Application of Fuzzy C-Means Clustering in Power System Model Reduction for Controller Design

SHU-CHEN WANG[1] PEI-HWA HUANG[2]
Department of Electrical Engineering
National Taiwan Ocean University
No. 2, Peining Rd., Keelung 20224
TAIWAN

CHI-JUI WU[3]
Department of Electrical Engineering
National Taiwan University of Science and Technology
No. 43, Sec. 4, Keelung Rd., Taipei 10607
TAIWAN

1

Abstract: -This paper presents the application of fuzzy c-means (FCM) clustering in the order reduction of dynamic models for controller design in a power system. Based on the fuzzy c-means algorithm, a method is proposed for clustering the poles and zeros of the original power system model into new clusters from which a reduced-order model can be obtained. Then the reduced-order model is used to design a proportional-integral type power system stabilizer to improve the damping in system oscillation after a system disturbance. The reduced-order model can contain the critical dynamic characteristics of the original model, but let it easier to design the controller. Results from a sample power system are presented to show the validity of the proposed method. The electromechanical mode of the power system can be improved by the designed power system stabilier from pole assignment.

Key-Words: **-Power system dynamics, Model reduction, Fuzzy c-means, Fuzzy Clustering, Pole assignment.**

1 Introduction

Model order reduction concerns the transformation of a higher-order model into a lower-order model through some sort of computation [1, 2]. A certain relationship between these two models is preserved and they are similar in the characteristics under consideration. In power system studies, creating a dynamic model is the first step for system stability research, dynamic behaviors analysis, or other system functional tests. As systems become larger, their complexity increases and power system analysis has to tackle high-order model analysis. However, computation on the high-order model is highly complex while the final analysis results may have unnecessary portions. In this case, having a low-order model that maintains the main characteristics of the high-order systemt can replace the original system and significantly simplify the computational problem [3-13].

If the stability performance of a power system is unable to satisfy the specification, the stabilizing controller can be used to improve the dynamic characteristics. Without stability disturbance compensation, steady-state performance and any other performance index are not possible. Therefore, a stabilizing controller of power system is needed. The most important application of the reduced order model let it easier to design of a suitable controller for the original high-order system. Many methods can used to design a power system stabilizier with output feedback scheme. The pole assignment design allows the power system for the electromechanical mode dynamic to be placed in desired location.

In this paper, the method based on fuzzy c-means clustering analysis [14-20] aims to group poles and zeros of a power system transfer function into some clusters. For each cluster, the original system poles (zeros) can be replaced by each cluster center that becomes the new member representative of the cluster. All new members representing their respective clusters jointly constitute a tentative reduced-order model of the original system. The reduced-order model is used to design a proportional-integral power stabilizer to improve the dynamic stability. The results obtained from a sample power system models will be illustrated and the effectiveness of the method is thus confirmed by the example.

2 Fuzzy c-means Cluster Analysis

The method proposed in this paper utilizes fuzzy c-means clustering (FCM) analysis [14- 16] to reduce the original high-order model into a low-order model. Cluster analysis [17-20], of which the task is to classify non-processed data into certain categories depending on various traits, is a basic tool commonly used in several scientific fields. Data in each category have the most resemblance while being very dissimilar with data from other categories.

Suppose there are n data points $\{x_j\}$, $1 \le j \le n$, to be clustered into c data clusters. Let μ_{ij} denote the degree of membership that x_j belongs to the i th cluster. It is noted that $0 \le \mu_{ij} \le 1$ and $\sum_{i=1}^{c} \mu_{ij} = 1$ for each j. Define the fuzzy partition matrix $U = [\mu_{ij}]$, $1 \le i \le c$, $1 \le j \le n$. Therefore, the objective of the fuzzy c-means algorithm is to determine all the elements of matrix U. The FCM algorithm is essentially an iterative procedure and can be formulated as the following six steps in which l denotes the iteration number.

(a) Set the number of clusters c. Initialize U randomly as $U^{(l)} = [\mu_{ij}]$, $l = 1$, $1 \le i \le c$.

(b) Compute the cluster center c_i of each cluster:

$$c_i = \frac{\sum_{j=1}^{n} \mu_{ij}^m x_j}{\sum_{j=1}^{n} \mu_{ij}^m} \quad (1)$$

Note that the value of m normally falls in the range of $1.5 \le m \le 3$.

(c) Select the weighting w_j of every data point, then the weighted data point W_j as

$$W_j = x_j \times w_j \quad (2)$$

(d) Compute the distance d_{ij} between the j th data point and the i th cluster center:

$$d_{ij} = \left\| c_i - W_j \right\| \quad (3)$$

(e) $l = l + 1$. Compute μ_{ij} in $U^{(l)}$ as

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} (d_{ij} / d_{kj})^{2/(m-1)}} \quad (4)$$

(f) If $\left| U^{(l+1)} - U^{(l)} \right| \le \varepsilon$, a preset accuracy, then stop; otherwise, return to Step (b).

