Support Position Optimization of Structural Fundamental Frequency Using Genetic Algorithm

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Abstract: An new generalized genetic algorithm was presented to optimize the support positions of structures in this paper. Many new theories such as integer code, real code, population isolation, arithmetical crossover and unequal random mutation were used. Numerical examples demonstrate that this generalized genetic algorithm can optimize support positions as both elastic and bending rigidity are considered. The method presented in this paper has extensive applicability in complicated optimization problem of support positions.

Key-Words: Genetic algorithm, Support position, Optimization of structure, Finite element

1 Introduction

Optimizing the support positions of a structure can improve the performances of structures,It is a usual engineering request for a structure to optimize the support posittions to maximize its fundamental frequency.Optimizing the support positions is widely used in many areas such as architecture, shipping industry and aircraft industry. Akesson and Olhoff [1] described the minimum stiffness, or critical stiffness, of an additional lateral support for maximum fundamental frequency of a cantilever beam. Hou and Chuang [2, 3] developed eigenvalue sensitivity equations with respect to the position of an intermediate simple support using both domain and boundary methods.Additionally, they used the material derivative concept to find the optimal position of an intermediate support for beams. Wang [4] used the classical normal modal method to derive the closed-form frequency sensitivity of an Euler–Bernoulli beam with regard to a support position by treating the support reaction as an external excitation imposed on a restrained structure.

Based on the Rayleigh’ s principle of stationary values, Liu *et al.* [5] derived the closed-form formulas for the frequency sensitivity using the Rayleigh quotient of the vibration system in conjunction with the Lagrange multiplier. Won and Park [6] illustrated that the optimal support positions rely heavily on the support stiffness. They pointed out that in many cases, the optimal support position is not unique for plates once the support is stiff enough. Sinha and Friswell [7] may be the first to apply the shape functions of an element to produce the global stiffness matrix of a support located within a beam element.With the preceding achievements, one can now predict the effect of a support movement on the structural frequencies without trial and error. WANG Dong [8] optimize the positions of simple or point supports to maximize the fundamental frequency of a beam or plate structure. Both elastic and rigid supports are taken into account. The supports are assumed to be massless, hold the structure at the nodes of the finite element (FE) model and act only on the transverse displacements of the supported points.

The previous approaches to optimize the positions of supports have the shortcoming that only the longitudinal stiffness of elastic support be considered.The main method to solve the optimization of support position is to obtain the frequency sensitivity equations with respect to the position.However, many supports of engineering structures not only provide the longitudinal stiffness but also cause bending stiffness.Many elastic supports can be expressed as the case shown in Fig.1.For this elastic support with bending stiffness the optimization of its support position is difficult to use mathematic optimal method because the frequency sensitivity with respect to the position is difficult to be obtained

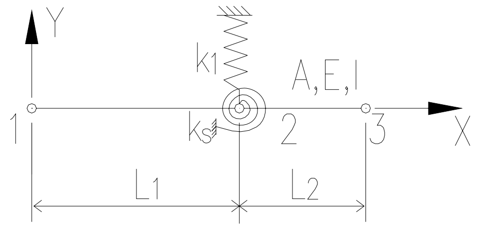

Fig.1 An elastic support with longitudinal and bending stiffness

Genetic algorithm is an optimization method based mainly on the concepts of natural selection and evolutionary process. The stochastic nature of the method and using a population of design points in each generation usually give rise to the global optimum. The details of the method can be found in many literatures. The classical genetic algorithm uses integer code, which has the disadvantages such as slow convergence and lower computational precision.In this paper the generalized genetic algorithm with real code is used to optimize the position of elastic supports to maximize the fundamental frequency of structures.

2 The outline of generalized genetic algorithm

2.1 creating seeds using population isolation

The whole parametric space is divided as several subdomains,each seed is created independently and synchronistically in every subdomain. Population isolation can ensure that the algorithm converges speedy to the global optimal solution and avoids the early mature.

