Filtration and Dispersion in a Porous Medium with Multiscale Conductivity and Porosity

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Abstract: Field measurements of conductivity, porosity, etc. have shown their are high heterogeneity, the bounds of such heterogeneity increasing as the scale of observations changes. This has led to the development of fractal models, renormgroup methods, and methods of subgrid modeling. The subgrid modeling approach, associated with problems of the subsurface hydrodynamics, is presented. We consider a single-phase flow of an incompressible fluid through a random porous medium. The joint multi-point probability distribution for porosity and permeability is supposed to be log-normal and satisfy the conditions of the Kolmogorov refined scaling hypothesis. A subgrid model is derived which is similar to the Landau-Lifschitz formula. The theoretical result is compared to the results of the direct numerical modeling and to the results of "ordinary" perturbation theory.

Key-Words: - random fields, effective parameters, subgrid modeling, scaling, multifractal

1 Introduction

When studying the mass transfer in a heterogeneous medium, the small-scale details of conductivity and porosity are unknown. They should be considered within the statistical approach, introducing effective parameters. The search for the effective coefficients in the equations for the large-scale components of filtration and dispersion may be regarded as a version of the subgrid modeling. Our use of the subgrid modeling method is motivated by the refined scaling properties [1] that were experimentally observed in the subsurface hydrodynamics [2]. In [3], [4], Kuz'min and Soboleva defined a conformal symmetric model of heterogeneous media and deduced the subgrid formulas for the effective permeability. In [4], the authors used the ideas of the Wilson renormalization group (RG) [5] and derived the Landau-Lifshits formula for the effective permeability coefficient. According to the arguments stated in [6], the renormalization group methods partially take account of high orders of perturbation theory and hence improve the accuracy of the derived formulas. The same arguments are also applicable to the subgrid modeling. The direct numerical verification of this statement is the main subject of this paper. If a medium is assumed to satisfy the refined scaling Kolmogorov hypothesis [1], the subgrid model equations take an especially simple form. In the present paper, we find the subgrid modeling formulas for solving problems of filtration and dispersion in a fractal porous media. In differential equations, which describe renormalization of effective parameters, one may abandon the self-similarity requirements. The RG methods for the filtration theory are developed many authors [7], [8]. The subgrid modeling methods propose to treat conductivity and porosity directly as a multifractal, but not to treat such parameters through the “window” of a logarithm. This is especially necessary, when parameters have a log-stable distribution. In this case, the variance of conductivity under certain values parameters is finite, although the variance of the logarithm conductivity is infinite. Such statements have been experimentally supported [9].

2 The refined scaling of the porous media

Let an incompressible fluid flow through a heterogeneous medium with a conductivity coefficient $\varepsilon(\mathbf{x})$. At low Reynolds numbers, the filtration velocity $\mathbf{v}$ and the pressure p are related by the Darcy law $\mathbf{v}=\varepsilon(\mathbf{x})\nabla p$. The condition of incompressibility $div\,\mathbf{v}=0$ yields the equation

$$\nabla_i \varepsilon(\mathbf{x}) \nabla_i p(\mathbf{x})=0,\ \ p(\mathbf{x})\big|_S = p_0(\mathbf{x}), \qquad (1)$$

