NUMERICAL SOLUTIONS BY USING FNL MODEL OF COMPLEX SHAPE CONCRETE STRUCTURES

SAULIUS VALENTINAVIČIUS, ANTANAS ŠAPALAS, LINA VASILIAUSKIENĖ
Department of Informatics Technology, Department of Graphical Systems,Department of Steel and Timber Structures
Vilnius Gediminas Technical University
Saulėtekio al.11, 10223 Vilnius
LITHUANIA

Abstract. *Various methods of calculation of physical non-linear systems under influence of modelling specific where investigated. Due the flexibility on modelling of complex shape structures by using composite material an incremental model of analysis was chosen. This paper reviews advantages of incremental FNL analysis method*

Keywords: *mechanics, FEM, mesh, FNL, concrete.*

1 Introduction

Advanced analysis of composite material structures poses a problem of non linear nature. This is due fact that dependence of the structure forces state and deformation of the structure itself during the loading process is nonlinear. Basically are two different methods to analyse the effects of second order, which are significant for the structural and/or physical non-linear analyses: the total load method [6] and the incremental method.

By using of a total load method, the structure is analysed for each loading based on total load. The stiffness matrix is modified on displacements, strains or forces until a state of balance has been achieved.

In case of an incremental method the load is increased step by step and analysed until the total load has been achieved. For each loading step the tangent stiffness is modified. The accuracy of the incremental method itself is dependent on number of steps, the influence of such a dependency could be sufficiently improved by using various incremental iterative solutions like Newton – Raphson (NR), modified Newton –Raphson (mNR) and others[1,4].

This article shows advantages of incremental FNL analysis method by modelling of destruction mechanisms on complex shape composite material structures.

2 Numerical approach of FNL calculation method

The physical non-linear aspect has been incorporated by the elementary stress-strain method by meshing 3D-beam structures using 2D-elements and by meshing 2D/1D-beam structures using 1D-elements (slicing). With this method a mesh will be generated along the cross-section (a division in strips). Based on the occurring strains and by means of the matching stress-strain diagrams of materials with integration, the stiffness and static moments are defined as

$$\begin{aligned}
EF &= \int_F E_b dF + \int_{F_a} E_a dF_a; \\
EI_y &= \int_F E_b z^2 dF + \int_{F_a} E_a z^2 dF_a; \\
EI_z &= \int_F E_b y^2 dF + \int_{F_a} E_a y^2 dF_a; \\
ES_y &= \int_F E_b z dF + \int_{F_a} E_a z dF_a; \\
ES_z &= \int_F E_b y dF + \int_{F_a} E_a y dF_a; \\
EI_{yz} &= \int_F E_b yz dF + \int_{F_a} E_a yz dF_a.
\end{aligned} \quad (1)$$

In general case the work law of a FNL model should match the following law:

$a(u) = f;$ (2)

where u – displacement to be calculated,
f – external loading.

The nonlinear function $a(u)$ correspond to calculated example, with $a(0) = 0$. Notation $a'(u)$ is describing derivative of function $a(u)$, so far u_0 - initial calculation point always equal to zero .

There are a few numerical methods to solve the equation $a(u) = f$:

Newton method:

- $a'(u_n)\cdot(u_{n+1} - u_n) = f - a(u_n);$ (3)

Simplified Newton method:

- $a'(u_0)\cdot(u_{n+1} - u_n) = f - a(u_n);$ (4)

Step method:

$a(u_n)\cdot(u_{n+1} - u_n) = (\lambda_{n+1} - \lambda_n)f,$ (5)

with $0=\lambda_0 < \lambda_1 < \ldots < \lambda_n = 1$.

Newton method derived calculation methods have convergence in case of:

$0 < C1 \leq a'(u) \leq C2;$

$-C3 \leq a''(u) \leq 0,$

where C1, C2, C3 – positive defined constants, only related to function a(u). It is proved, that mentioned inequalities are satisfied as truth by FNL calculation case they are satisfying the constitutive conditions. (e.g. σ(ε) diagram of concrete)

In case the mentioned inequalities are not satisfied as it is mentioned in figure 1, convergence will be never reached.

