol{x} \\ {}_2\boldsymbol{x} \end{bmatrix} = z^{-1}\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}\begin{bmatrix} {}_1\boldsymbol{x} \\ {}_2\boldsymbol{x} \end{bmatrix} + z^{-1}\begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix}\boldsymbol{u} \ , \quad (7)$$

where ${}_1\boldsymbol{x}$ is vector of controlled state variables, ${}_2\boldsymbol{x}$ is vector of not controlled state variables.

The description (7) is augmented with state variables $\boldsymbol{x}_0$ of the integrators (see Fig.2a)

$$\begin{bmatrix} \mathrm{x}_0 \\ {}_1\mathrm{x} \\ {}_2\mathrm{x} \end{bmatrix} = z^{-1}\begin{bmatrix} \boldsymbol{I} & K^{-1}\boldsymbol{A}_1 \\ \boldsymbol{0} & \boldsymbol{A}_1 \\ \boldsymbol{0} & \boldsymbol{A}_2 \end{bmatrix}\begin{bmatrix} \mathrm{x}_0 \\ {}_1\mathrm{x} \\ {}_2\mathrm{x} \end{bmatrix} + z^{-1}\begin{bmatrix} K^{-1}\boldsymbol{B}_1 \\ \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{bmatrix}\mathrm{u} \ . \quad (8)$$

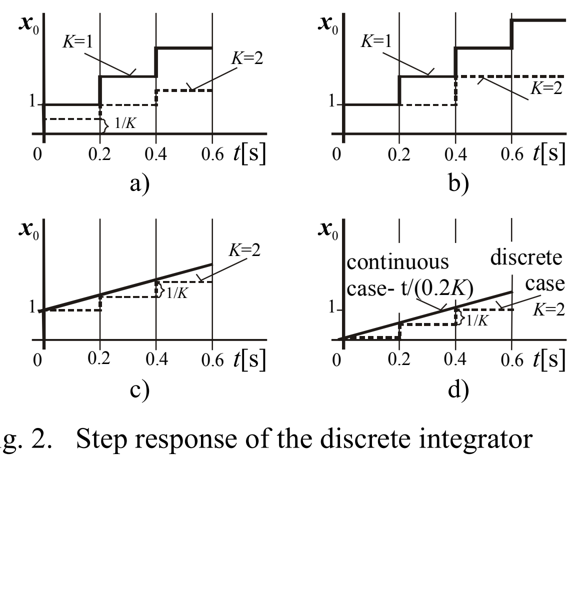

Fig. 2. Step response of the discrete integrator

a) $\frac{1}{K}+\frac{z^{-1}}{K\left(1-z^{-1}\right)}$, c) $1+\frac{z^{-1}}{K\left(1-z^{-1}\right)}$, d) $\frac{z^{-1}}{K\left(1-z^{-1}\right)}$,

b) $\frac{1}{1-z^{-1}}$, $\Delta T = 0.2s$ and $\Delta T = 0.4s$.

The proposed hybrid control structure was tested with the discretized model ((5) and (6)) of the real continuous system (4). Parameter K of the model can be changed. The output controlled variables $\boldsymbol{y}_S$ of the model are connected permanently to the estimators (E_{ARX}, E_{IN}, E_{OUT}), witch are tuned for the step disturbances ($\boldsymbol{d}_{ARX}$, $\boldsymbol{d}_{IN}$, $\boldsymbol{d}_{OUT}$). Control system switches over the estimator functions ($E_{IN} \leftrightarrow E_{ARX}$, $E_{OUT} \leftrightarrow E_{ARX}$) and connects the estimator to the controller and the controller to the disrete integrator.

The control performance was tested with different values of K and different step disturbance inputs in the controlled system. The responses are compared with the control responses of a controlled system with all state variables measurable. The controllers of both control systems are equally tuned according to the performance criterion (1) and to the parameter K of the controlled system. Every single test is carried out for one nominal regime of the controlled system (K = 1 or K = 2) and for one-step disturbance ($\boldsymbol{d}_{IN}$, $\boldsymbol{d}_{OUT}$ and $\boldsymbol{w}$) in the time 1s. The test includes the responses of $\boldsymbol{y}=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\boldsymbol{u}=\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ on the system with measurable state variables and the responses $\boldsymbol{y}_S=\begin{bmatrix} y_{S1} \\ y_{S2} \end{bmatrix}$, $\boldsymbol{u}_S=\begin{bmatrix} u_{S1} \\ u_{S2} \end{bmatrix}$ on the system with estimators, the differences $y_{S1}-y_1$ and $y_{S2}-y_2$ and the changes of the nominal regime and of the type of the estimators during the process. The testing results are in Fig. 3 to Fig.14.