It is worth noting that in the above algorithm, the cluster center c_i of each cluster is referred to as the prototype of the cluster and can be considered as the representative of that cluster.

3 Design Method

Given a state space linear model, dynamic characteristics of the system can be best revealed from its poles and zeros. The following steps comprise the proposed model reduction method and controller design.

<u>*Step1:*</u>

After configuring all the parameters of the power system and linearizing the system state equations, the following system dynamic equations are obtained as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (5)$$

where A, B, C are the state, input, and output matrices of the system; x, u and y denote the state, input and output vectors, respectively.

<u>*Step2:*</u>

From Equation (5), the transfer function is found to be

$$\begin{aligned} G(s) &= C(sI - A)^{-1} B \\ &= \frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n} \end{aligned} \quad (6)$$

Based on the transfer function, the poles and zeros can be computed.

<u>*Step3:*</u>

Using the fuzzy c-means algorithm, it can cluster separately the poles and zeros in the complex plane to obtain the corresponding cluster centers. In order to keep the system oscillatory behaviors, poles with and without imaginary parts are clustered into distinct groups, and zeros are processed likewise.

<u>*Step4:*</u>

The calculated cluster centers replace the respective groups of poles and zeros of the original system and collectively constitute the set of poles and zeros for the reduced-order model. The tentative reduced-order model transfer function is thus set as

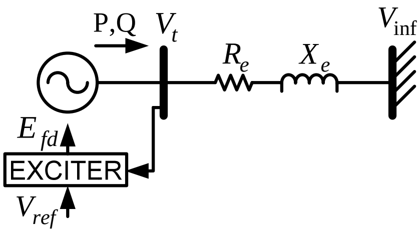

Figure 1. Single-machine infinite bus power system

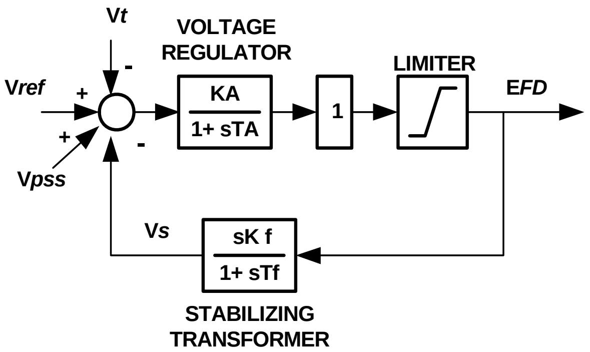

Figure 2. Block diagram of static excitation system

Table 1 The parameters of generator

X_{d} = 2pu	$X_d^{'}$ = 0.244pu	$T_{do}^{'}$ = 4.18sec
X_{q} = 1.91pu	$X_q^{'}$ = 0.17pu	$T_{qo}^{'}$ = 0.55sec

Table 2 The parameters of static excitation system

K_{A} = 400	T_{A} = 0.05
K_{F} = 0.025	T_{F} = 1.0

$$R(s) = \frac{\prod_{i=1}^{t}(s+z_i)}{\prod_{i=1}^{r}(s+p_i)} \tag{7}$$

Step5:

In order to make the time response of the reduced-order model compatible with that of the original higher order model, a gain adjustment factor defined by

$$k = \left.\frac{G(s)}{R(s)}\right|_{s=0} \tag{8}$$

is used to adjust the steady state value of the reduced order model.

Step6:

The parameters of a proportional-integral power system stabilizier (PSS) are to be determined. The power system stabilizier has the transfer funtion as

$$V_{PSS} = k_P\omega + \frac{k_I}{s}\omega \tag{9}$$

Then the closed-loop transfer function of the system is

$$\frac{y}{v_{PSS}} = \frac{sG(s)}{s - sk_P G(s) - k_I G(s)} \tag{10}$$

4 Example

Consider the single-machine infinite bus power system shown in Figure 1.

The generator can be represented by the two axis model. The equations are obtained:

$$\dot{E}_d^{'} = \frac{1}{T_{qo}^{'}}[-E_d^{'} - (X_q - X_q^{'})I_q] \tag{11}$$

$$\dot{E}_q^{'} = \frac{1}{T_{do}^{'}}[E_{FD} - E_q^{'} - (X_d - X_d^{'})I_d] \tag{12}$$

The parameters of generator are shown in Table 1.

The block diagram of static excitation system is displayed in Figure 2. The parameters of static excitation system are shown in Table 2.

