# Hybrid Image Compression Using Fractal-Wavelet Prediction

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*Abstract:* - Based on the standard fractal transformation in the spatial domain, simple relations may be found relating coefficients in detail(high-pass) subbands in the wavelet domain. In this paper, we devise a hybrid image compression using fractal-wavelet prediction where the causal similarity among blocks of different subbands in a wavelet decomposition of the image is exploited. The proposed coding scheme consists of predicting fractal code in one subband from fractal code in lower resolution subband with the same orientation. By linear adjusting the fractal code parameters (including matched domain block position of current range block,eight isomorph type, contrast scaling $\alpha$ and the offset $o$ ) in lower resolution subband, fractal code in the adjoin high resolution subband with the same orientation is approximately forecast achieved. The experimental results show that the performance of our scheme is superior for both acceptable visual decoding image quality, an average of 20 % reduction in encoding time and higher compression ratio,compared with standard Jacquin fractal coders. As a side effect, our work motivates shortened encoding time and improved bit allocation strategies for fractal coding.

*Key-Words:* - Fractal predict, Wavelet decomposition, *S*elf-similarity, Multiresolution, Encoding

## 1 Introduction

Fractal image compression techniques, introduced by Barnsley and Jacquin[1],[2],[3],are the product of the study of iterated function system(IFS)[4].These techniques involve an approach to compression quite different from standard transform coder-based methods.Fractal coding consists of the representation of image using the self-similarity concept. They store images as contraction mapping of which the images are approximate fixed points. Images are decoded by iterating these maps to their fixed points. Wavelet transform[5] provides a multiscale representation of signals and the redundancy of this representation has been clearly demonstrated for the case of fractal image.

Wavelet and fractal image compression have become very popular in the last years as new image compression schemes. Hybrid image compression techniques have a long tradition-the basic idea of combining different compression techniques in a way that advantages of both schemes are preserved while disadvantages are avoided looks very promising.

The link between fractal image coding and wavelet is not a new one.The first mention of the connection was by Pentland and Horowitz in [6]. However, consists of a within-subband fixed vector quantizer that uses cross-scale conditioning for entropy coding vector indices, and it only loosely related to Jacquin-style schemes.

An important paper linking wavelet and fractal image coding is that of Rinaldo and Calvagno[7].The coder in [7] uses blocks from low frequency image subbands as a vector codebook for quantizing blocks in higher frequency subbands. Davis[8]showed that the fractal contractive mapping could be considered as a prediction operation in the wavelet domain. A fractal coder in the wavelet domain called the self-quantization of subtrees was developed in [9].As an extension zerotree wavelet coding schemes exploit the correlation between different pyramid levels by setting entire insignificant subtrees to zero,so their coefficients do not have to be transmitted[10]. Li and Kuo[11] use the fractal contractive mapping to predict interscale wavelet coefficients and then encode the prediction residue with a bitplane wavelet coder.

In this work,we propose a coding procedure for general images that aims at exploiting similarities among detail signals in a multiresolution decomposition of the image. To make good use of similarity among adjoining subbands in a wavelet decomposition of image ,our coding scheme consists of predicting current high frequency subband fractal code parameters by linear changing adjoining fractal code parameters in lower subband with the same orientation. As is included: compared with fractal code parameters in lower subband, the match domain block position of adjoining high frequency subband with the same orientation is on the double, isomorph type and contrast scaling $\alpha$ is the same, and offset $o$ is on the double. Without searching,this is used to obviously reduce the block coding time and gives acceptable experimental results. From a subjective point of view, compared with standard Jacquin fractal block coder, in terms of shortener coding time, improved compression ratio and available encoding image quality, our predict scheme is feasible and effective.

The balance of the paper is organized as follows: an overview of a basic fractal block coding scheme is given in Section 2. Fractal predict coding based on wavelet domain is described in Section 3. The performance of our coding scheme is demonstrated with extensive experiments in Section 4. Some concluding remarks are given in Section 5.

## 2 Fractal Coding

Fractal image coding makes good uses of image self-similarity in space by ablating image geometric redundant.Fractal coding process is quite complicated but decoding process is very simple,which makes use of potentials in high compression ratio. Main theory of fractal image coding is based on iterated function system,attractor theorem,and collage theorem.Regard original compressible image as attractor,how to get IFS parameters is main problem settled of fractal coding.