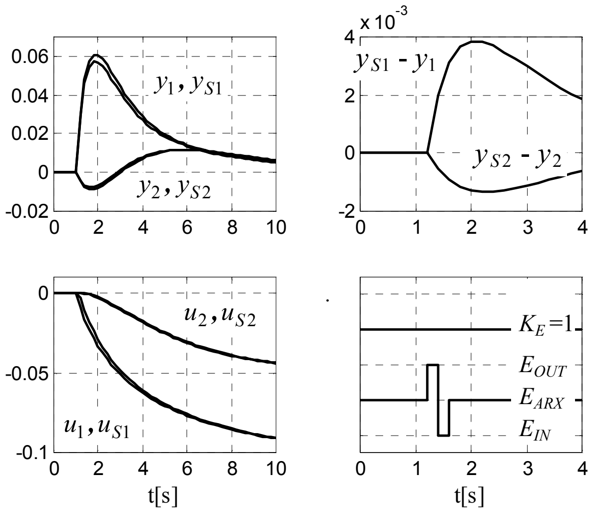

Fig.3. $K_S = 1$, disturbance $\boldsymbol{d}_{IN}=\begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

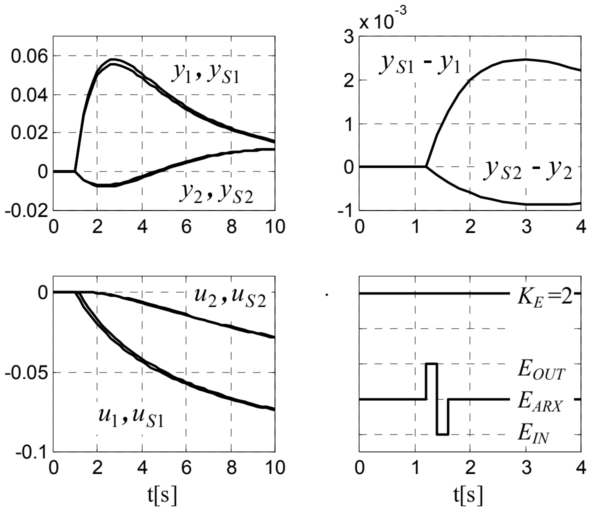

Fig. 4. $K_S = 2$, disturbance $\boldsymbol{d}_{IN}=\begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

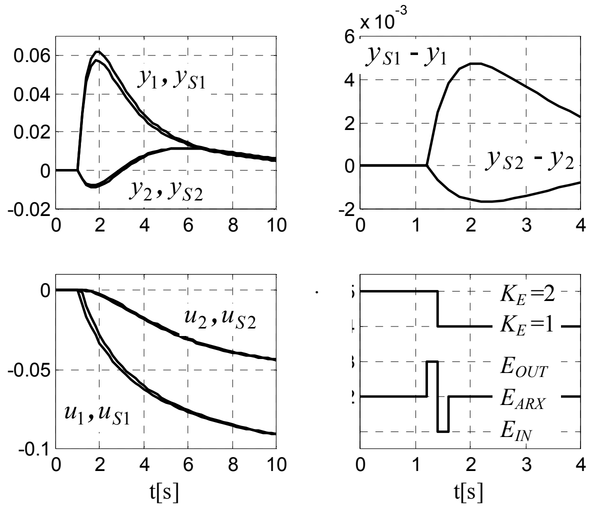

Fig. 5. $K_S = 1$, disturbance $\boldsymbol{d}_{IN}=\begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