2.2 Selection operator

The method of 2/4 alternative optimum is used, in which the parent generation is permitted to compete with child generation in crossover and mutation and the fine individuals of parent generation step into the next generation.Using this method can endure that the algorithm is stabe and has the capability to realize local optimum.

2.3 Crossover operator

In the paper the arithmetic crossover operator is used. Suppose $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_j^t$ are two individuals that will cross in the $\boldsymbol{t}$ th generation，After crossing, they become as follows

$$\begin{cases} x_i^{t+1} = x_i^t + \tau_1(x_i^t - x_j^t) \\ x_j^{t+1} = x_j^t + \tau_2(x_i^t - x_j^t) \end{cases} \quad (1)$$

where $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are random numbers distributed uniformly in [-1,1].

arithmetic crossover operator can ensure that the searching space covers the whole neighborhood of $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_j^t$, and x_i^{t+1} and x_j^{t+1} are between $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_j^t$ with large probability.

2.4 Mutation operator

The nonuniform and random mutation is used in the paper.The child generation after mutation can be expressed as follows

$$\boldsymbol{x}_i^{t+1} = \boldsymbol{x}_i^t + \tau_3 \boldsymbol{b} \quad (2)$$

whereare $\boldsymbol{\tau}_3$ is a random number distributed uniformly in [-1,1], $\boldsymbol{b}$ is the radius of mutation operator.

This mutation operator can ensure that $\boldsymbol{x}_i^{t+1}$ is obtained in the neighborhood $\boldsymbol{U}(\boldsymbol{x}_i^t, \boldsymbol{b})$ of $\boldsymbol{x}_i^t$. Radius $\boldsymbol{b}$ is a variable with the number of cycles ,expressed as follows

$$\boldsymbol{b} = \frac{1}{2^c} \boldsymbol{b}_s \quad (3)$$

where $\boldsymbol{c}$ is the current number of cycles， $\boldsymbol{b}_s$ is the maximal value of mutation.The radius of mutation is gradually minished.

3 Main steps of the generalized genetical algorithm

The main steps of the generalized genetic algorithm employing above are as follows

(1)Initialize all variables in the algorithm.

(2)Initialize a population using population isolation.

(3)Evaluate the fitness value of each individual in the population. If stopping criterion is met,the cycle is exited,otherwise go to next step.

(4)Select a half individuals of the population to apply crossover.

(5)Selecting a half individuals from the child generation created by crossover and parent generation which was applied crossover according to selection operator combines the other half individuals of the parent generation which are not applied crossover to form a middle population.

(6)Select randomly a half individuals of the middle population to apply mutation
(7)Selecting a half individuals from the child generation created by mutation and parent generation which was applied mutation according to selection operator combines the other half individuals of the middle population which are not applied mutation to form a new population.
(8)return step (3).

4 Examples

A uniform cantilever beam of length L = 10m, with a lumped mass attachment m = 500 kg at its mid-point, is shown in Figure 2. The beam, discretized with 20 regular beam elements, is required to maximize its first frequency by introducing an additional support. The cross-section of the beam is a square with its side of H = 0.2m. Young' s modulus is E = 210 GPa and material density is ρ = 7800 kg/m^3.This cantilever beam is the same as that in the paper [8] in which only the longitudinal stiffness of elastic support was taken into account,but in the paper the bending stiffness of elastic support will be taken into account. To execute the optimization procedure, the FE method is utilized as an analyser to calculate the natural frequency and associated vibration mode.The first natural frequency of the cantilever beam without an additional support is f_1=1.6164Hz, the second natural frequency f_2=9.1686Hz.We will introduce the optimizing process considering two cases due to different supports.