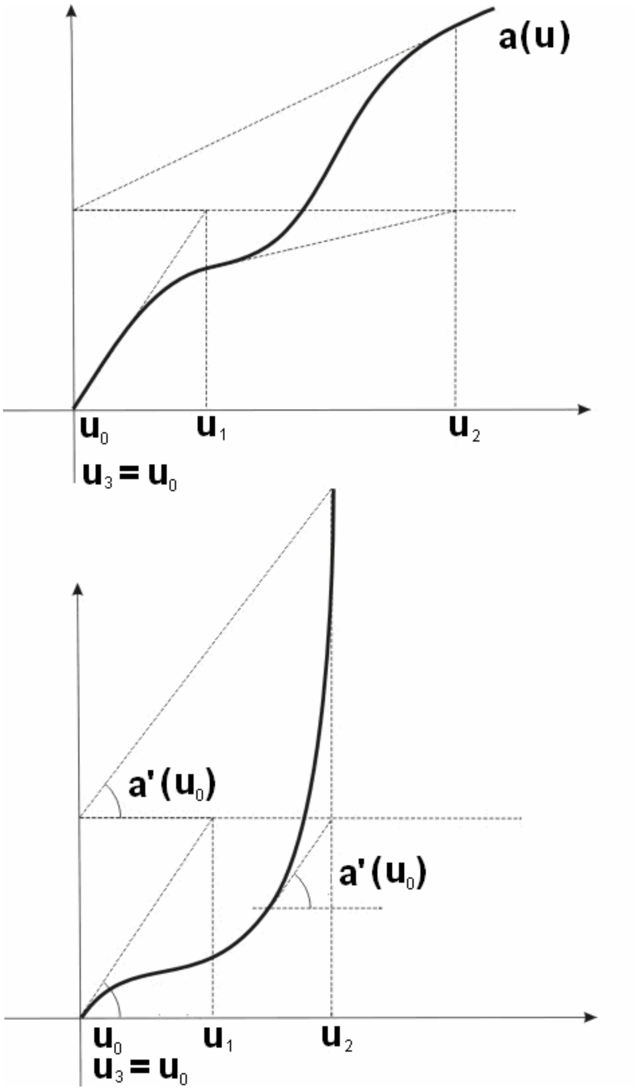


Fig 1. Divergating functions.

Such an effect wouldn't happen by using a simple step-method based FNL calculation, what convergence criteria must be satisfied only as $\text{abs}(a''(u)) \leq C3$ and $0 < C1 \leq a'(u) \leq C2$ conditions.

This method approaches very well the inconsistent curves in the working conditions law M-κ diagram of reinforced concrete (figure 2).

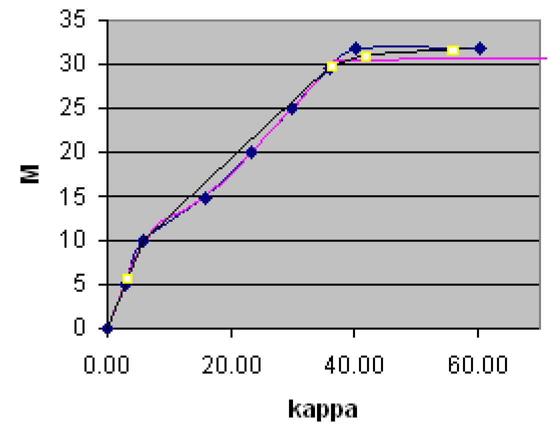


Fig 2. Implementation of working conditions law M-κ for reinforced concrete cross-sections under bending and compression

Because of the powerful influence of the stress-strain method, occurring second-order effects, among which cross-section reduction and the shifting of the neutral line, could be incorporated smoothly in the system as is mentioned in figure 3.