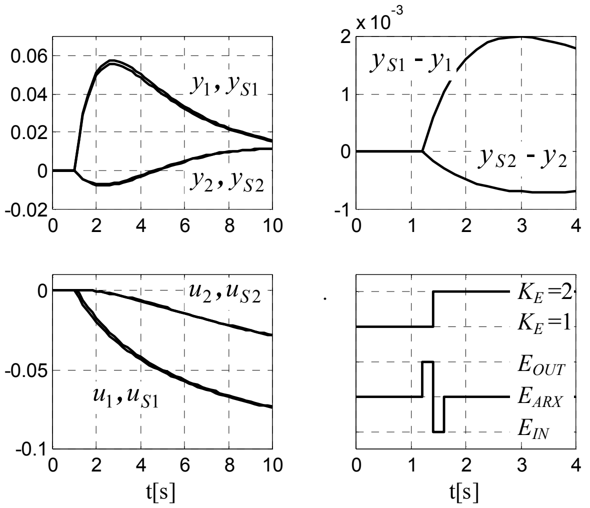

Fig.6. $K_S = 2$, disturbance $\boldsymbol{d}_{IN}=\begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

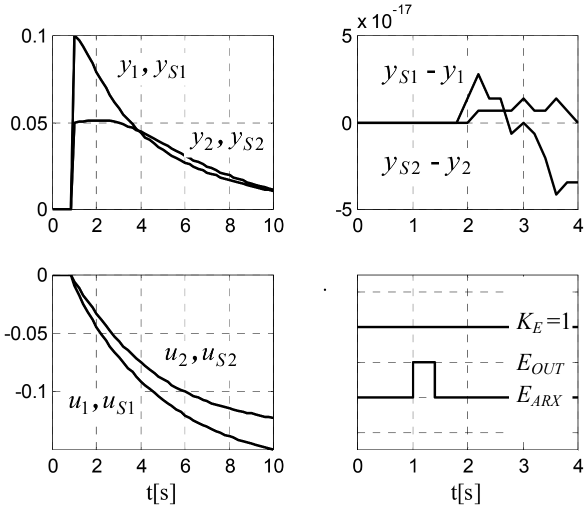

Fig.7. $K_S = 1$, disturbance $\boldsymbol{d}_{OUT} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

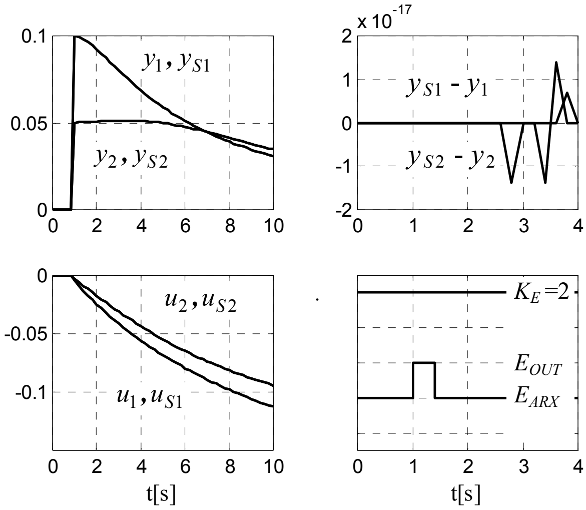

Fig.8. $K_S = 2$, disturbance $\boldsymbol{d}_{OUT} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

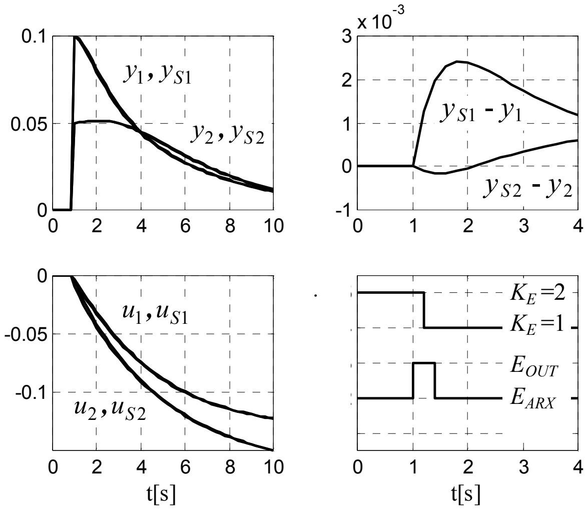

Fig.9. $K_S = 1$, disturbance $\boldsymbol{d}_{OUT} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

