Fuzzy Dynamic Characteristic of Concrete Material under Impact Loads

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Abstract:- For different impact velocity (high, middle and low), the concrete material shows different characteristics. To describe such a characteristic effectively, the concept of fuzzy dynamic characteristic of concrete material is presented. Based on this concept, a fuzzy model to predict the penetration is given and a total penetration process of a projectile against semi-infinite concrete target is analyzed and calculated. From the comparison with the tests, the good agreement is obtained.

Keywords: concrete; fuzzy characteristic; fuzzy model; penetration

1 Introduction

For a semi-infinite concrete target or a thick concrete slab, the instantaneous penetration velocity of a projectile will be decreased until to zero because of the effect of resistance applied on the projectile-nose. For different penetration velocity, the concrete material shows different characteristics.

In general, the resistance of the concrete target applied on the projectile consists of two parts. One is from the inertial effect, the other is from the strength effect of the material. The inertial resistance depends mainly on the mass of the material (i.e. the density of material). The strength resistance depends mainly on the strength of the material including the dynamic strength. When the striking velocity is high, the great majority of resistance is the inertial force. When the striking velocity is middle, in addition to inertial force, the majority of resistance includes also the dynamic strength force. With the decrease of the striking velocity, the strength force becomes more major. In the point of view of military engineering, the high velocity ranges from 800m/s to 1500m/s, the middle velocity ranges from 100m/s to 800m/s, and the low velocity ranges from 0m/s to 100m/s. In this paper, the striking velocity is confined in the range mentioned above, the velocity more than 1500m/s is beyond the scope of this study.

The concrete material is characterized by brittle nature which differs from isotropic tenacious metal material. It has a great difference between the compressive strength and the tensile strength. In addition, compressibility of concrete plays an important role in the deformation for high pressure and high striking velocity because of the existence of the air voids. It is shown from a great deal of theoretical and experimental investigations by previous author that, for high striking velocity, the concrete material shows the characteristic which is the same as ideal compressible fluids; for middle striking velocity, it shows the characteristic of ideal or strain-hardening plastic solids; and for low striking velocity, it shows the characteristic of general elastic-plastic solids. Therefore, for the total procedure of deep penetration, for different velocity ranges, different models must be used to predict the total penetration effectively.

In view of the difference in resistance between inertial effect and the strength effect for different striking velocities, a fuzzy model of resistance is presented in this paper.

2 Description of the fuzzy model

During the penetration, the resistances of target applied on the projectile are mainly from the normal pressure and consequent slide friction force on the surface of the projectile-nose. The normal pressure consists of the following four portions. The first is the static strength. The second is the dynamic strength. The third is the solid inertia. The fourth is the fluid inertia. The general expression of normal compressive stress can be written by

$$\sigma_{sum} = \sigma_{ss} \oplus \sigma_{ds} \oplus \sigma_{si} \oplus \sigma_{fi} \qquad (1)$$

where σ_{ss} is the static strength stress, σ_{ds} is the dynamic strength stress, σ_{si} is the solid inertial compressive stress, and σ_{fi} is the fluid inertial compressive stress. $\oplus$ stands for logical sum. Equation (1) can not be arithmetic sum because σ_{si} and σ_{fi} come from different models. That is, the concrete material can not be meanwhile (in the same condition) considered as both solid and fluid. To obtain an accurate correct model about compressive stresses, the concept of fuzzy dynamic characteristic of concrete material is established. By means of the methods of fuzzy mathematics, we define a variable μ as the grade of membership of the concrete material. This variable can represent a degree belonging to which deformation and damage (that is, dynamic fluids damage, dynamic solids damage, static solids damage, etc.) the material will subject to, when it is penetrated by a projectile.

By means of the grade of membership of the material, the arithmetic form of equation (1) can be rewritten as

$$\sigma_{sum} = \mu_{ss}\sigma_{ss} + \mu_{ds}\sigma_{ds} + \mu_{si}\sigma_{si} + \mu_{fi}\sigma_{fi} \qquad (2)$$

where μ_{ss}, μ_{ds}, μ_{si} and μ_{fi} are components of the vector of the grade of membership. They are grades of membership of static elastic-plastic solid strength damage, dynamic elastic-plastic solid strength damage, dynamic elastic-plastic solid inertial damage and dynamic fluid inertial damage, respectively. They are satisfied by the following normalized relationships

$$\mu_{ss} + \mu_{ds} = 1 \qquad (3a)$$

$$\mu_{si} + \mu_{fi} = 1 \qquad (3b)$$

The grade of membership $\mu = \{\mu_{ss}, \mu_{ds}, \mu_{si}, \mu_{fi}\}$ depends mainly on the striking velocities.

