Identification of composite plate elastic properties by displacement field measurement

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Abstract: - In this work a method combining finite element analysis and genetic algorithms (GAs) is adopted to inversely determine the elastic constants from the full-field measurement of the surface displacements of plates under flexural loads. The principle of this method is to measure the displacements of the plate and to adjust the parameters put in a numerical model in such a way that measured and computed values match as precisely as possible. The unknown parameters are simultaneously identified by a single test and without damaging the structure. Numerical investigations were performed on different thin and moderately thick square orthotropic laminates. Results have permitted to test the effectiveness of the procedure and identify its applicability limitations as well as it robustness to the noise effects.

Key-Words: - Composite materials; Elastic constants; FEM; Genetic algorithms; NDT; NDE.

1 Introduction

The knowledge of the elastic properties is essential for design and application of composite materials [1]. In recent years many researchers have investigated the feasibility of determining the whole set of rigidities of anisotropic materials from a single plate subjected to a reduced number of testing configurations. Both static [2-5] and dynamic [6] approaches have been adopted. The former approach involves the measurement, almost always by optical techniques, of the superficial displacements field of a properly loaded specimen, while in the second it is necessary to measure the frequencies and/or shape of the first modes of vibration of plates or beams. Extracting elastic constants from these experimental measurements requires the use of the so-called inverse approaches [7]. The present paper describes an inverse method, which combines finite element analysis and genetic algorithms to identify the elastic properties of isotropic or anisotropic materials by the full-field measurement of the surface displacements of a plate under suitable flexural loads. An optimising process updates the elastic constants in a finite element model so that the outputs from the numerical analysis fit the experimental data. In this way, the unknown parameters can be identified simultaneously in a single test and without damaging the structure.

In the last three decades coherent optics has provided several interferometric techniques which enable the full-field surface displacement of an object to be determined with an accuracy of a few tens of nanometers without any contact with the investigated surface [7]. The amount of data that optical interferometric whole-field methods provide is much greater than the number of the unknowns to be estimated. This fact makes the elastic characterization an over-posed inverse problem and its solution can be easily obtained when the problem is well-posed. For this reason, great care needs to be taken in choosing the geometry and the way of loading and constraining the specimen [5]. The resulting interferometric fringe pattern must contain sufficient information for determining all the unknown parameters quickly and unequivocally. Besides, to reduce the effects of noise that inevitably disturb the displacement field on the solution, it must also be sufficiently sensitive to the variation in each elastic parameter.

In this paper, a very practical loading and constraining configuration for a plate specimen is proposed by which the elastic constants of the material are identified from measuring the out-of-plane component (normal to the surface) of the displacement field. This configuration can be successfully employed for the elastic characterization of thin square isotropic or orthotropic plates. In the paper, the results of numerical simulations carried out on laminates with different composite materials are reported.

2 Problem Formulation

Recently, the feasibility of using iterative strategies to identify the elastic constants of a material from the interferometric fringe pattern, observed on the surface of a properly loaded plate, has been investigated [3-5]. In such studies, the unknown parameters (elastic constants) are identified by iteratively adjusting their values in a numerical model until an error function φ is minimized. φ is usually, related to the difference between the computed displacement field (numerical fringe pattern) and the measured one (experimental fringe pattern). Note that it is not always easy to minimize the error function φ, especially in the case of orthotropic materials, and that the right choice of minimization procedure is fundamental. In any case, it is desirable to use a robust and reliable optimization procedure able to converge to the target values of elastic properties regardless of load type, initial guess values, boundary conditions etc.