Based on the above-described method, the reduced order model and the controller design for the study system is obtained as follows:

Step1:

Choose the state vector x as

$$x^T = \begin{bmatrix} \Delta E_d^{'} & \Delta E_q^{'} & \Delta\omega & \Delta\delta & \Delta E_{FD} & \Delta V_S \end{bmatrix}$$

The definitions for each state variable are

$\Delta E_d^{'}$	direct-axis transient voltage
$\Delta E_q^{'}$	quadrature-axis transient voltage
ω	speed
δ	rotor angle
E_{FD}	exciter output voltage
V_S	stabilizier transformer output voltage

The system matrices are

$$A = \begin{bmatrix} -8.94 & 0 & 0 & -2.79 & 0 & 0 \\ 0 & -1.19 & 0 & -0.93 & 0.239 & 0 \\ -0.136 & -0.34 & 0 & -0.367 & 0 & 0 \\ 0 & 0 & 377 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & -800 \\ 0 & 0 & 0 & 0 & -0.5 & -201 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 8000 & 200 \end{bmatrix}$$

$$U = \begin{bmatrix} \Delta V_{ref} \end{bmatrix}$$

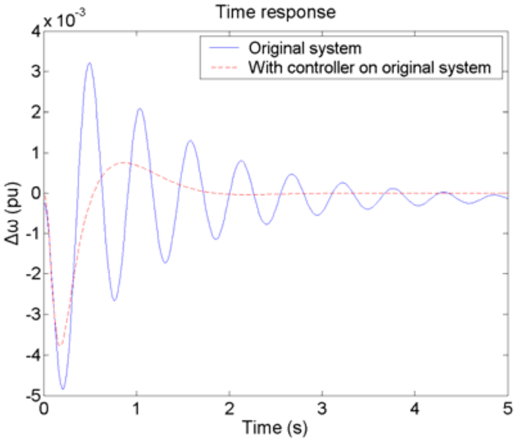

Figure 3. Comparison the original system and the reduced model

Table 3 Poles and zeros of the original model

Poles	Zeros
-220.9	-8.940
-8.302	-1.000
-0.213	0.000
-0.091	$-5.030 \pm j7.562\times10^{7}$
$-0.808 \pm j11.53$	

Table4. Poles and zeros of the reduced model

Clustered Poles	Clustered Zeros
-28.75	$-4.875 \pm j7.562\times10^{7}$
$-0.808 \pm j11.53$	

Table 5 Electromechanical modes of the power system

Eigenvalue without PSS	Eigenvalue with PSS
$-0.808 \pm j11.53$	$-2 \pm j11$

Step2:

The transfer function of the original model is calculated as

$$G(s)=\frac{-1.137\times10^{-13}s(s+1.000)(s+8.940)(s+5.030\pm j7.562\times10^{7})}{(s+0.808\pm j11.53)(s+220.9)(s+8.302)(s+0.213)(s+0.091)}$$

The poles and zeros of the original model are displayed in Table 3.

Step3:

Using fuzzy c-means algorithm, the poles and zeros of the original models are processed to obtain some cluster centers to be used for representing the original poles and zeros.

Table 4 shows the poles and zeros after clustering. In Table 4, the poles (-28.75) are obtained from clustering the poles of the original model, (-220.9), (-8.302), (-0.213), and (-0.091). The poles ($-0.808\pm j11.53$) of the electromechanical mode are retained. Regarding the zeros, the clustered zeros are ($-4.875\pm j7.562\times10^{7}$).

Step4:

The cluster center is obtained after computation and is used to replace the poles and zeros of the original system to become the reduced model. The tentative transfer functions for the reduced model are

$$R(s)=\frac{-1.137\times10^{-13}(s+4.875\pm j7.562\times10^{7})}{(s+28.75)(s+0.808\pm j11.53)}$$

Step5:

The gain adjustment factor is used to adjust the system response to make the reduced-order model compatible with the original model. For the study system, the gain adjustment factors are calculated as

$$k=\left.\frac{G(s)}{R(s)}\right|_{s=\lambda}=0.135$$

where λ is the electromechanical mode. After the above steps, the transfer function of the reduced order model is $R'(s)=kR(s)$ which is given below

$$R'(s)=\frac{1.530\times10^{-14}(s+4.875\pm j7.562\times10^{7})}{(s+28.75)(s+0.808\pm j11.53)}$$

Step6:

The reduced-order model is used to design a proportional-integral power system stabilizer. If the electromechanical mode of the closed-loop system is to be assigned at $\lambda=-2\pm j11$, the parameters of power system stabilizier are obtained as

$$\begin{bmatrix}k_P & k_I\end{bmatrix}=\begin{bmatrix}-7.112 & -89.80\end{bmatrix}$$

From Table5, the electromechanical mode of of the original model system are obviously improved.