For high velocity impact, the concrete material can be considered as ideal compressible fluid [1], the grade of membership can be taken as,

$$\mu = \{0.0, 1.0, 0.0, 1.0\} \qquad (4a)$$

For low striking velocities, the concrete material can be considered as general elastic-plastic solid. The quasi-static strength plays an important role in the resistant force. The grade of membership can be taken as

$$\mu = \{1.0, 0.0, 1.0, 0.0\} \qquad (4b)$$

For middle striking velocity, the concrete material can not be simply considered as ideal compressible fluid or elastic-plastic solid. Instead of, it is necessary to consider the concrete as a fuzzy material. The grade of membership must be taken as

$$\mu = \{\mu_{ss}, \mu_{ds}, \mu_{si}, \mu_{fi}\} \qquad (5)$$

where the components must satisfy the normalized relationship (3a) and (3b). Their further analysis will be made below.

3 Analyses of normal compressive stresses

Much effort has been directed at modeling the resistance of concrete target on the projectile-nose. The empirical formulae include Petry formula, ACE formula, NDRC formula, BRL formula, etc. [2,3,4]. The analytical formulae are derived mainly by Forrestal, Luk, Brar, etc. [5,6,7,8,9,10,11] based on the dynamic cavity expansion. Their formulae are considerably similar each other. They all consist of two portions. One is the static resistance, the other is the inertial resistance of solid. The empirical formulae are based on the dimensional analysis and test investigation. The analytical formulae of dynamic cavity-expansion are based only on the assumption of spherical (or cylindrical) cavity and on the assumption of constant-velocity expansion. In addition, this analytical theory has not considered the mass-variation effect of the projectile caused by response medium. To deal with such a problem, for high striking velocity, Gao [12,13,15,16,17] has presented a normal expansion theory in which the boundary of cavity is not confined to spherical surface and its propagation velocity is not confined to constant. In this theory, the majority of stresses is the hydrostatic pressure and the majority of deformation of the concrete targets is the volumetric strain because the air voids will be firstly gradually compressed out of concrete. Holmquist, Johnson and Cook [14] gave a dynamic strength model for concrete subjected to large strain, high strain rates, and high pressures.

3.1 The solid resistant force model

There are many empirical or analytical formulae to describe the axial resistant force on the projectile nose. One is the Forrestal's [8] analytical and empirical formula which consequently developed by Li and Chen [4], whose form is as follows

$$\sigma_n = \tau_0 A + B\rho v^2 \cos^2 \varphi \qquad (6)$$

where φ is angle between axis and the normal direction of projectile nose surface, v is the instantaneous velocity of the projectile, ρ is the density of concrete targets, $(\tau_0 A)$ and B are constants that involve only material parameters obtained from triaxial tests, which are suggested for concrete by Forrestal et al. [8] as $B = 1.0$ and $\tau_0 A = Sf_c$ where S is an empirical constant obtained from triaxial tests, and f_c is the unconfined compressive strength..

In equation (6), the right hand side consists of two terms, one is the static strength term and the other is the inertial term.

It is obvious that, in this resistant force model, only the static strength portion and solid inertial portion have been considered.

Because the concrete material is considered as a elastic-plastic solid in this model, the above two terms will represent the static strength stress and the solid inertial compressive stress.

3.2 The dynamic fluid resistant force model

For high striking velocity, the concrete material can be considered as ideal compressible fluid, in view of which, Gao [12,13,15,16,17] has presented a normal expansion model considering the wave velocity as a variable and the projectile mass as a variable. The normal pressure on the nose surface is expressed as

$$\sigma_n = \rho_0 c_n v_n + \rho^* \frac{dv_n}{dt} l \qquad (7)$$

where ρ_0 is the original density of concrete material, ρ^* is the ultimate density of the concrete material, c_n is the wave velocity of response medium boundary which depends on the instantaneous normal velocity of the nose surface v_n, and l is the propagation distance of response medium wave.

The first term of equation (7) is the dynamical fluid resistant force, the second term is the effect of mass variation. In this model the static strength is neglected.

3.3 Dynamic strength resistant force model of solid

For large strains, large strain rates, and high pressures, the strength of the concrete material will change relevant to static state. Holmquist et al. [14] presents a computational constitutive model for concrete subjected to large strains, high strain rates, and high pressures. In this model, the normal dynamical strength resistant force on the nose surface is expressed as

$$\sigma_n = f_c[A_1(1-D)+B_1P^{*N_1}][1+C_1\ln\dot{\varepsilon}^*] \quad (8)$$

where f_c is the quasi-static compressive strength, D is the damage ($0 \le D \le 1.0$), $P^* = P/f_c$ is the normalized pressure (where P is the actual pressure), $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0$ is the dimensionless strain rate (where $\dot{\varepsilon}$ is the actual strain rate and $\dot{\varepsilon}_0 = 1.0s^{-1}$ is the reference strain rate), and A_1, B_1, N, and C_1 are the material constants (where A_1 is the normalized cohesive strength, B_1 is the normalized pressure hardening coefficient, N_1 is the pressure hardening exponent, C_1 is the strain rate coefficient).