Genetic algorithms (GAs) are able to find the global optimum even for ill-conditioned functions. Therefore, they appear to be highly suitable for the characterization of anisotropic materials for which φ is a highly non-smooth function and gradient-based optimization methods would not work well. The fundamental idea may be found in [9]. Based on probabilistic rules, GAs use the processes of natural selection by imitating the concept of survival of the fittest. Due to the way the genetic algorithm explores the region of interest, it avoids getting trapped at a particular local minimum and locates the global optimum. Genetic algorithms, unlike the gradient-based method, do not require initial estimates, but instead work within a suitable set of bounds that can often be rather broad. For these reasons in the recent years many researchers have used GAs for the elastic characterization of materials [5, 10-12].

The first step of the genetic algorithm is the creation of a population of individuals (initial population) chosen from a set of potential solutions of the problem. Each individual is subjected to evaluation based on a given fitness function. Then, a selection process permits those individuals of superior fitness to reproduce and create a new population, which combines the desirable characteristics of the old population. The reproduction is generally based on two operators; crossover and mutation. The new population then replaces the old one and the process restarts. New generations of solutions are created through the genetic manipulation, and this iterative process is repeated for a fixed number of generations or for a fixed number of analyses until there is no improvement in the best solution. The diagram of the genetic algorithm used for the identification of the elastic constants in this paper is shown in Fig. 1.

It differs from that used in [5] with regard to an adaptive range module [13] that was added to

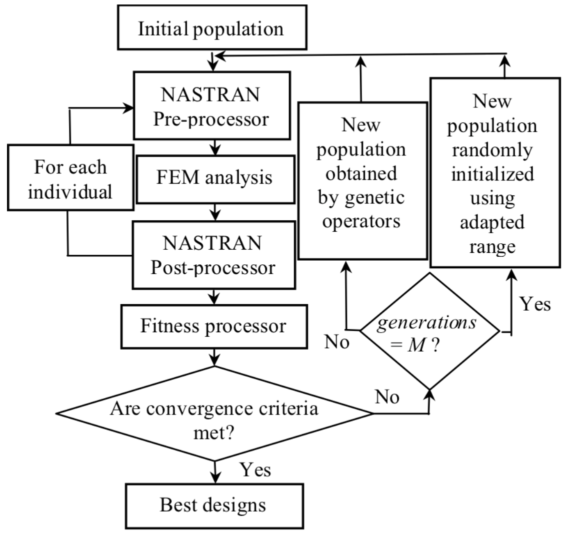


Fig. 1 Flow chart for optimal design by genetic algorithm.

explore the search space more efficiently. The algorithm was developed on a personal computer in MatLab environment (distributed by MathWorks Inc). It applies the general-purpose numerical code MSC-NASTRAN to carry out the static analysis.

The process starts with the generation of a random initial population of sets of elastic constant values. Each design is formed randomly by choosing the elastic constant values within an interval of positive values set by the user. To take the proper set of elastic constants into account, for each design of the population, in the NASTRAN pre-processor stage, the MSC/NASTRAN input file is adjusted by modifying the MAT1 or MAT8 bulk data entry, defining isotropic and two dimensional orthotropic stress-strain relationships, respectively. Then the actual static analysis is carried out.

In the post-processing stage, by using the displacement field extracted from the NASTRAN result file, for each design, the error function (fitness) is evaluated. After that, both the fitness and the relative elastic constants are saved, all the FEM output files are removed to release the computer memory and, the cycle restarts.

The fitness processor begins to operate at the end of the processing of the population arranging the fitness values of the population in decreasing order and checking the convergence criteria. If the convergence criteria are not reached, the most

suitable solutions are selected and then processed by means of the genetic operators to create the new population. The process is repeated for a fixed number of analyses until there is no further improvement in the best solution.
To explore the search space more efficiently, the algorithm described above was provided with an adaptive range procedure [13] by which the entire population is regenerated every M (with M>1) generations. Then, three additional steps were incorporated into the structure of the algorithm. In the first step, every M generations, the top half (the fittest individuals) of the previous generation is collected as a group; for each elastic constant the average (μ_{group}) and standard deviation (σ_{group}) of this group is calculated and then a new average and standard deviation for each elastic constant are obtained updating the previous values according to the following equations:

$$\mu_{new} = \mu_{old} + \omega_\mu \; (\mu_{group} - \mu_{old}) \qquad (1)$$

$$\sigma_{new} = \sigma_{old} + \omega_\sigma \; (\sigma_{group} - \sigma_{old}) \qquad (2)$$

where ω_μ and ω_σ are relaxation factors that provide robustness during the range adaptation. In the second step, a new search range (l_{min}, $l_{,max}$) for each elastic constants is calculated using average and standard deviation computed in the previous step by the following equations

$$l_{min} = \mu_{new} - \kappa \, \sigma_{new} \qquad (3)$$

$$l_{max} = \mu_{new} + \kappa \, \sigma_{new} \qquad (4)$$

where κ ($1 \le \kappa \le 10$) is a measure of the overlap between the group and the new generation. In the final step, almost all but two individuals in the population are generated randomly according to the new range. The population is completed including the two fittest individuals of the old population. The goal of the optimisation is to minimize the error function defined as

$$\varphi = \sum_{i=1}^{n} \left| s_i - \overline{s_i} \right| \qquad (5)$$

which accounts for the sum of the absolute errors between the calculated displacements and the measured ones. s_i and $\overline{s_i}$ represent the generic calculated and measured component of the displacement, respectively, at a point i on the surface of the plate. n is the number of sampling points considered. A proportional selection scheme was adopted for the reproduction of the child generation and two procedures (arithmetical and replacing types) used to carry out the crossover operation. In order to speed up the evolution and to improve the convergence performance of the GA a mutation and elitism selection have also been introduced.
Greater details on these operators can be found in [8] and the values of the parameters involved (population size, probability of mutation and crossover, etc.), selected on the basis of systematic trials, are reported in table 1.

Tab. 1 Genetic algorithm parameters

Population number	120	M	35
Crossover probability	80%	$\omega_\mu = \omega_\sigma$	0.5
Mutation probability	5%	κ	10

3 Loading and constraining conditions

Optical full-field techniques provide an enormous number of information. This information is much more than that necessary for the determination of the elastic constants of the specimen investigated. Consequently, the elastic characterization problem is an over-posed inverse problem. Obviously, solutions of this problem can be obtained if it is well-posed, and therefore great care needs to be taken in choosing how to load and constrain the plate so that the resulting displacement field contains enough information to determine all the unknown parameters, quickly and unequivocally.
In [5], a numerical procedure for optimizing the loading and constraining conditions of the specimen is proposed. Basically, the procedure consists in determining the conditions minimizing the "correlation index" i_c. This index represents the degree of statistical correlation between the variation of the displacement fields due to a variation of the elastic constants and its absolute value is for definition ≤ 1. In case of isotropic plate the correlation index coincides with the usual correlation coefficient, while for orthotropic plate the correlation index is the mean of the absolute values of the correlation coefficients. In [5], to illustrate the procedure, a referenced configuration for testing square plates under flexural loads was analyzed. Such a configuration proved to be suitable for solving the problem with two unknowns, but it was not very appropriate for solving the problem with four unknowns (mainly, because of the practical difficulty of applying the load at the location which corresponds to the lower i_c).
In the present paper, we propose an alternative configuration which also solves the problem with four unknowns properly. This configuration (see figure 2) requires loading and constraining conditions similar to those of the previous configuration, but which are easier to set up. The square plate is simple supported on three points P_1,

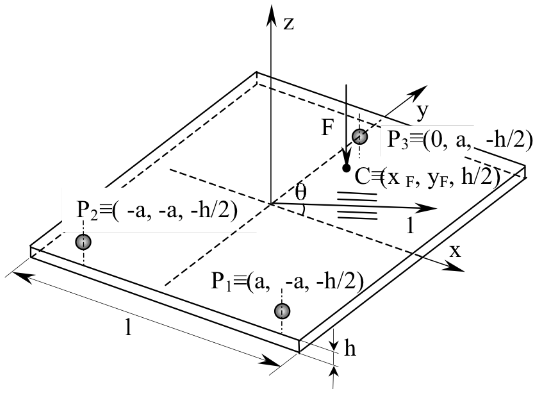


Fig. 2 The loading configuration

P_2 and P_3 lying on the corners of an isosceles triangle. Considering a rectangular coordinate system, 0xyz, with the origin located at the centre of the plate and the axes parallel to the sides of the plate, the locations of the three support points are completely definite by means of the length a.