where S is a boundary of the domain V. Let the field of conductivity be known. This means that it is measured at each point $\mathbf{x}$ as the fluid is pumped through a sample of small size l_0. A random function of spatial coordinates $\varepsilon(\mathbf{x})$ is considered as limit of the conductivity $\varepsilon(\mathbf{x})_{l_0}$. As $l_0 \to 0$, we have $\varepsilon(\mathbf{x})_{l_0} \to \varepsilon(\mathbf{x})$. To pass to a coarser grid l_1, one can smooth the resultant field $\varepsilon(\mathbf{x})_{l_0}$ using the scale $l_1 > l_0$. The obtained field is not the true conductivity that describes filtration in the interval of scales (l_1, L), where L is a maximum scale of heterogeneities. To find conductivity on a coarser grid, one has to repeat the measurements, pumping the fluid through a larger sample of size l_1. This procedure is necessary since the conductivity fluctuations within the scale interval (l_0, l_1) have correlations with the pressure fluctuations induced by them. Similar to [1, 3], we consider a dimensionless field ψ equal to the ratio of conductivity smoothed using two different scales $\psi(\mathbf{x}, l, l_1) = \varepsilon(\mathbf{x})_{l_1} / \varepsilon(\mathbf{x})_l$, where $\varepsilon(\mathbf{x})_l$ is the conductivity $\varepsilon(\mathbf{x})_{l_0}$ smoothed over scale l, $l_1 < l$. The field $\psi(\mathbf{x}, l, l_1)$ has too many arguments. We define a simpler field $\varphi(\mathbf{x}, l) = \partial \psi(\mathbf{x}, l, l\lambda) / \partial \lambda \,|_{\lambda=1}$, $\lambda = l_1 / l$, that contains the same information, then we have the relation

$$\frac{\partial \ln \varepsilon(\mathbf{x})_l}{\partial \ln l} = \varphi(\mathbf{x}, l). \quad (2)$$

The solution to Eq. (2) has the form

$$\varepsilon(\mathbf{x})_{l_0} = \varepsilon_0 \exp\left[-\int_{l_0}^{L} \varphi(\mathbf{x}, l_1) \frac{dl_1}{l_1} \right]. \quad (3)$$

We suppose that the conductivity has heterogeneities of the scale l_1 from the interval (l_0, L), where l_0 is minimal and L is maximal scales of the measurements, $L^3 \ll V$, $\varepsilon(\mathbf{x}) = \varepsilon(\mathbf{x})_{l_0}$. The field $\varphi(\mathbf{x}, l)$ is assumed to be statistically homogeneous and isotropic and then a correlation function is as follows

$$\langle \varphi(\mathbf{x}, l) \varphi(\mathbf{y}, l_1) \rangle - \langle \varphi(\mathbf{x}, l) \rangle \langle \varphi(\mathbf{y}, l_1) \rangle = \Phi\left((\mathbf{x} - \mathbf{y})^2, l, l_1 \right),$$

where $\langle \; \rangle$ is the probability averaging. For simplicity, we use the same notation Φ in the right-hand side. If the function φ is statistically invariant to the scale transform, its correlation is satisfied by $\Phi\left((\mathbf{x} - \mathbf{y})^2 / l^2 \right)$. The approximation, the fields $\varphi(\mathbf{x}, l), \varphi(\mathbf{y}, l_1)$ with different scales l, l_1 at any $\mathbf{x}, \mathbf{y}$ are considered to be statistically independent. This is usually assumed in the scaling models and reflects the decay of statistical dependence, when the scales of fluctuations become different in magnitude. This means that $\varphi(\mathbf{x}, l), \varphi(\mathbf{y}, l_1)$ are delta correlated in the logarithm of scale. The latter was proposed in [1]. A simple model without such a request is a conformal symmetric model [3]. To describe the probability distribution for the integral from (7) for large L / l_0, we use the theorem about sums of independent variables. If the variance of $\varphi(\mathbf{x}, l)$ at a given point exists, then the theorem says that the integral from (7) for very large L / l_0 tends to a normal field. In the opposite case (the second correlation function does not exist), the integral tends to a field described by a stable distribution [10]. The case of a stable distribution is considered in [11]. In this paper, for simplicity, it is assumed that $\varphi(\mathbf{x}, l)$ has normal distribution.