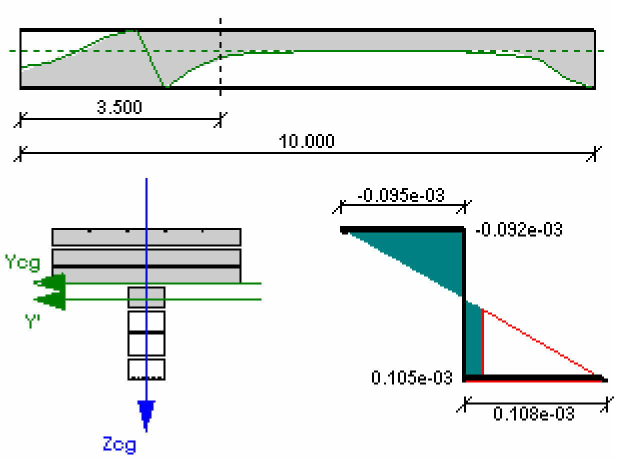


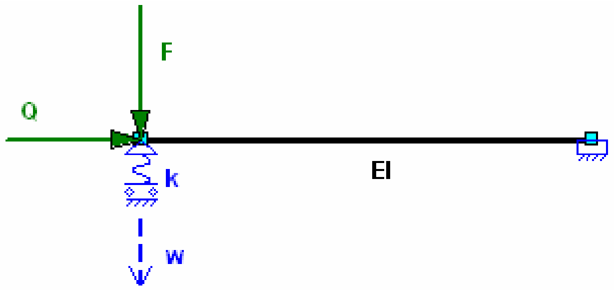
Fig 3. Modelling of cross section

3. Numerical results/evaluation

Numerical calculation results are evaluated based on a number of academic cases especially for the non-linear behaviour[6,7,8,9].

3.1. Structural and Physical non-linear behavior, an academic case

When defining the distribution of forces the deformed position of the structure has to be taken into account.. This second order effect has been tested by running an uncomplicated basic case with familiar results (figure 4). Initially the behaviour of the spring support is linear-elastic with a spring stiffness k [kN/m].



F

ig 4. Testing case for GNL approach

From this structure a first-order analysis is performed (F = 0 kN). The second order analysis (GNL) is performed with the classical theory, based on fourth-order differential equations for bending-buckling, in which the influence of the axial deformation is not taken into account.

With the values shown in fig. 4, this solution is compared with the numerical results of GNL-analysis by using MatrixFrame© software package is developed by the authors of this paper.

The results are shown in fig. 5. This figure represents the relation between the pressed spring and the axial load Q. The final results of the GNL analysis correspond extremely well with the theoretical solution with the estimated error less then 0.1%.

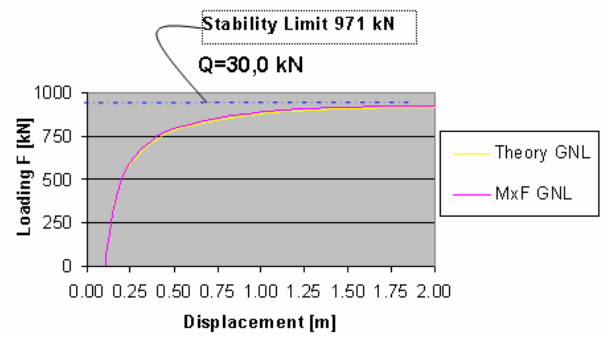


Fig 5. GNL calculation testing results.

When an elastic-plastic spring behaviour (FNL) has been taken into account for the spring support, the ultimate load can be considerable lower than the buckling load.

The results of a physical and structural non-linear analysis with MatrixFrame are shown in figure 5. When the spring support turns into a plastically one, the maximum load F_c has been achieved. In the figure a rough description is given of the downward equation track for larger spring deformations. This track, however, can't be traced since MatrixFrame© is a load driven analysing program. When the critical value Fc of the load has been achieved the analysis stops.

The corresponding deformation of the spring u_c is also shown in figure 6.

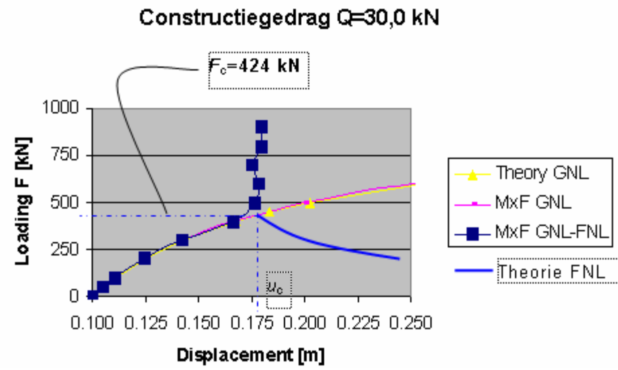


Fig 6. Elasto-plastic geometrical non- linear approach.

3.2 Physical non-linear behavior for concrete cross-sections

A reinforced concrete cross-section shows a highly non-linear behaviour due to cracking or possible plastic deformation of the reinforcement. This is a well known phenomenon which can be made clear by M-N-κ diagrams and by the stress-strain (step) method used by authors. The concrete model due numerical investigation, has been tested by a meshed model (meshed column), for the concrete model under tension and pressure there are several constitutive models available. The tension stiffening behaviour of the cross-section can be modelled as well. A concrete slab with a rectangular b(1000 mm) x h(120 mm) can be used as a verification model (figure 7).