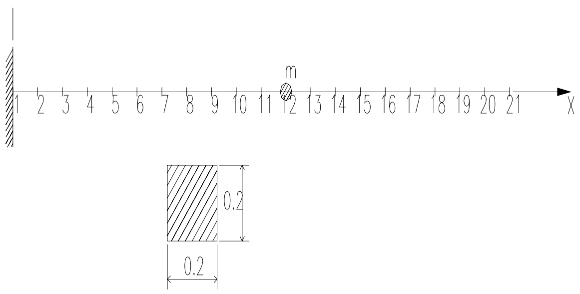

Fig.2 Finite element model of uniform cantilever beam

4.1 Elastic support without bending stiffness

As shown in Fig.1, Suppose the elastic support has no bending stiffness and its longitudinal stiffness is $\boldsymbol{k} = 2.8\times10^{6}\,\boldsymbol{N/m}$.Real code is used in the generalized genetic algorithm.

As shown in Fig.2, The position of the support is located by its x-coordinate that the zero is at the cantilevered end of the beam.,so the x-coordinate of the support is the design variable. We Use population isolation to select four seeds as 1.772m、2.833m、6.239m and 8.660m，calculating the corresponding fundamental frequencies and substituting them into the program of the generalized genetic algorithm. After four cycles,the optimal position of the support is obtained as $\boldsymbol{l}_{opt} = 8.244\boldsymbol{m}$,and the maximal fundamental frequeny is $\boldsymbol{p}_{opt} = 6.6292\,\boldsymbol{Hz}$.Fig.3 shows the process of the optimization.

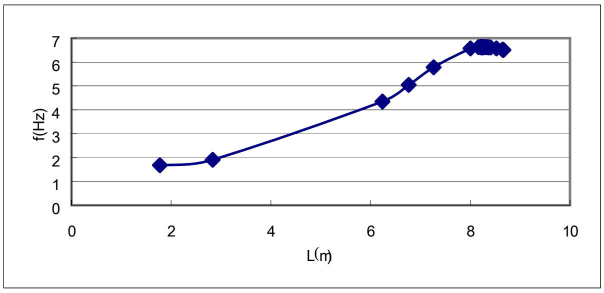

Fig.3 Support position/fundamental frequency curve

From Fig.3 it can be seen that the calculating points in the area that is far from the optimum value are sparse,but ones in the neighborhood of the optimum value are very dense, which shows the characteristic that the generalized genetic algorithm can rapidly reach the neighborhood of the optimum value and carry through local seeking.
Fig.4 and Fig.5 show the optimal results for another stiffness when there are eleven different stiffness between $2.8\times10^{6}\,N/m$ *and* $8.4\times10^{6}\,N/m$.

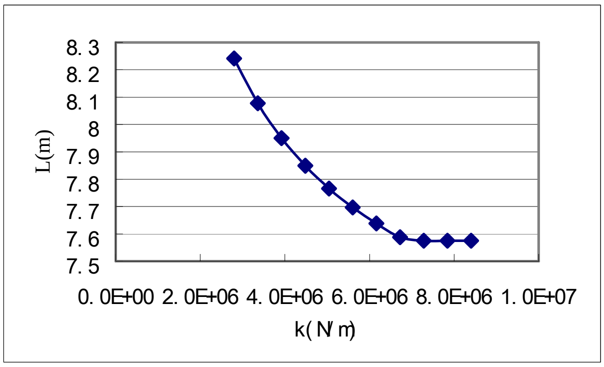

Fig.4 Support rigidity/optimum support position curve

It can be seen from Fig.4 and Fig.5 that the optimum position of support and the fundamental frequency depend on the stiffness of elastic support without bending stiffness,the fundamental frequency approaches to the second natural frequency f_2=9.1686Hz of the cantilever beam without the additional support and the optimum position of support approaches to the node point of the second mode of the original structure. When the stiffness is large enough (over critical stiffness nearly 7×10^6 N/m), the optimal position is at the node point exactly,and the fundamental frequency reaches its maximal limit f_2=9.1686Hz,even though the support become rigid. These results are same as that in the paper [8].