3 A subgrid model

The conductivity function $\varepsilon(\mathbf{x})$ is divided into two components with respect to the scale l. The large-scale component (ongrid) $\varepsilon(\mathbf{x}, l)$ is obtained by statistical averaging over all $\varphi(\mathbf{x}, l_1)$ with $l_0 < l_1 < l$, where $dl = l - l_0$ is small. A small-scale (subgrid) component is equal to $\varepsilon'(\mathbf{x}) = \varepsilon(\mathbf{x}) - \varepsilon(\mathbf{x}, l)$:

$$\varepsilon(\mathbf{x}, l) = \varepsilon_0 \exp\left[-\int_l^L \varphi(\mathbf{x}, l_1) \frac{dl_1}{l_1} \right] \left\langle \exp\left[-\int_{l_0}^{l} \varphi(\mathbf{x}, l_1) \frac{dl_1}{l_1} \right] \right\rangle,$$

$$\varepsilon'(\mathbf{x}) = \varepsilon(\mathbf{x}, l) \left[\frac{\exp\left(-\int_{l_0}^{l} \varphi(\mathbf{x}, l_1) \frac{dl_1}{l_1} \right)}{\left\langle \exp\left(-\int_{l_0}^{l} \varphi(\mathbf{x}, l_1) \frac{dl_1}{l_1} \right) \right\rangle} - 1 \right]. \quad (4)$$

A large-scale (ongrid) component of the pressure $p(\mathbf{x}, l)$ is obtained as averaging solutions of Eq.(1), in which a large-scale component of conductivity is fixed, and a small component $\varepsilon'(\mathbf{x})$ is a random variable. A subgrid component of the pressure is $p'(\mathbf{x}) = p(\mathbf{x}) - p(\mathbf{x}, l)$. Substituting the expression for $\varepsilon(\mathbf{x}), p(\mathbf{x})$ in Eq.(1) and averaging over a small-scale component, we obtain:

$$\nabla_i \left[\varepsilon(\mathbf{x}, l) \nabla_i p(\mathbf{x}, l) + \langle \varepsilon'(\mathbf{x}) \nabla_i p'(\mathbf{x}) \rangle_{\varepsilon(\mathbf{x}, l)} \right] = 0, \quad (5)$$

where $\langle \; \rangle_{\varepsilon(\mathbf{x}, l)}$ is averaging over $l_1 < l$ when $\varepsilon(\mathbf{x}, l)$ is fixed. The second term in Eq.(5) is unknown. This cannot be rejected without preliminary estimation, since the correlation between the conductivity and the pressure gradient may be substantial. The choice of the form of the second term in (5) determines the subgrid model. This expression is estimated using perturbation theory. Subtracting (5) from (1) and ignoring the terms of second order of smallness including $\varepsilon'(\mathbf{x})\nabla_i p'(\mathbf{x}) - <\varepsilon'(\mathbf{x})\nabla_i p'(\mathbf{x})>_{\varepsilon(\mathbf{x},l)}$, we obtain the subgrid equation for the pressure:

$$\Delta p'(\mathbf{x}) = -\frac{1}{\varepsilon(\mathbf{x},l)}\nabla_i\varepsilon'(\mathbf{x})\nabla_i p(\mathbf{x},l). \qquad (6)$$

The variables $\varepsilon(\mathbf{x},l)$, $p(\mathbf{x},l)$ in the right-hand side of Eq.(6) are considered to be known, these variables and their derivatives varying slower than $\varepsilon'(\mathbf{x})$ and its derivatives. Therefore,

$$p'(\mathbf{x}) \approx \frac{1}{4\pi\varepsilon(\mathbf{x},l)}\int_V \frac{1}{r}\nabla_j\varepsilon'(\mathbf{x}')d\mathbf{x}\nabla_j p(\mathbf{x},l), \qquad (7)$$

where $r = |\mathbf{x}-\mathbf{x}'|$. From (7), obtain

$$<\varepsilon'(\mathbf{x})\nabla_i p'(\mathbf{x})>_{\varepsilon(\mathbf{x},l)} =$$

$$\frac{1}{4\pi}\int\frac{\partial^2}{\partial x'_i\,\partial x'_j}\frac{1}{r}\left\langle\varepsilon'(\mathbf{x})\varepsilon'(\mathbf{x}')\right\rangle d\mathbf{x}'\frac{\nabla p_j(\mathbf{x},l)}{\varepsilon(\mathbf{x},l)}. \qquad (8)$$