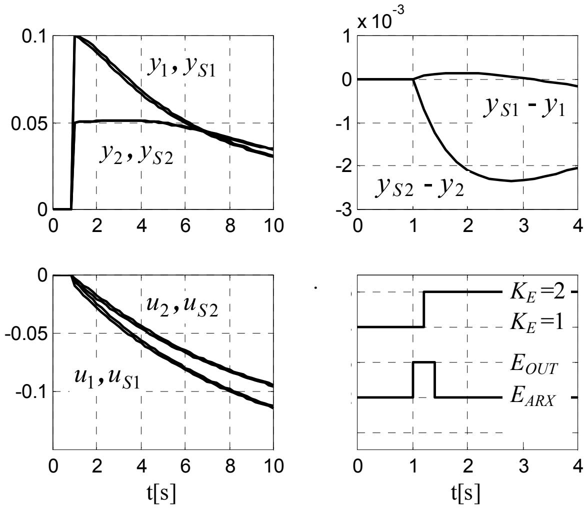

Fig.10. $K_S = 2$, disturbance $\boldsymbol{d}_{OUT} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

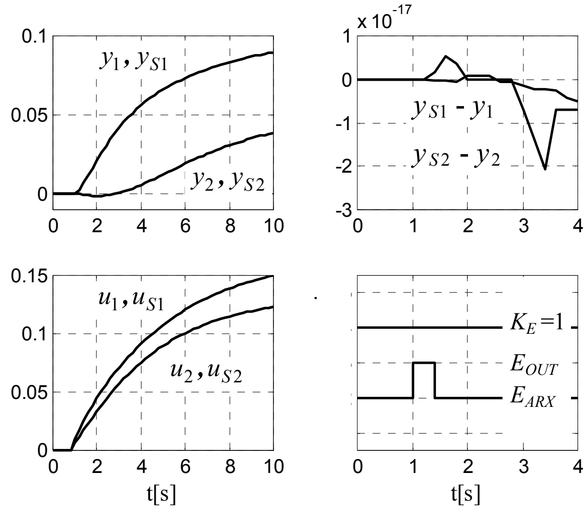

Fig.11. $K_S = 1$, disturbance $\boldsymbol{w} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

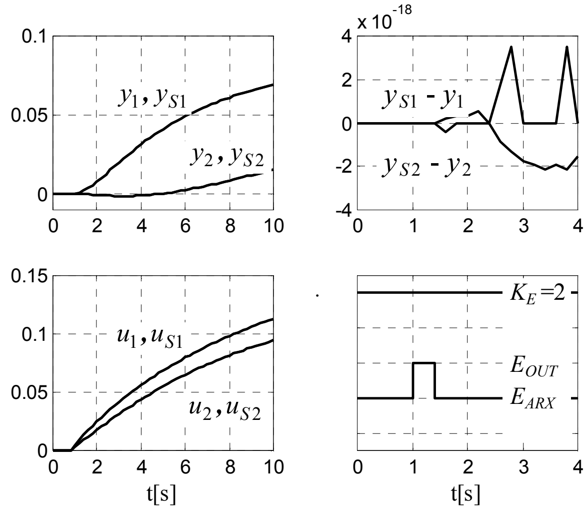

Fig.12. $K_S = 2$, disturbance $\boldsymbol{w} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

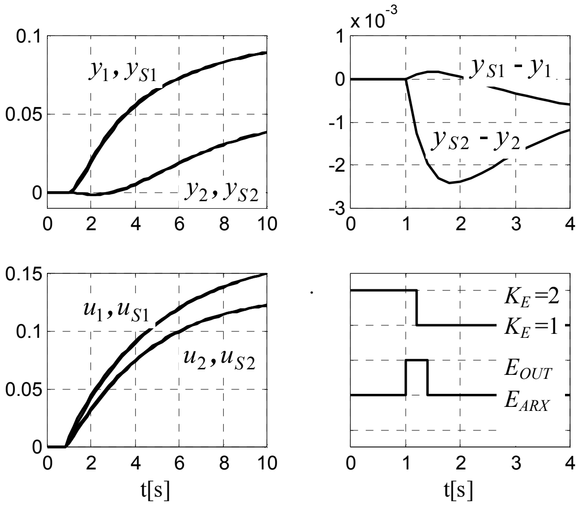

Fig.13. $K_S = 1$, disturbance $\boldsymbol{w} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

