Variable Causality Dynamics System Approach for Robot Control

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Abstract: - The paper presents the mathematical model of a walking robot controlled by a shoulder position and its joining angle. It is considered that first leg angles from the active pair of legs are both external controlled and second leg has the knee joint free and the shoulder joint external controlled. The mathematical model is determined considering all the points in the xz-plane as being complex numbers. Taking into account a possible symmetrical structure, only the vertical xz-plane evolution is considered. The model is based on the so called Variable Causality Dynamic Systems (VCDS) approach.
In this causality ordering, the whole structure is controlled in open circuit or in manual operating conditions. The results of this paper are implemented and verified in RoPa, a platform for simulation and design of walking robots control algorithms. The model is implemented in MATLAB environment and some evolutions examples are presented.

Key-Words: - causality ordering, control algorithms, legged locomotion, walking robots, mathematical model.

1 Introduction

Behavior of walking robots from the biped structure till the multileg structures is characterized by a specific type of movement called legged locomotion, [1], [2].

When compared with wheeled or tracked locomotion, legged locomotion is recognize as superior in its capability to cross irregular and difficult terrains.

Most approaches to robot walking begin by considering the issue of walking on flat ground as the basic one, solving the problems of stability and gait generation for this case, usually avoiding the use of sensors. Then, the problem of walking on uneven terrain is solved by altering the basic walking routine in order to cope with the obstacles detected using different kinds of sensorial information. Thus, for example, [3] in the OSU Hexapod vehicle, Klein et al. [4] use force and attitude sensors in order to modify a basic control algorithm to permit adaptation to irregular terrain, Ozguner et al. [5] make use of visual information with the same purpose and Gorinevsky and Shneider [6] working with a small hexapod, compute appropriate corrections to commanded forces and leg positions depending on the soil properties. In an analysis of the design of the robot Attila, Binnard [7] emphasizes the differentiation made between walking and climbing as two different tasks with different hardware and software requirements. In agreement with this view, Ferrell [8] designed a control for this robot in which the aspects of walking and climbing are clearly differentiated, providing specific strategies for different kinds of obstacles.

In this present work it is presented a different approach, a systemic approach of a walking robot as Variable Causality Dynamic System (VCDS). Taking into account a possible symmetrical structure, only the vertical xz-plane evolution is considered. The mathematical model of the robot is determined considering all the points in the xz-plane as being complex numbers.

It is well known that the evolution of a walking robot is characterized by changing the causality ordering. Depending on the evolution context, some variables are causes and the other are effects, their structure could be changed in other context.

It is considered a robot leg with two joints. The angles of these joints can be free outputs or effects of other causes, or could be controls, depending on context. Because of the rigid body of the robot, these variables interact respecting some kinematics restrictions due to the finite length of the body. Thus, the degrees of freedom are decreased by one in order to achieve these restrictions.

In this paper it is considered a causality structure with one degree of freedom. The active pair of legs has four angles, which can be controlled, but one of these is free and has to accomplish the kinematics restriction of finite length.

A three numbers vector presents the causality structure that express the state of each leg

$$c=[c^i, c^j, c^p],$$

where i, j represents the active pair, c^i denotes the leg i, c^j denotes the leg j, c^p represents the leg p.

The main difficulty in conceiving this mathematical model was the ununivocity of the input-output dependence of the TJA (Two Joint Arm) structure in inverse kinematics.

2 Geometrical structure

It is considered a walking robot structure as depicted in Fig. 1, having three normal legs L^i, L^j, L^p and a head equivalent to another leg L^0 containing the robot centre of gravity G, placed in its foot. The robot body RB is characterized by two position vectors O^0, O^1 and the leg joining points denoted R^i, R^j, R^p. The joining point of the head L^0 is the central point O^0, $R^0 = O^0$, so the robot body RB is univocally characterized by the set,