From (4), as $\varepsilon'(\mathbf{x})$ has log-normal distribution, we have

$$\left\langle\varepsilon'(\mathbf{x})\varepsilon'(\mathbf{x}')\right\rangle \approx \varepsilon(\mathbf{x},l)^2\Phi(r,l)dl/l. \qquad (9)$$

Using (9) and the equality

$$\int n_i n_j\, d\omega = 4\pi/3\delta_{ij},\ n_i = x_i/r,$$

where ω is a full solid angle, obtain

$$<\varepsilon'\nabla_i p'>_{\varepsilon(\mathbf{x},l)} \approx -\Phi_0(l)\varepsilon(\mathbf{x},l)\nabla_i p(\mathbf{x},l)dl/3l, \qquad (10)$$

where $\Phi_0(l) = \Phi(0,l)$. Here, the integration over the finite volume V in (7) is replaced by the integration with infinite limits, because the correlation function Φ is small outside the domain of scale L. Such a substitution gives a coarse estimation near to the boundary, but this does not affect the determined mean values, because $L^3 \ll V$. Substituting (10) in Eq. (5) in the limit $l \to l_0$, we come to the expression for the effective coefficient, which correctly describes a mean value of the filtration velocity:

$$\varepsilon(\mathbf{x})_{ef} = \varepsilon^1{}_{0l}\exp\left[-\int_l^L\varphi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right],$$

$$\frac{d\ln\varepsilon^1_{0l}}{d\ln l} = -\langle\varphi\rangle + \frac{1}{6}\Phi_0(l). \qquad (11)$$

If a function φ is statistically invariant to the scale transform, the solution to Eq.(11) has especially a simple form: $\varepsilon^1_{0l} = \varepsilon_{0L}(l/L)^{-\langle\varphi\rangle+\Phi_0/6}$, where the constant ε_{0L} describes the filtration velocity for the largest scale $\langle\mathbf{v}\rangle = -\varepsilon_{0L}\nabla\langle p\rangle$.

3 A subgrid model for dispersion

Let at the initial time a colored liquid flow-in into a volume filled with a pure liquid. The interface is labeled with passive particles, which are moved by a stationary velocity field. Since both liquids have the same physical parameters, their filtration velocities satisfy Darcy equation (1). The movement of the labeled particles is described by the equation

$$m(\mathbf{x})\frac{d\mathbf{x}_i}{dt} = \varepsilon(\mathbf{x})\nabla p, \quad \mathbf{x}_i(0) = \mathbf{x}_{i0}, \qquad (12)$$

where $i = 1,...,N$ is the number of a particle. The porosity coefficient $m(\mathbf{x})$ is constructed similar to the conductivity coefficient:

$$m(\mathbf{x})_{l_0} = m_0\exp\left[-\int_{l_0}^L\chi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right]. \qquad (13)$$

The function $\chi(\mathbf{x},l_1)$ is assumed to have a normal distribution and to be delta correlated in the logarithm of scale. In fact, the distribution cannot be exactly Gaussian, because the normal density has a negative tail. The distribution of the integral in (13) is more likely an asymmetric stable distribution or the positive part of the normal distribution. This case should be considered separately, because of cumbersome formulas. Here, for the numerical modeling, a natural limit for the porosity $0 < m(\mathbf{x}) < 1$ is satisfied by selecting normal distribution parameters. The correlation between the porosity and the conductivity fields is determined via the correlation of the fields $\Phi^{\chi\varphi}(\mathbf{x},\mathbf{x},l,l_1) = \Phi_0^{\chi\varphi}(l)\delta(\ln l - \ln l_1)$. In this case, the self-similarity of these fields is not violated. Let us derive a formula for the joint contribution of small-scale fluctuations of conductivity and porosity to the evolution of a large-scale velocity of labeled particles. The field of porosity .is divided, as in the previous section, into two components with respect to a scale l.Here $m(\mathbf{x},l)$ is determined in the same way as conductivity

$$m(\mathbf{x},l) = m_0\exp\left[-\int_l^L\chi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right]$$

$$\left\langle\exp\left[-\int_{l_0}^l\chi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right]\right\rangle,$$

$$m'(\mathbf{x}) = m(\mathbf{x},l)\left[\frac{\exp\left(-\int_{l_0}^{l}\chi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right)}{\left\langle\exp\left(-\int_{l_0}^{l}\chi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right)\right\rangle}-1\right], \quad (14)$$