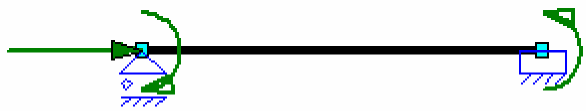


Fig 7. Testing example by using bending moment and axial force.

A value of 22.5 kN is used for the axial force in the test slab. The M-N- κ diagram resulted from the meshed column is shown in figure 2, in which the four characteristic points of the M-N-κ - diagram according to the method of Bruggeling are also marked. For concrete under tension a softening model is used, indicated on the left side of the figure 8. The M-N-κ diagram of the tested slab for the tension softening option is shown in figure 8, the results correspond well with the indicated verifications based on the meshed model.

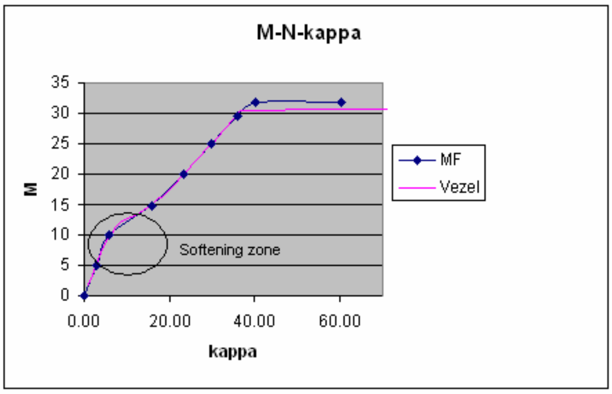


Fig 8. M-N-κ diagram for 2 sided reinforced concrete beam.

Strictly speaking a softening model is only advisable if the analysis procedure also allows exemptions. With the load driven system in MatrixFrame© , however, it's impossible to follow the downward branches in the load-displacement area [3].

For this reason the decision was taken to ignore the softening branch in the constitutive model for concrete under tension. Without this option the constitutive behaviour of the cross-section has no stiffening effect on the concrete part under tension (the tension stiffening) see figure 8. The LE-phase and the ultimate moment of resistance are described correctly with this model.

Apart from the represented tests for non-linear behaviour there are also tests based on structural level in which the structures are loaded till the level of collapse. The occurring ultimate load together with the mechanisms corresponds well with the predicted results[5].

6. Conclusions

During the mentioned investigation the step method of a meshed structure was chosen. This approach was taken as the primary calculation method while it let:

- take in to account detailed redistribution of plastic zones in a plastic hinge;
- smoothly take in to account discontinuous points in working conditions of structures;
- take in to account various forms of cross sections;
- implement composite material in a whole structure as far as in a separate cross section;
- take in to account the shift of the neutral axis during calculation;
- calculate 3D structural models.

References

[1] Hinton, E. *NAFEMS Introduction to Nonlinear Finite Element Analysis, NAFEMS,* 1992, 380 p.

[2] Hinton,E. *Recent Advances in Non-Linear Computational Mechanics,* Pine Ridge Press, 1982, 277 p.

[3] Collins, M.P. and Mitchell, D. *Prestressed Concrete Structures,* Prentice Hall, englewood Cliffs, New Jersey, 1992.

[4] Bruggeling, A.S.G. *Theory and Practise of Reinforced Concrete,* Professor Bakker Fonds, Delft, 1986

[5] NEN 6720, *Regulations for concrete, TGB 1990, Structural requirements and calculation methods*, dec 2001

[6] D. Bačinskas, G. Kaklauskas, E. Geda, *FE software ATENA applications to non-linear analysis of RC beams subjected to high temperatures,* Journal of Civil Engineering and Management. Vilnius: Technika, 2004, Vol X, Suppl 1, p. 11-18.

[7] Welleman, J.W. *Concrete structures under impulsive loads,* TU-Delft report no 25.2.92-2-14, 1992

[8] Welleman, J.W. *Flexure shear interaction in beams under distributed loads,* TU-Delft report no. 25.2.92-2-13, 1992

[9] Welleman, J.W. *Numerical Simulations of RC Panels with the Modified Compression Field theory,* TU-Delft report no. 03.21.1.22.26, 1995

[10] Welleman, J.W. *Dynamic load characteristics and constitutive models for RC beams,* TU-Delft report no. 03.21.1.22.01, 1995