$$RB = \{O^0, O^1, \lambda^i, \lambda^j, \lambda^p, \lambda^0\} \tag{1}$$

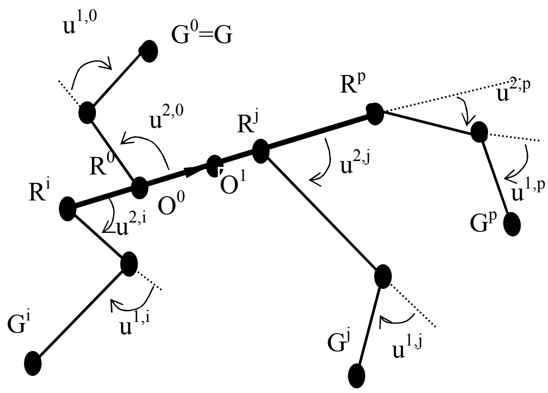

Fig. 1. The robot geometrical structure

The robot has a rigid body if the three scalars ($\lambda^i, \lambda^j, \lambda^k$) are constant in time.

The geometrical structure of the robot is defined by the following relations:

$$O^1 - O^0 = e^{j\theta} \tag{2}$$

$$R^i = O^0 + \lambda^i \cdot e^{j\cdot\theta} \tag{3}$$

$$R^j = O^0 + \lambda^j \cdot e^{j\cdot\theta} \tag{4}$$

$$R^p = O^0 + \lambda^p \cdot e^{j\cdot\theta} \tag{5}$$

$$R^0 = O^0 + \lambda^0 \cdot e^{j\cdot\theta} = O^0 \tag{6}$$

from which

$$R^i - R^j = (\lambda^i - \lambda^j) \cdot e^{j\cdot\theta} \tag{7}$$

$$R^p - R^j = (\lambda^p - \lambda^j) \cdot e^{j\cdot\theta} \tag{8}$$

$$R^i - R^p = (\lambda^i - \lambda^p) \cdot e^{j\cdot\theta}. \tag{9}$$

The robot position in the vertical plane is defined by the pair of the position vectors O^0, O^1 where $|O^1 - O^0| = 1$, or by the vector O^0 and the scalar θ, the angular direction of the robot body.

Each of the four robot legs: L^i, L^j, L^p, L^0 is characterized by a so-called Existence Relation $ER(L)$ depending on specific variables as it is presented in [9], [10].

The mathematical model of this object is a Variable Causality Dynamic Systems VCDS and it is analyzed from this point of view.

A pair of legs $\{L_i, L_j\}$ constitutes the Active Pair of Legs (APL) if the robot body position is the same irrespective of the feet position of all the other legs different of L_i and L_j. A label is assigned to each possible APL. The APL label is expressed by a variable q called Index of Activity (IA), which can take N_a values, numbers or strings of characters.

All the other legs that at a time instant do not belong to APL are called Passive Legs (PL). The leg in APL, having a free joining point (FJP) is called slave leg, the opposite of the motor (or master) leg whose both joining points are external controlled (EC).

3 Causality ordering of an active pair of legs with one free joint

The kinematics structure of the robot can be followed in Fig. 1. In this structure, only one angle is free so three joints are external controlled (EC). Considering also the pair $\{L_i, L_j\}$ as APL, this is denoted by

$q = 'ij', c = [\text{motor12, motor01, } c^p]$ or

$q = 'ij', c = [\text{motor12, motor02, } c^p]$.

In this paper, it is considered the causality ordering $c = [\text{motor12, motor02}, c^p]$ corresponded to the state having the leg L^i as a motor (master) leg, external controlled (EC) by its angles $u^{1,i}$, $u^{2,i}$, which controls two degree of freedom, and the leg L^j, a slave leg, which can control only one scalar component. In this causality ordering, the angle $u^{2,j}$ is external controlled (EC) and the angle $u^{1,j}$ is free.