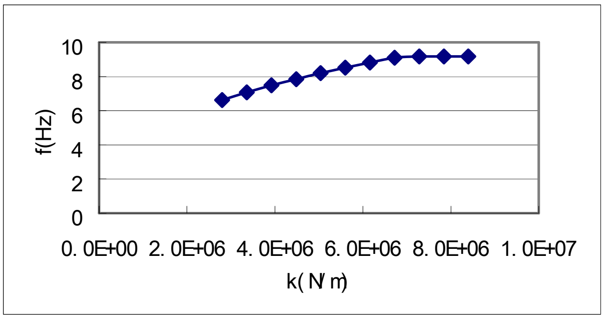

Fig.5 Support rigidity/optimum fundamental frequency curve

4.2 Elastic support with bending stiffness

It must be clear in engineering that a support has or has not bending stiffness,which will affect the performances of structures.However the genetic algorithm is the same for the support with or without bending stiffness,which shows the adaptability of genetic algorithm.In this example the longitudinal stiffness of the elastic support is the same as first example but there is a bending spring,the bending stiffness is $\boldsymbol{k_\theta} = 2.8\times10^6\,\boldsymbol{N\cdot m/rad}$ .As the same in the first example,the longitudinal stiffness is divided uniformly eleven points between $2.8\times10^6\,N/m$ *and* $8.4\times10^6\,N/m$,optimizing the positin of support to maximize the fundamental frequency.

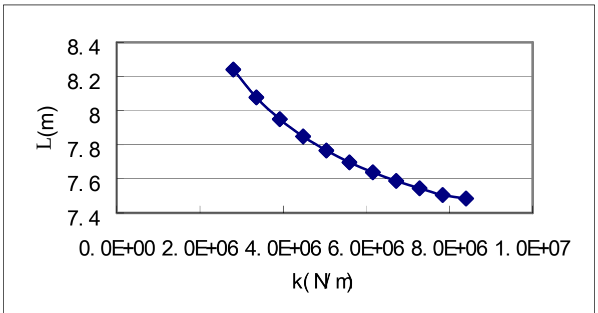

Fig.6 Support rigidity/optimum support position curve

Fig.6 and Fig.7 show the optimal position and fundamental frequency vary with the longitudinal stiffness of support which has a bending stiffness $\boldsymbol{k_\theta} = 2.8\times10^6\,\boldsymbol{N\cdot m/rad}$.

It can be seen from Fig.6 and Fig.7 that the optimal position of support moves toward the cantilever end and the fundamental frequency becomes larger and exceeds the second natural frequency of the original structure with the increase of longitudinal stiffness.

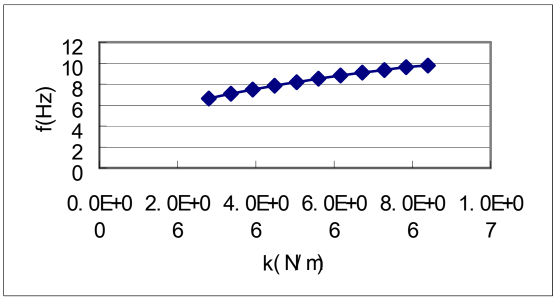

Fig.7 Support rigidity/optimum fundamental frequency curve

Changing the bending stiffness,another optimal results can be obtained.For small longitudinal stiffness the optimal position is at the point between the node point of second mode of the original structure and the free end, increasing the bending stiffness will make the fundamental frequency increased but less than the second natural frequency of the original structure and the optimal position moves toward the node point..When the longitudinal stiffness is larger than the critical stiffness, the fundamental frequency will exceed the second natural frequency of the origianal structure,the optimal position will pass through the node point of the second mode of the original structure and moves toward the cantilever end slowly with the increasing of bending stiffness.When the longitudinal stiffness is equal to the critical stiffness,the optimal position is at the node point of second mode of the original structure and the bending stiffness will not affect the optimal results.

5 Conclusion

In this paper we find the optimization of position of a elastic support with bending stiffness is difficult to use the mathematical optimal method through the sensitivities.The generalized genetic algorithm was presented to solve the optimization of position of a elastic support with bending stiffness. The validity of the method was proved by the examples.Future work will extend the approach to the case there are more supports.

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