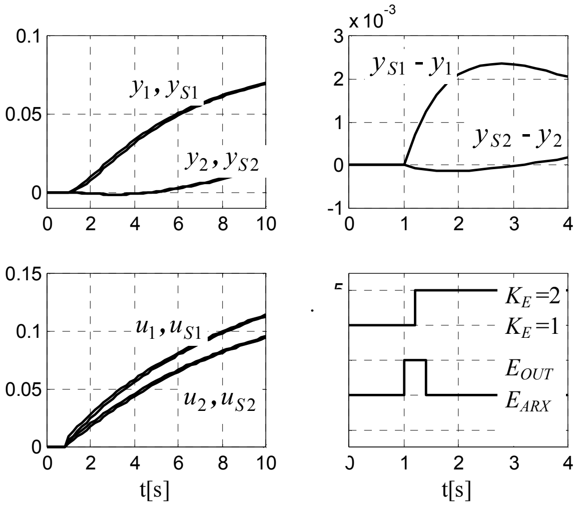

Fig.14. $K_S = 2$, disturbance $\boldsymbol{w} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$.

5. Conclusion

A novel hybrid control architecture based on controller and estimator switching was described and tested using the computer simulation with a model of linear multivariable fourth order system with two nominal models. The control system consists of two blocks with the estimators tuned for the expected disturbances ($\boldsymbol{d}_{IN}$, $\boldsymbol{d}_{OUT}$, $\boldsymbol{d}_{ARX}$), each block with one of the nominal transfer functions. The value of the disturbances is evaluated from the estimator errors in the first sampling instant (using matrix **M**). The type of the disturbance and the nominal model of the controlled system are estimated in the second sampling instant by means of the comparison of the estimator output and of the response of the controlled system. The reference value w changes are controlled without error by means of the estimator of any type. The error values from the following sampling instants can be accumulated for further evaluation. The sampling intervals of the estimators and of the controllers may be different. The testing results are promising and they confirm the expectations. Further testing of this control architecture will be done using the laboratory plant for experiments with hybrid control that was described by the authors in [4].

Acknowledgement:

This research has been supported by the Czech Science Foundation within project 101/04/1182.

References

[1] Fu M., and Barmish B. R., Adaptive stabilization of linear systems via switching control, *IEEE Transactions on Automatic Control*, Vol. 31, 1986, pp. 1097-1103.

[2] Hanuš, B. and Tůma L., Estimators for hybrid multidimensional control system with variable structure, *Proceedings of the 15th International Conference on Process Control*, Štrbské pleso, Slovak Republic May 2005

[3] Hanuš, B. and Tůma, L., Hybrid Control Scheme With Discrete Estimator for Efficient Disturbance Rejection, *Preprints of the 16th IFAC World Congress*, Prague, Czech Republic, July 2005

[4] Hlava J., and Šulc, B., An Experimental Approach to Verification of Control Algorithms for Hybrid Continuous-Discrete Systems, *WSEAS Transactions on Systems*, Vol. 5, No. 11, 2006, pp. 2645-2650.

[5] Liberzon D., *Switching in Systems and Control*, Birkhäuser 2003

[6] Mårtensson, B., The order of any stabilizing regulator is sufficient a priori information for adaptive stabilization, *Systems & Control Letters*, Vol. 6, No. 2, 1985 pp. 87 –91

[7] Morse, A.S., Supervisory control of families of linear set-point controllers part 1: exact matching, *IEEE Transactions on Automatic Control*, Vol. 41, No. 10, 1996, pp. 1413-1431.

[8] A.S. Morse. Supervisory control of families of linear set-point controllers - part 2: robustness. *IEEE Transactions on Automatic Control*, Vol. 42, No. 11, 1997, pp. 1500-1515.

[9] Mosca, E., Capecchi, F. and Casavola, A., Designing predictors for MIMO switching supervisory control, *International Journal of Adaptive Control and Signal Processing Special*, Vol. 15, No. 3, 2001, pp. 265 –286.

[10] Safonov, M.G., and Tsao, T.-C., The unfalsified control concept and learning, *Proceedings of the 33rd Conference on Decision and Control*, pp. 2819 –2824, Dec.1994.