4 The fuzzy axial resistant force model

4.1 The axial resistant force

According the analysis mentioned above, from equations (6) (7) and (8), the different effective stresses can be expressed as

$$\sigma_{ss} = Sf_c \quad (9a)$$

$$\sigma_{ds} = f_c[A_1(1-D)+B_1P^{*N_1}][1+C_1\ln\dot{\varepsilon}^*] \quad (9b)$$

$$\sigma_{si} = B\rho v^2\cos^2\varphi \quad (9c)$$

$$\sigma_{fi} = \rho_0 c_n v_n + \rho^*\frac{dv_n}{dt}l \quad (9d)$$

The fuzzy model for sum normal compressive stress can be expressed as

$$\begin{aligned}\sigma_{sum} = {} & \mu_{ss}Sf_c + \mu_{ds}f_c[A_1(1-D)+B_1P^{*N_1}] \\ & [1+C_1\ln\dot{\varepsilon}^*] + \mu_{si}B\rho_0 v^2\cos^{2\varphi} \quad (10) \\ & + \mu_{fi}(\rho_0 c_n v_n + \rho^*\frac{dv_n}{dt})\end{aligned}$$

The fuzzy model for the axial resistant force can be expressed as

$$F_x = \iint_{S_A}\sigma_{sum}\bullet\mathbf{e}_{\mathbf{z}^*}ds \quad (11)$$

where $\mathbf{e}_\mathbf{z}$ is the unit vector in axial direction of the projectile and S_A is the curve surface of projectile nose which has penetrated the target.

4.2 The analysis of the grade of membership

The grade of membership vector μ can describe the characteristic variation of the concrete material with the striking velocity.

Equation (4a) and (4b) give us the both extreme cases for the high striking velocity and low striking velocity respectively. For middle striking velocity (100m/s-800m/s), the general form is as equation (5).

In order to determine the components of the grade vector of membership, the linear function of velocity is chosen. They are called the membership functions which can be expressed by

$$\mu_{ss} = \mu_{si} = \frac{8}{7} - \frac{v}{700}$$

$$\text{for } 100m/s \le v \le 800m/s \quad (12)$$

$$\mu_{ds} = \mu_{fi} = \frac{v}{700} - \frac{1}{7}$$

where is the instantaneous penetration velocity of the projectile.

5 Calculations and comparison with

experiments

By means of the method, the calculation and comparison with the experiments are made on the deceleration of penetration characteristics of ogive-nose projectile normally penetrating into thick concrete target. We conducted the deceleration experiments of penetration with the ogive-nose, steel projectile as shown in Fig.1

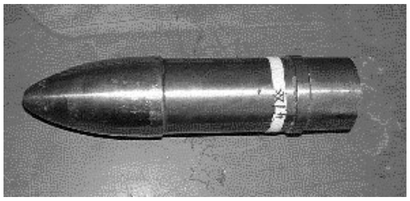

Fig.1. The projectile used in the experiments

The diameter of the projectile $2a = 0.062m$. The radius of the ogive nose $R = 0.177m$. The caliber-radius-head (CRH) $\psi = 2.724$ .The mass of the projectile is $m = 3.777kg$. The initial striking velocity of the projectile is $763m/s$.

The density of concrete target is $\rho = 2400kg/m^3$ and the compressive strength of it is $f_c = 3.0\times10^7 N/m^2$. The limit density of concrete target is $\rho^* = 2460kg/m^3$.

The penetrated concrete target in tests is shown in Fig.2.

Fig.2. The penetrated target

The curves of deceleration vs time of the projectile are shown in Fig.3. In Fig.3, there are four curves. One is the from experiments. The other three curves are from calculations repectively by the Forrestal's model [8] (considering the concrete material as elastic-plastic solids), by the model [12] (considering the concrete material as compressible ideal fluid), and by the model in this paper. It is obvious that, for the high velocity, the compressible ideal fluid model is suitable. For low velocity, the Forrestal's solid model should be suitable, but the parameter S must be reasonably determined. In [8], the static strength resistant force was too significantly considered that it can not predict the peak of deceleration suitably. Every one of the method mentioned above can not singly be suitable for the total penetration process. The fuzzy model in this paper is suitable for the total penetration process during which the velocity of the projectile varies from the initial velocity to zero. It is also shown that, the calculated results by the fuzzy model in this paper are in good agreement with the experiment results.

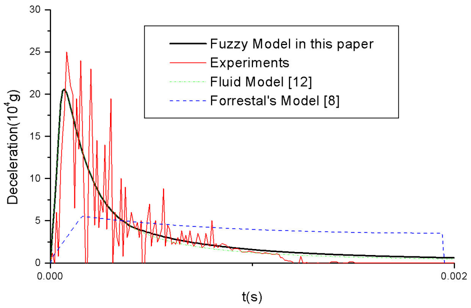

Fig.3. The deceleration curves vs time

6 Conclusion

In terms of the comparison of the results from the experiments with the results from the calculation, it is obvious that, in Forrestal's model, the effect of static strength was too emphatically considered that the peak of deceleration is inconsistent with the reality; in the fluid model, the effect of inertia of the material was too emphatically considered that the depth of penetration is inconsistent with the reality of experiments. To overcome their defects, the fuzzy model in this paper is suggested for the analysis of total process of penetration.

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