The time responses of the original system with and without controller after a small distrubance is shown in Figure 3.

5 Conclusion

A model reduction method for reducing the order of power system dynamic models in controller design has been proposed in this paper. Based on the fuzzy c-means algorithm, the proposed method performs clustering on the poles and the zeros of the original system model into new clusters from which a reduced-order model can be derived. The reduced-order model that maintains the main characteristics of the high-order system can significantly simplify the design of a power system stabilizer. Results from applying the method to a sample power system have been demonstrated to show the validity of the proposed method.

Acknowledgments

This work was supported in part by the National Taiwan Ocean University and the National Science Council of Taiwan.

References:

[1] A. Bergen, *Power Systems Analysis*, Prentice Hall, New York, 2000.

[2] D. Trudnowski, Order reduction of large-scale linear oscillatory system models, *IEEE Trans. on Power Systems*, Vol. 9, No. 1, 1994, pp. 451-458.

[3] F. Saleh and M. Mahmoud, Design of power system stabilizers using reduced-order models, *Electric Power System Research*, Vol. 33, 1995, pp. 219-226.

[4] A. Feliachi, X. Zhang and C. Sims, Power system stabilizers design using optimal reduced order models, *IEEE Trans. on Power Systems*, Vol. 3, No. 4, 1988, pp. 1670-1684.

[5] R. Castro and J. de Jesus, A wind park reduced-order model using singular perturbations theory, *IEEE Trans. on Energy Conversion*, Vol. 11, No. 4, 1996, pp. 735-741.

[6] N. Nihei and T. Oyama, A study on decomposition and model reduction for wide area power system stability assessment, *Power Engineering Society 1999 Winter Meeting of the IEEE*, Vol. 1, 1999, pp. 651-654.

[7] N. Sinha and J. Pal, Simulation based reduced order modeling using a clustering technique, *Computer & Elect. Engineering*, Vol. 16, No. 3, 1990, pp. 159-169.

[8] N. Sinha, Reduced-order models for linear systems, *Proc. of the IEEE Conference on Systems, Man and Cybernetics*, 1992, pp. 537-542.

[9] M. Duric, Z. Radojevic and E. Turkovic, A reduced order multimachine power system model suitable for small signal stability analysis, *Electrical Power & Energy Systems*, Vol. 20, No. 5, 1998, pp. 369-374.

[10] G. Obinata and B. Anderson, *Model Reduction for Control System Design*, Springer, 2001.

[11] P. Benner, R. Mayo, E. Quintana-Orti, and G. Quintana-Orti, A service for remote model reduction of very large linear systems, *Proc. of the International Parallel and Distributed Processing Symposium*, 2003, pp. 22-26.

[12] H. Ukai, H. Masubara, M. Kobayashi, and H. Kandoh, Stabilizing control of series capacitor compensated power system on the basis of reduced order model, *Proc. of the 4th International Conference on Advances in Power System Control, Operation and Management*, 1997, pp. 603-608.

[13] F. Saleh and M. Mahmoud, Design of power system stabilizers using reduced-order models, *Electric Power System Research*, Vol. 33, 1995, pp. 219-226.

[14] N. Pal, K. Pal, J. Keller, and J. Bezdek, A possibilistic fuzzy c-means clustering algorithm, *IEEE Trans Fuzzy Systems*, Vol. 13, No. 4, 2005, pp. 517–530.

[15] S. Nascimento, B. Mirkin and F. Moura-Pires, A fuzzy clustering model of data and fuzzy c-means, *Proc. of the IEEE Conference on Fuzzy Systems*, 2000, pp. 302-307.

[16] J. Bezdek, J. Keller, R. Krisnapuram, and M. Pal, *Fuzzy Models and Algorithms for Pattern Recognition and Image Processing*, Kluwer Academic, 1999.

[17] F. Hoppner, F. Klawonn, R. Kruse, and T. Runkler, *Fuzzy Cluster Analysis: Methods for Classification, Data Analysis and Image Recognition*, John Wiley & Sons, 1999.

[18] M. Aldenderfer and R. Blashfield, *Cluster analysis*, Sage Publications, Beverly Hills, 1984.

[19] L. Kaufman and P. Rousseeuw, *Finding Groups in Data: An Introduction to Cluster Analysis*, John Wiley & Sons, 1990.

[20] J. Chiang and Y. Chen, Incorporating fuzzy operators in the decision network to improve classification reliability, *Computer & Elect. Engineering*, Vol. 28, 2002, pp. 547-560.