The kinematics restriction

$$\left|R^j - R^i\right| = r_{ij} \tag{10}$$

is consumed by changing the angle $u^{1,j}$ at a value

$$u^{1,j} = \tilde{u}^{1,j} = F_{1j}(u^{1,i}, u^{2,i}, u^{2,j}, G^i, G^j, s^j). \tag{11}$$

From the kinematics restriction moved to the input,

$$F_{ji}(\cdot) = \left|\lambda^j - \lambda^i + \tilde{AB}{}^j_1 - AB^i\right| - \left|G^j - G^i\right| = 0 \tag{12}$$

where

$$AB^i = e^{j\cdot u^{2,i}} \cdot [b^i + a^i \cdot e^{j\cdot u^{1,i}}] \tag{13}$$

$$\tilde{AB}{}^j_1 = e^{j\cdot u^{2,j}} \cdot [b^j + a^j \cdot e^{j\cdot \tilde{u}^{1,j}}]. \tag{14}$$

it is obtained the angle $u^{1,j} = \tilde{u}^{1,j}$.

So, $F_{ji}(\cdot) = 0$

$$\Rightarrow u^{1,j} = \tilde{u}^{1,j} = F_{1j}(u^{1,i}, u^{2,i}, u^{2,j}, G^i, G^j, s^j). \tag{15}$$

The kinematics restriction (12) can be also written

$$\left|\lambda^j - \lambda^i - AB^i + b^j \cdot e^{j\cdot u^{2,j}} + a^j \cdot e^{j\cdot u^{2,j}} \cdot e^{j\cdot \tilde{u}^{1,j}}\right| = \left|G^j - G^i\right| \tag{16}$$

(17)

$$\left|\frac{\lambda^j - \lambda^i - AB^i + b^j \cdot e^{j\cdot u^{2,j}}}{a^j \cdot e^{j\cdot u^{2,j}}} + e^{j\cdot \tilde{u}^{1,j}}\right| = \left|\frac{G^j - G^i}{a^j}\right| \tag{18}$$

It is noted,

$$W_{1j} = a^j \cdot e^{j\cdot u^{2,j}} \tag{19}$$

$$Q_{1j} = \frac{\lambda^j - \lambda^i - AB^i + b^j \cdot e^{j\cdot u^{2,j}}}{W_{1j}} = q^x_{1j} + jq^z_{1j} \tag{20}$$

$$g_{1j} = \left|\frac{G^j - G^i}{a_j}\right| \tag{21}$$

$$q^x_{1j} = \mathrm{Re}\{Q_{1j}\} = \mathrm{Re}\left\{\frac{\lambda^j - \lambda^i - AB^i + b^j \cdot e^{j\cdot u^{2,j}}}{W_{1j}}\right\} \tag{22}$$

$$q^z_{1j} = \mathrm{Im}\left\{\frac{\lambda^j - \lambda^i - AB^i + b^j \cdot e^{j\cdot u^{2,j}}}{W_{1j}}\right\} \tag{23}$$

With these notes, the equation (18) becomes

$$\left|Q_{1j} + e^{j\cdot u^{1,j}}\right| = g_{1j} \tag{24}$$

having the angle $u^{1,j}$ as unknown.

Using the following notations:

$$a_{1j} = 2 \cdot q^x_{1j} \tag{25}$$

$$b_{1j} = 2 \cdot q^z_{1j} \tag{26}$$

$$c_{1j} = g^2{}_{1j} - 1 - (q^x_{1j})^2 - (q^z_{1j})^2 = g_{1j} - 1 - \left|Q_{1j}\right|^2. \tag{27}$$

from (24) is obtained a trigonometric equation

$$a_{1j} \cdot \cos(u^{1,j}) + b_{1j} \cdot \sin(u^{1,j}) = c_{1j} \tag{28}$$

with the solutions given by the relation (15).

It is calculated,

$$\varphi_{1j} = \begin{cases} \mathrm{arctg}\left(\frac{a_{1j}}{b_{1j}}\right) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), & b_{1j} \neq 0 \\ 0\ , & b_{1j} = 0\ ,\ a_{1j} = 0 \\ \frac{\pi}{2}\ , & b_{1j} = 0\ ,\ a_{1j} > 0 \\ -\frac{\pi}{2}\ , & b_{1j} = 0\ ,\ a_{1j} < 0 \end{cases} \tag{29}$$

The case b_{1j}=0 and a_{1j}≠0.