We carry out similar partitions for the displacement and the pressure

$$\mathbf{x}(t) = \mathbf{x}(t,l)+\mathbf{x}', \quad p(\mathbf{x}) = p(\mathbf{x},l)+p'(\mathbf{x}),$$

where $p(\mathbf{x},l)$, $\mathbf{x}(t,l)$ are, respectively, the solutions to Eq.(1), Eq.(12) averaged over the small-scale fields ε', m'. Averaging equation (12) over ε', m' with given ε, m yields the ongrid equation for the particles velocities:

$$m(\mathbf{x},l)\frac{d\mathbf{x}_i(t,l)}{dt} = -\varepsilon(\mathbf{x},l)\nabla p(\mathbf{x},l) - \langle\varepsilon'(\mathbf{x})\nabla p'(\mathbf{x})\rangle - \left\langle m'(\mathbf{x})\frac{d\mathbf{x}'_i(t)}{dt}\right\rangle. \quad (15)$$

The velocity of simulated fluctuations $\frac{d\mathbf{x}'_i(t)}{dt}$ is found from the equation:

$$\frac{d\mathbf{x}'_i}{dt} = -\frac{\varepsilon'(\mathbf{x})}{m(\mathbf{x},l)}\nabla p(\mathbf{x},l) - \frac{\varepsilon(\mathbf{x},l)}{m(\mathbf{x},l)}\nabla p'(\mathbf{x}) - \frac{m'(\mathbf{x})}{m(\mathbf{x},l)}\frac{d\mathbf{x}_i(t,l)}{dt}. \quad (16)$$

The substitution of (16) into Eq.(15) gives

$$-\left\langle m'(\mathbf{x})\frac{d\mathbf{x}'_i}{dt}\right\rangle = \frac{\langle m'(\mathbf{x})\varepsilon'(\mathbf{x})\rangle}{m(\mathbf{x},l)}\nabla p(\mathbf{x},l) + \frac{\varepsilon(\mathbf{x},l)}{m(\mathbf{x},l)}\langle m'(\mathbf{x})\nabla p'(\mathbf{x})\rangle + \frac{\langle m'(\mathbf{x})^2\rangle}{m(\mathbf{x},l)}\frac{d\mathbf{x}_i(t,l)}{dt}. \quad (17)$$

The second statistical moments in Eq. (17) are calculated under the same assumptions with respect to $m'(\mathbf{x})$ and the scale l as for obtaining appropriate moments when deriving effective conductivity coefficients

$$\begin{aligned} &\langle m'(\mathbf{x})m'(\mathbf{x}')\rangle \approx m(\mathbf{x},l)^2\Phi^{\chi\chi}(r,l)\,dl/l,\\ &\langle m'(\mathbf{x})\varepsilon'(\mathbf{x}')\rangle \approx m(\mathbf{x},l)\varepsilon(\mathbf{x},l)\Phi^{\varphi\chi}(r,l)\,dl/l,\\ &\langle m'(\mathbf{x})\nabla p'(\mathbf{x})\rangle \approx -\Phi^{\varphi\chi}{}_0(l)\,m(\mathbf{x},l)\nabla p(\mathbf{x},l)\,dl/3l, \end{aligned} \quad (18)$$

where $\Phi^{\varphi\chi}{}_0(l) = \Phi^{\varphi\chi}(0,l)$. For the ongrid component $m(\mathbf{x},l)$ from (14) follows

$$m(\mathbf{x},l) \approx m(\mathbf{x})_l\left[1+\left(\frac{1}{2}\Phi^{\chi\chi}(l)-\langle\chi\rangle\right)\frac{dl}{l}\right]. \quad (19)$$