Fractal block coders are essentially based on the work done by Jacquin in [1], with several improvements and variations presented in [12, 13]. In standard fractal block coders, the image f is partitioned into a set of nonoverlapping range blocks $\{r_i, i = 1,......, N_R\}$ such that

$$\bigcup_{i=1}^{N_R} r_i = f, r_i \cap r_j = \Phi, \text{for } i \neq j. \quad (1)$$

Each of the range blocks has size $B \times B$ pixels. As proposed by Jacquin, similarities between range blocks and overlapping domain blocks of another size usually $D \times D (D = 2 \times B)$, taken from other parts of the image, are exploited. The set of domain blocks $\{d_j, j = 1,......N_D\}$ plays the role of a codebook for the range blocks $\{r_i, i = 1,.....N_R\}$ as in vector quantization,with the important difference that the domain blocks $\{d_j\}$ are taken from the image itself.

When exploiting similarities between a range block $r_i$ and a domain block $d_j$, the domain block is spatially scaled from size $D \times D$ to $B \times B$ size, it is isometrically transformed (i.e., reshuffling of the domain block pixels is carried out), it is contrast scaled by a factor $\alpha_i$ and added to an offset value $o_i$. The mapping from the domain block $d_j$ to the range block $r_i$ is given by

$$\hat{r}_i = \tau_i(d_j) = \alpha_i T_i(S_i(dj)) + o_i \quad (2)$$

Here, $\tau_i$ contains three parts: geometry, eight isomorph and gray transform.

Geometry transform $S_i$ is near neighbours compression.In the range of $d_j$ can be get a compressive block with the same size of $r_i$. Eight isomorph transform $T_i$ is composed of rotate $0^0$, $90^0$, $180^0$ and $270^0$ angle,vertical certerline reflex,horizontal certerline reflex,and diagonal reflex.In gray transform ,$\alpha_i$ is the contrast scaling and $o_i$ is the offset.

The map $\tau_i$ is chosen in order to minimize the distance between the range block $r_i$ and its approximation $\hat{r}_i$ .Typically,we want to minimize the mse distance

$$D(r_i, \hat{r}_i) \equiv \frac{1}{B_i^2}\left[\sum_{l=1}^{Bi}\sum_{m=1}^{Bi}(r_i(l,m) - \hat{r}_i(l,m))^2\right] \quad (3)$$

Where l,m denote the pixel position of range block.

So, the fractal coding process of $r_i$ can be concluded as finding a domain block $d_j$, which has minimum distortion mse after the transformation $\tau_i$. Fractal code of the range block $r_i$ is composed of $T_i$, $\alpha_i$, $o_i$ and position of mapping $d_j$.

The goal of a fractal encoding scheme is to define the image $f(x, y)$ as the fixed point of a transformation $W : F \to F$ from a complete metric space F of images to itself [3]. The map W is obtained by composing the action of the individual block maps $\tau_i$. An approximation $\hat{f}$ of the image f is obtained as the fixed point of the transformation W.

$$\hat{f} = \lim_{n \to \infty} W^{on}(f_0), \forall f_0 \in F \quad (4)$$

Here, “on” denotes the n-fold composition of the map W with itself. Note that in order for (4) to converge, W must be contractive with respect to the metric induced by (3).

# 3 Description Of Fractal Predict Coding Based On Wavelet Domain

We try to exploit a causal interdependence of the subband images in a multiresolution decomposition of general image. In this section we will describe in detail our scheme.

## 3.1 Image Wavelet Decomposition And Reconstruction

The image is first subband decomposed,using a pyramid QMF decomposition. In a subband decomposition,the input image y is decomposed into four subimages (or subbands) $y^{lh}, y^{hl}, y^{hh}, y^{ll}$, where the pair of superscript letters denotes the row-column filtering operations performed to obtain the subimage. For instance, subimage $y^{lh}$ is obtained by low-pass filtering the rows and high-pass filtering the columns of y, followed by a factor two subsampling in each direction.

This procedure can be iterated to obtain a multilevel pyramidal decomposition of the image y. Denoting with $y_0^{ll}$ the input image y,at each decomposition level i ,the image $y_{i-1}^{ll}$ is decomposed into the four subimages $y_i^{lh}, y_i^{hl}, y_i^{hh}, y_i^{ll}$ ,for $i = 1, \ldots\ldots m$ .The result of such a decomposition is a set of subimages which are localized in scale, orientation and space. So,Fig.1 and Fig.2 show that image pyramid wavelet decomposition of three level.

| | | | |
|---|---|---|---|
| LL3 | HL3 | HL2 | $HL_1$ horizontal high-pass vertical low-pass |
| LH3 | HH3 | | |
| LH2 | | HH2 | |
| $LH_1$ horizontal low-pass vertical high-pass | | | $HH_1$ horizontal high-pass vertical high-pass |

**Fig.1 pyramid wavelet decomposition of three level**

Energy relation of subband images in a multiresolution decomposition is as follows.

$$|I_0|^2 = |I_{0LL}|^2 + \sum_{i=1}^{k} (\| I_{0HL_i}|^2 + |I_{0LH_i}|^2 + |I_{0HH_i}|^2 |),$$

k is wavelet decomposition level.