The fields W of the out-of-plane components of the displacements, undergone by the upper surface of the plate, were considered. The correlation index was calculated for each possible location of the force on the surface (all the nodes of the mesh). The area around each location was coloured with a grey, proportional to the corresponding value of the calculated correlation index (from black for $i_c = 0$ to white for $i_c \geq 0.5$), and maps similar to that of Fig. 3 were obtained.

Isotropic material

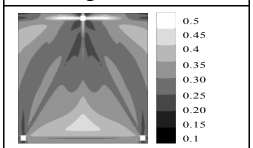


Fig. 3 Aluminium correlation index map.

To obtain the maps, a variation of the elastic constants equal to 10% was considered. The calculus of the displacement fields was carried out using a finite element code, the side l, the length a and the applied force F were 50 mm, $23/50\ l$ mm, and 1 N, respectively. The thickness h of the plates was $1/50\ l$ mm. The plates were discretized into 2500 (50x50) quadratic eight node QUAD elements. The usefulness of these maps will be clarified in the following. An intensive numerical analysis revealed that plates of different isotropic materials have very similar correlation index maps.

[0^0] unidirectional laminates

C/E | G/E | P/E

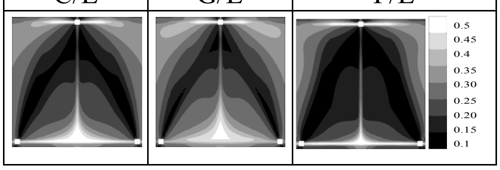


Fig 4 Correlation index maps of unidirectional laminates.

In Fig. 3 the map relative to an aluminium plate is reported to show the behaviour of this class of materials. In addition, little difference has been observed among the maps of unidirectional thin laminates of different material. In Fig. 4, for example, the maps of the correlation index of three different laminates with fibers parallel to the x axis ($\theta = 0^0$) are illustrated.

The properties of the material examined, Aluminium, Carbon/Epoxy (C/E), Glassy/Epoxy (G/E) and Pitch/Epoxy (P/E), respectively, are reported in Tab. 2.

Table 2 Material properties.

material	E_1 GPa	E_2 GPa	ν_{12}	G_{12} GPa
Aluminium	70.0	70.0	0.33	26.3
Carbon/Epoxy	134.0	8.9	0.24	7.0
Glassy/Epoxy	38.6	8.3	0.26	4.14
Pitch/Epoxy	462.0	6.2	0.31	5.58

In all the cases, the distributions of the correlation index are characterized by low-level values and common areas can be distinguished, where the correlation index is minimal. Such areas can be more easily identified by observing the mean and the standard deviation of all the maps (Fig. 5).

Mean | Standard deviation

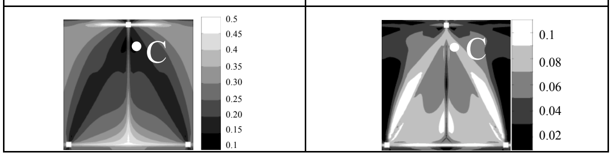


Fig. 5 Mean and standard deviation of the correlation maps.

From such figures an area can be identified, around the point C of coordinates $x_F = 1/25\,l$ mm and $y_F = 7/25\,l$ mm, characterized by the lower values of the mean correlation index (≤ 0.14) and with a negligible standard deviation (≤ 0.05). This area represents one of the better location for the application of the force to profitably characterize both isotropic and unidirectional laminates.