If we substitute (18) into (17), we arrive at:

$$-\left\langle m'(\mathbf{x})\frac{d\mathbf{x}'_i}{dt}\right\rangle \approx \frac{2}{3}\varepsilon(\mathbf{x},l)\Phi_0^{\varphi\chi}(l)\frac{dl}{l}\nabla p(\mathbf{x},l) + m(\mathbf{x},l)\Phi_0^{\chi\chi}(l)\frac{dl}{l}\frac{d\mathbf{x}_i(t,l)}{dt}.$$

Thus, the estimation for the last term in ongrid equation (15) has been obtained. As a result, the equation takes the form:

$$m(\mathbf{x},l)\left[1-\Phi_0^{\chi\chi}\frac{dl}{l}\right]\frac{d\mathbf{x}_i(t,l)}{dt} = \varepsilon(\mathbf{x},l)\left[\frac{\Phi_0^{\varphi\varphi}+2\Phi_0^{\varphi\chi}}{3}\frac{dl}{l}-1\right]\nabla p(\mathbf{x},l).$$

Substituting the values $\varepsilon(\mathbf{x},l)$, $m(\mathbf{x},l)$ from (19) and neglecting the second order terms with respect to *dl*, yields

$$m(\mathbf{x})_l\left[1-\left(\langle\chi\rangle+\frac{1}{2}\Phi_0^{\chi\chi}\right)\frac{dl}{l}\right]\frac{d\mathbf{x}_i(t,l)}{dt} = \varepsilon(\mathbf{x})_l\left[1-\left(\langle\varphi\rangle-\frac{1}{6}\Phi_0^{\varphi\varphi}+\frac{2}{3}\Phi_0^{\varphi\chi}\right)\frac{dl}{l}-1\right]\nabla p(\mathbf{x},l).$$

In the limit $l\to l_0$, we obtain the equation for the effective conductivity and porosity

$$\begin{aligned} \frac{d\ln\varepsilon_{0l}}{d\ln l} &= -\langle\varphi\rangle+\frac{1}{6}\Phi_0^{\varphi\varphi}(l)-\frac{2}{3}\Phi_0^{\varphi\chi}(l),\\ \frac{d\ln m_{0l}}{d\ln l} &= \langle\chi\rangle+\frac{1}{2}\Phi_0^{\chi\chi}(l). \end{aligned} \quad (20)$$

If the functions φ, χ are statistically invariant to the scale transform, the parameters $\langle\varphi\rangle$, $\langle\chi\rangle$, $\Phi_0^{\chi\chi}$, $\Phi_0^{\varphi\chi}$, $\Phi_0^{\varphi\varphi}$ are constant, we obtain

$$\begin{aligned} \varepsilon_{0l} &= \varepsilon_{0L}(l/L)^{-\langle\varphi\rangle+\Phi_0^{\varphi\varphi}/6-2\Phi_0^{\varphi\chi}}\\ m_{0l} &= m_{0L}(l/L)^{-\Phi_0^{\chi\chi}/2-\langle\varphi\rangle}, \end{aligned} \quad (21)$$

where the constant m_{0L} satisfies the equation

$$\left\langle m_{0L}\frac{d\mathbf{x}_i}{dt}\right\rangle = \varepsilon_{0L}\nabla p.$$