It is calculated,

$$\psi_{1j} = \arccos\left(\frac{c_{1j}}{a_{1j}}\right) \in [0, \pi] \tag{30}$$

$$\tilde{u}^{1,j}(t) = \begin{cases} \psi_{1j} \text{ daca } u^{1,j}(t-\varepsilon) > 0 \Leftrightarrow s^j = \text{'sus'}, \\ \qquad b_{1j} = 0,\ a_{1j} \neq 0 \\ -\psi_{1j} \text{ daca } u^{1,j}(t-\varepsilon) \leq 0 \Leftrightarrow s^j = \text{'jos'}, \\ \qquad b_{1j} = 0,\ a_{1j} \neq 0 \end{cases} \tag{31}$$

The case a_{1j}=0 and b_{1j}=0 is possible only if c_{1j}=0.

It is chosen,

$$\tilde{u}^{1,j} = 0\ ,\ a_{1j}=0,\ b_{1j}=0 \ \rightarrow\ c_{1j}=0 \rightarrow \tilde{u}^{1,j} = 0. \tag{32}$$

The case b_{1j}≠0.

It is calculated,

$$\sin(u^{1,j} + \varphi_{1j}) = \frac{|b_{1j}|}{b_{1j}} \cdot \frac{c_{1j}}{\sqrt{a^2_{1j} + b^2_{1j}}} \tag{33}$$

$$\psi_{1j} = \arcsin\left(\frac{|b_{1j}|}{b_{1j}} \cdot \frac{c_{1j}}{\sqrt{a^2_{1j} + b^2_{1j}}}\right) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \tag{34}$$

$$\tilde{u}^{1,j} = -\varphi_{1j} + \psi_{1j} + k\pi. \tag{35}$$

To respect the continuity condition of the angle $u^{1,j}$ time evolution there are chosen the solutions above

If $\quad u^{1,j}(t-\varepsilon) \leq 0 \Leftrightarrow s^j = \text{'down'}$

$$\tilde{u}^{1,j}(t) = \begin{cases} -\varphi_{1j} + \psi_{1j} & \text{dacă } -\varphi_{1j} + \psi_{1j} \leq 0 \\ -\varphi_{1j} + \psi_{1j} - \pi & \text{dacă } -\varphi_{1j} + \psi_{1j} > 0 \end{cases} \tag{36}$$

If $\quad u^{1,j}(t-\varepsilon) > 0 \Leftrightarrow s^j = \text{'up'}$

$$\tilde{u}^{1,j}(t) = \begin{cases} -\varphi_{1j} + \psi_{1j} + \pi & \text{dacă } -\varphi_{1j} + \psi_{1j} \leq 0 \\ -\varphi_{1j} + \psi_{1j} & \text{dacă } -\varphi_{1j} + \psi_{1j} > 0 \end{cases} \tag{37}$$

Therefore, in the causality structure, $c = [\text{motor12, motor02, } c^p]$ the kinematics restriction $\left|R^j - R^i\right| = r_{ij}$ is accomplished by changing the value of $u^{1,j}$ at $\tilde{u}^{1,j}$ giving by the equation (31), (36), (37).

4 Experimental results

The ordering c=[motor12, motor02, c^p], developed in this paper is implemented together with other causal orderings on RoPa, an experimental platform for design and simulation of walking robots.

The RoPa platform is a complex of Matlab programs to analyze and design walking robots evolving in uncertain environments, according to a new control approach called Stable State Transition Approach (SSTA), conceived by the authors.

The causality ordering developed in this paper is activated by selecting the causal variable $cz = [1\,2\,2\,0]$.

The RoPa platform allows animation, recording of the evolutions and playback them.

In the following there are presented some experimental results of walking robot behavior considering this causality ordering.