It is important to underline that when the fibers are oriented parallel to the y axis ($\theta = 90°$) the values of the correlation index are generally higher than those obtained in laminates with the 0° orientation (see, in Fig. 6, the mean map). As a consequence this last orientation (θ=0°) is

[90^0] unidirectional laminates

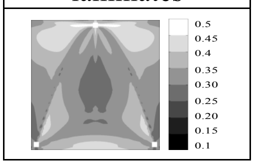


Fig. 6 Map of the mean correlation index.

certainly preferable for a faster and more accurate characterization.
To investigate the limits of the approach proposed some multidirectional laminates were also analysed. In Fig. 7, the correlation index maps related to three of the simpler thin multidirectional laminates, a

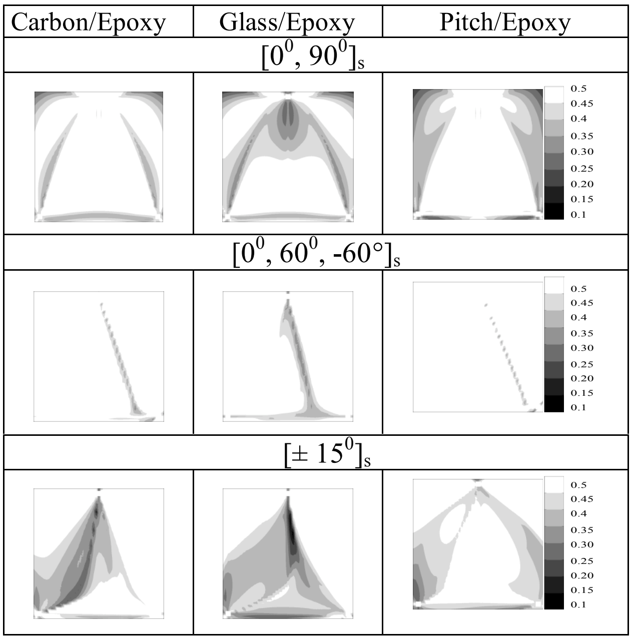


Fig. 7 Correlation maps of multidirectional laminates.

cross-ply laminate $[0°, 90°]_s$, an angle-ply laminate $[\pm 15°]_s$ and a quasi-isotropic Pi/3 laminate $[0°, 60°, -60°]_s$, respectively, are shown. It can be observed that the correct point of application of the load depends on the type of laminate and point C is not more suitable. Similar results were obtained for other multidirectional laminates.

4 Examples of identification

The inverse procedure was developed and tested by means of a series of numerical simulations. First, the FEM forward solver evaluated the behaviour of a plate. The component of the displacement along the direction normal to the upper surface of the plate was calculated at each node of the mesh and, the resulting displacement field was used in substitution of the experimental data. Then, the genetic algorithm, using a part or the whole nodal displacement field, identified the elastic constants. A comparison, between the results and the values of the elastic properties used to simulate the experimental displacement field, allowed us to refine the procedure and verify its accuracy.
The contours of the computer-simulated displacement field (fringe patterns obtained, for example, by an interferometric technique with a sensitivity of 0.266 μm/fringe) due to a load force equal to 0.45 N are reported, for each plate, in Fig. 8.