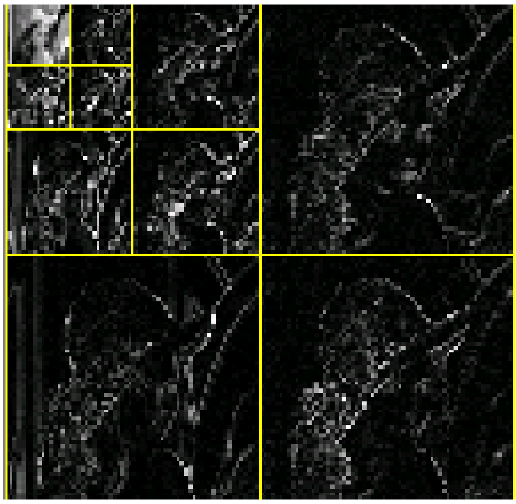

**Fig. 2 image lena wavelet decomposition of three level**

Note that , total energy is conservation in wavelet decomposition and reconstruction, original image energy is equal to all of the multiresolution subbands energy. Usually subband of wavelet image decomposition has much remarkable characteristic as follows:

(1)Subband of image wavelet decomposition is not compression but new energy allocated over again. subband of the lowest frequency focuses most of the image energy and very similar to the original image on lessened scale.

(2)The result of the decomposition is a set of subbands which are localized in scale, orientation(including horizontal, vertical and cross direction)and space, which is anastomosed with human vision characteristic.

(3) The wavelet representation can be seen as a multilevel pyramid tree-structure with the coarsest scale coefficients at the top and the finest scale coefficients at the bottom of the pyramid.

## 3.2 Coding And Decoding Algorithm

Our coding scheme consists of predicting subband fractal coding parameters of current higher frequency by linear changing lower subband fractal coding parameters with the same orientation.In the following,the coder organization is described for $256 \times 256$ images,but it can be easily adapted to images with different dimensions.

Hers is coding procedure.

Firstly, we consider a pyramid subband decomposition with two level, so $Y_2^{ll}, Y_2^{hl}, Y_2^{lh}, Y_2^{hh}, Y_1^{hl}, Y_1^{lh}, Y_1^{hh}$ seven subbands are obtained. Both tightknit, smoothing orthogonal and symmetric of the wavelet base are considered, the filters used to compute the subband decomposition are D7/9 double orthogonal, synthesize rank seven and analysis rank nine[14].Talbe1 shows fitlers coefficient.

Table 1. filter D7/9 coefficient

| N | 0 | ±1 | ±2 | ±3 | ±4 |
|---|---|---|---|---|---|
| h(n) | 0.812 | 0.384 | -0.108 | -0.021 | -0.011 |
| h1(n) | 0.766 | 0.408 | -0.041 | -0.063 | 0 |

Here, wavedec2 of Matlab function is used to decompose the image, appcoe2f and detcoef2 functions are used to get low subband coefficient and high subband coefficient respectively.

Secondly, the lowest subband $Y_2^{ll}$ is coded independently using DCT, at the same time subband $Y_2^{hl}, Y_2^{lh}, Y_2^{hh}$ of two level are coded with standard Jacquin method. For instance, subband $Y_2^{hl}$ is divided into $4\times 4$ range block $\{r_i\}$ .For each block $r_i$ ,a $8\times 8$ domain block $d_j$ is searched in the $64\times 64$ region of subband $Y_2^{hl}$ .Once the domain block $d_j$ and the optimal transformation are found, we record fractal code parameters of current range block $r_i$ . An optimal bit allocation strategy is that,12 bits for the address of the domain block(usually the positon of top left corner),3 bits for eight isomorph types,5 bits for contrast scaling and 7 bits for the offset. For each of block $r_i$ ,fracal code needs 27 bits storage space.