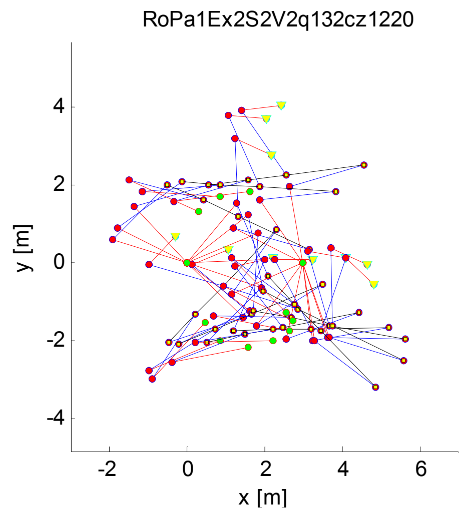

Fig. 2. The robot kinematics evolution

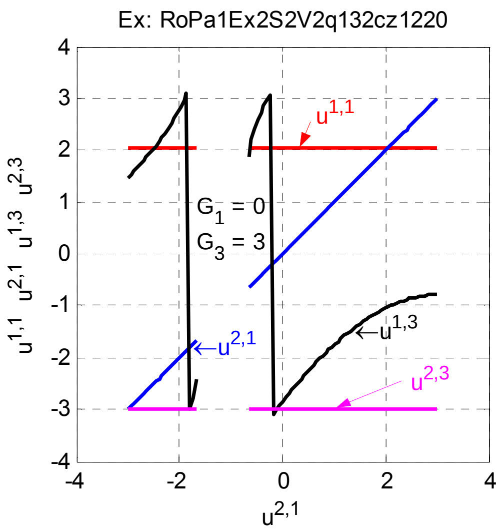

Fig. 3. Controlled angles with respect to the input angle

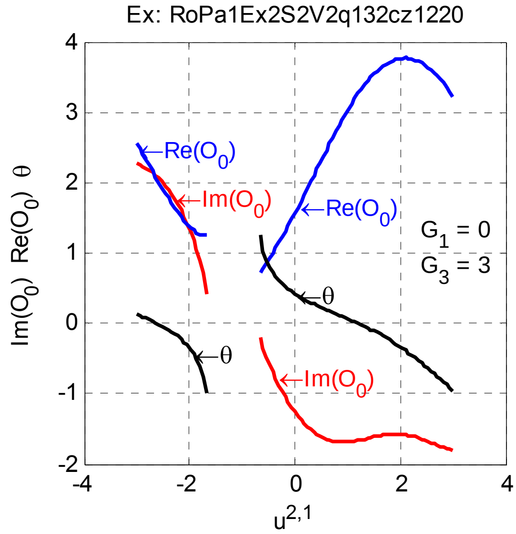

Fig. 4. Robot body position with respect to the input angle

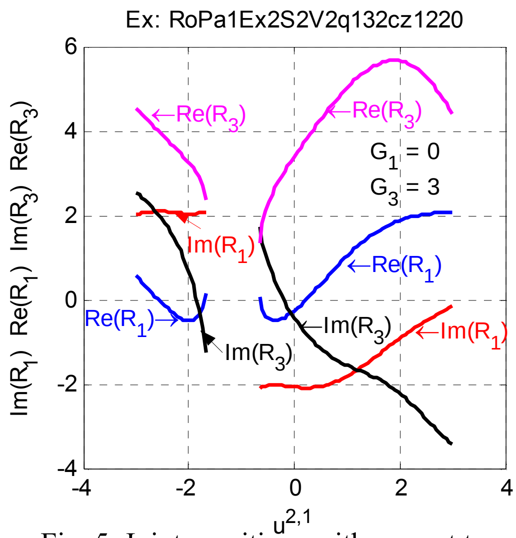

Fig. 5. Joints positions with respect to the input angle

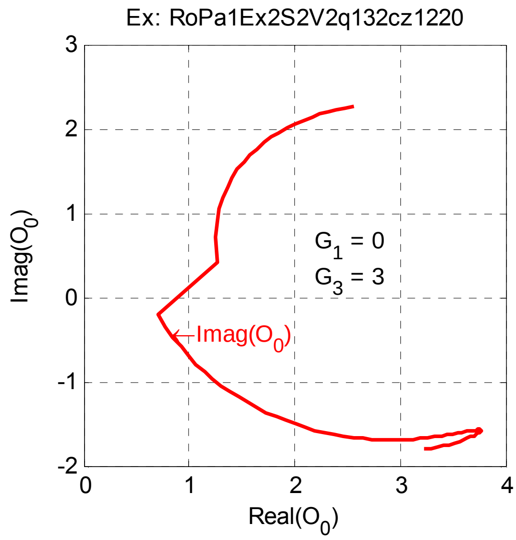

Fig. 6. Reference point O^0 evolution scene

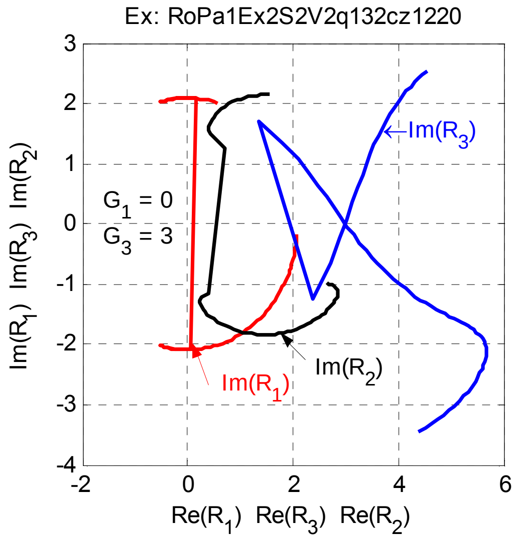

Fig. 7. Joints points' locus in evolution scene

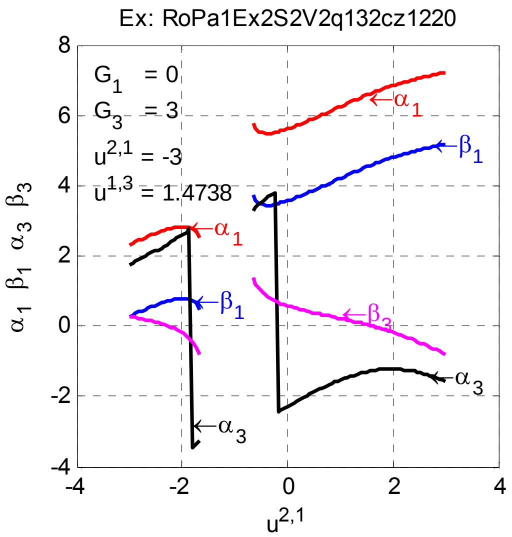

Figura 8. Legs angular coordinates with respect to the input angle

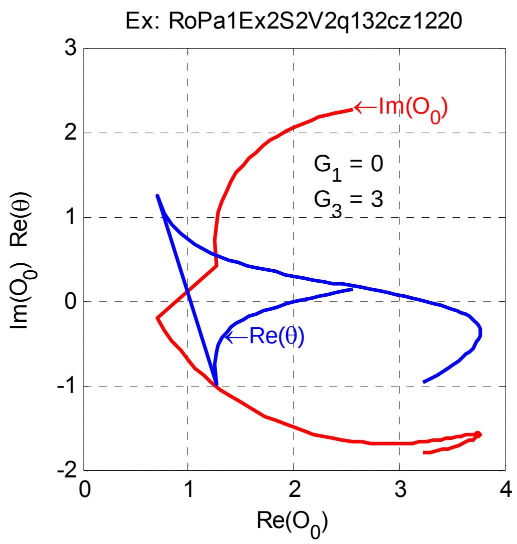

Figura 9. Robot body angular position with respect to horizontal position

5 Conclusion

Robot behavior with controlled attitude of legs represents one of the possible causality orderings of the robot, considered as a Variable Causality Dynamic System (VCDS). This evolution state is necessary for legs control in order to avoid the impact of the robot body and legs joints with the ground.

As a result of such evolutions, the robot reaches a state that is taken over in this way by the other causality orderings operated by the walking algorithm. In this way are avoided the possible random evolutions due to the initials states where the change of causality orderings is realized.

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