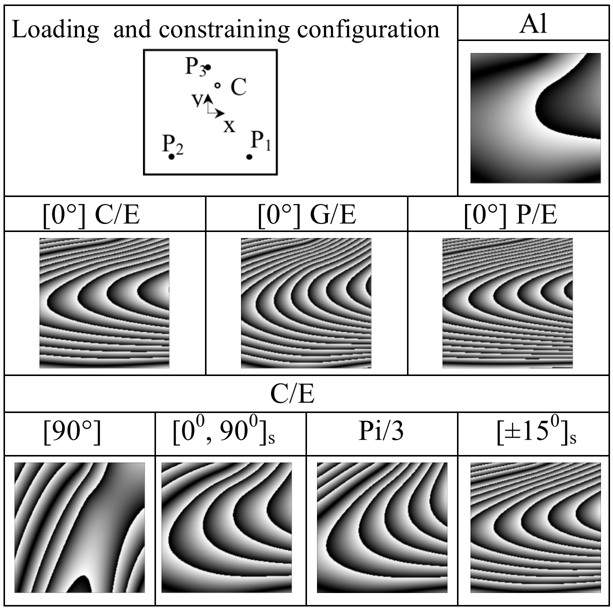


Fig. 8 Computer-simulated fringe patterns.

In Tab. 3 the mean values of the number of executions of the FEM code to converge to the solution, with an error less than 0.1%, are reported for each case. The bounds on parameters were set at approximately ±100% from the true values. For each case three GA runs were performed. Obviously, in case of aluminium plate, even if, fine results can be obtained solving for four unknowns, the identification of only two elastic constants is recommended because it requires shorter execution times.

Table 3 Number of FEM runs for each material.

Al			
Two unknowns	Four unknowns		
380	4721		
[0°] C/E	[0°] G/E	[0°] P/E	
4171	4002	7861	
[90°] C/E	Pi/3 C/E	$[0^0,90^0]_s$ C/E	$[\pm 15^0]_s$ C/E
*	*	*	*

* After 20000 runs convergence was not reached

The stability of the inverse procedure was checked using simulated measurements with artificial Gaussian noise. In particular, a vector of random numbers was generated from a Gaussian distribution with the mean $\overline{w}$ set to zero and standard deviation s_w equal to a percentage of the sensitivity of the interferometric technique. s_w can be defined as 0.266 p_{cnlv} where p_{cnlv} is the value to control the level of noise contamination. For example, p_{cnlv} =0.5

means a 50% noise level, in terms of fringe order this noise level is equivalent to half fringe. The Gaussian noise was directly added to the computer-generated fringe patterns.

In figure 9, the fringe patterns affected by two levels of noise, 15% and 30%, respectively, are illustrated for the cases of carbon-epoxy plate, glass-epoxy plate and pitch-epoxy plate, respectively. Similar pattern was obtained and analyzed for the other materials. For each case, three GA runs were performed. The GA was stopped if no improvements were obtained after 35 generations.

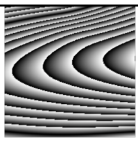
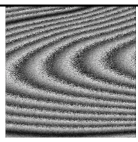
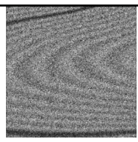
[0°] Carbon/Epoxy		
Noise free	15% Noise	30% Noise
		

Fig. 9 Computer-simulated fringe patterns with noise.

The mean values of material properties obtained by the identification procedure are shown in Table 4. It was found that the solutions are very stable and accurate not only for the noise-free case, but also for the other noise levels examined. Results with similar accuracy were obtained for all the other unidirectional composite plates.

Table 4. Results for the [0°] C/E thin laminate based on computer-simulated responses with different noise levels.

	Noise free		15% Noise		30% Noise	
Material Constant	Mean	% sd	Mean	% sd	Mean	% sd
E_x	134.0	0.0	133.6	0.3	134.7	0.6
E_y	8.9	0.0	8.9	0.0	8.9	0.0
ν_{xy}	0.24	0.0	0.24	0.2	0.24	1.0
G_{xy}	7.0	0.0	7.0	0.0	7.0	0.1

4 Conclusion

The procedure proposed has proved to be suitable for the elastic identification of unidirectional laminates but less appropriate for multidirectional ones. In the paper, it was applied to square thin plates but other tests will have to be executed in order to verify if the procedure is also suitable for the characterization of plates with generic shapes subjected to in-plane or out-plane loading and constraining configurations.

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