Finally, fractal code parameters of subband $Y_1^{hl}, Y_1^{lh}, Y_1^{hh}$ are gradually predicting obtained by correspondingly changing fractal code parameters of subband $Y_2^{hl}, Y_2^{lh}, Y_2^{hh}$ in a linear way. For instance, subband $Y_1^{hl}$ is divided into $8\times 8$ range block $\{r_i\}$. So range block numbers of $Y_1^{hl}$ is equal to $Y_2^{hl}$.Thus here is the predicting strategy. Comparing with $Y_2^{hl}$ for each range block $\{r_i\}$ in $Y_1^{hl}$, the match domain block position is double of $Y_2^{hl}$, isomorph type is the same of $Y_2^{hl}$, contrast scaling $\alpha_i$ is the same of $Y_2^{hl}$ ,and offset $o_i$ is double of $Y_2^{hl}$ .The same method is used to apply in subband $Y_2^{lh}, Y_2^{hh}$ .Thus the coding procedure is finished.

Decoding procedure is considerably simple. According to computing and predicting obtained fractal code, we choose unlimited same size image as the decode original image to recursive ten-fold iterate. So attractor presented and the additional DCT decoding image composes new wavelet coefficient domain. In the end, the approximate decoding image is obtained by making wavelet inverse transform. As for no strict constringency in prediction coding, there may be several block of subband dispersed, we use to four adjoining domain average value to conquer.

# 4 Experimental Results

In this section, we present some experimental results to evaluate the performance of the proposed our coding scheme.Original image is classical $256\times 256$ gray-level Lena image, coded with 8 bits per pixel. All experiments are carried out on a computer with Intel 2.5Ghz and 512M RAM in the Win2000Professional operating system, hybrid Matlab7.0 and VC6.0 language is used to coding.

Fig.3 shows simulation image in the different conditions.Table2 shows the detail experimental data.

**Fig.3 Examples of the reconstructed images: original image(top-left), reconstructed image Jacquin method range 4*4 (top-right), reconstructed image our method range 4*4(bottom-left), reconstructed image our method range 2*2 (bottom-right).**

Table 2 experimental ontrasting data

| Coding method | Coding time(S) | Compression ratio | PSNR (dB) |
|---|---|---|---|
| Jacquin range 4*4 | 1060 | 4.2:1 | 32.1 |
| Our method Range 4*4 | 112 | 25.3:1 | 24.3 |
| Jacquin Range 2*2 | 3560 | 1.2:1 | 35.7 |
| Our method Range 2*2 | 203 | 6.3:1 | 28.1 |

PSNR is defined as follows and measured in decibel(dB):

$$psnr = 10\log_{10}^{\frac{255^2}{e_{ms}^2}} \quad (5)$$

Where 255 is the maximal allowed gray-value of the original image and $e_{ms}^2$ is the average sample mean-square error. Where $f(i,j)$ and $\hat{f}(i,j)$ represent the $N \times N$ original and the reproduced images, respectively.

$$e_{ms}^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}(f(i,j) - \hat{f}(i,j))^2 \quad (6)$$

Our work motivates shortened encoding time and improved bit allocation strategies for fractal coding. As seen, the total coding time for "lena" was about 2 minutes(range 4*4) about twenty percent of Jacquin method, for only three sixteenths of the all subbands in a wavelet image decomposition are computing coded by Jacquin method in our coding scheme(as before subband $Y_2^{hl}, Y_2^{lh}, Y_2^{hh}$ ).The report states that image compression time is obviously shortened though decoding image is available. Moreover,our coding scheme improves compression ratio greatly for only fractal code of subband $Y_2^{hl}, Y_2^{lh}, Y_2^{hh}$ are stored and the rest subband $Y_1^{hl}, Y_1^{lh}, Y_1^{hh}$ can be obtained by predicting from fractal code of subband $Y_2^{hl}, Y_2^{lh}, Y_2^{hh}$ respectively.

As for no strict constringency in our prediction coding scheme, a little blocking effect is visible in the reconstructed image. Two ways of our coding scheme are used to conquer it. One is four adjoining domain average value for decoding when several block of subband dispersed. The other is range block of subband $Y_2^{hl}, Y_2^{lh}, Y_2^{hh}$ split into smaller dimension. if range block $\{r_i\}$ of subband $Y_2^{hl}, Y_2^{lh}, Y_2^{hh}$ is allowed to be divided smaller dimension(such as 2*2),the approximate decoding image has less block effect and higher PSNR but coding time and compression ratio influences.

# 5 Conclusion

In this work, we devised a fractal image predict coding based on wavelet domain, the proposed coding scheme consists of predicting current higher subband fractal code parameters by linear changing lower subband fractal code parameters with the same orientation without searching.The experimental results show that with the available visual decoding image quality, compression time is obviously shortened and compression ratio is greatly improved, which explores a new way in hybrid fractal-wavelet image coding. Exploiting similarities among fractal-wavelet relation deeply and using statistics similarities or fractal geometry similarities to represent similarities of different subbands in a image wavelet decomposition is our main work for the future.

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