4 Numerical modeling

For the numerical calculation, we use dimensionless variables**.** The problem is solved for $\varepsilon_0 = 1$ in a unit cube. On the edges of the cube $y=0$ and $y=1$, the pressure is set constant $p|_{y=0} = p_1$, $p|_{y=1} = p_2$, $p_1 - p_2 = 1$. On the other edges of the cube, the pressure is specified by the linear relation for y: $p = p_1 + (p_2 - p_1)y$. The main filtration flow is directed along Y-axis. The integrals in (3), (13) are replaced by a finite difference formula, in which it is convenient to pass to the logarithm with base 2:

$$\varepsilon(\mathbf{x})_{l_0} = \exp\left[-\ln 2 \sum_{i=-6}^{-4} \varphi(\mathbf{x},\tau_i)\Delta\tau\right],$$
$$m(\mathbf{x})_{l_0} = \exp\left[-\ln 2 \sum_{i=-6}^{-4} \chi(\mathbf{x},\tau_i)\Delta\tau\right]. \qquad (22)$$

For the spatial variables, we use $256\times256\times256$ grid, the scale step $\Delta\tau=1$, $\tau_i=(i-1)\Delta\tau$, $i=-6,...-4$, $l_i=2^{\tau_i}$. The delta correlation in the scale logarithm means that the fields are generated. Independently, on each scale τ_i we use the correlation functions:

$$\varphi(\mathbf{x},\tau_i)=\sqrt{\frac{\Phi_0^{\varphi\varphi}}{\ln 2}}\xi(\mathbf{x},\tau_i)+\langle\varphi\rangle\ln 2,$$
$$\chi(\mathbf{x},\tau_i)=\sqrt{\frac{\Phi_0^{\chi\chi}}{\ln 2}}\gamma\xi(\mathbf{x},\tau_i)+\sqrt{1-\gamma^2}\sigma(\mathbf{x},\tau_i)+\langle\chi\rangle\ln 2,$$

where $-1\le\gamma\le1$ γ is a correlation coefficient. The independent Gaussian fields $\xi(\mathbf{x},\tau_i)$, $\sigma(\mathbf{x},\tau_i)$ have a unit variance, zero mean and the correlation function:

$$\left\langle\zeta(\mathbf{x},\tau_i)\zeta(\mathbf{y},\tau_j)\right\rangle-\left\langle\zeta(\mathbf{x},\tau_i)\right\rangle\left\langle\zeta(\mathbf{y},\tau_j)\right\rangle$$
$$=\left\langle\zeta(\mathbf{x},\tau_i)\sigma(\mathbf{y},\tau_j)\right\rangle-\left\langle\zeta(\mathbf{x},\tau_i)\right\rangle\left\langle\sigma(\mathbf{y},\tau_j)\right\rangle \quad (23)$$
$$=\exp\left[-(\mathbf{x}-\mathbf{y})^2/2^{2\tau_i}\right]\delta_{ij}.$$

The structure of the correlation matrix allows us to represent it in the form of direct product of four matrices of lower dimensionality and apply the algorithm “along rows and columns” for the numerical simulation [12].

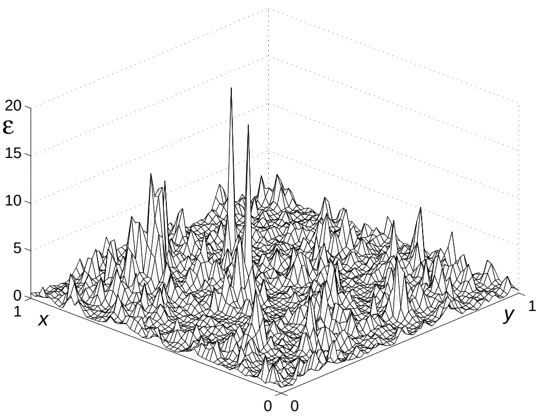

Fig.1 The conductivity for three scales in the mid-span section, $\langle\varphi\rangle=0.15$, $\Phi_0^{\varphi\varphi}=0.3$.

The constants $\langle\varphi\rangle,\langle\chi\rangle,\Phi_0^{\varphi\varphi},\Phi_0^{\chi\chi}$ should be chosen from experimental data for natural media [2]. In Fig.1, we have the self-similar conductivity in the mid-span section for formula (22). The scale of the extreme fluctuations is $L=1/8$. This allows us to replace the statistical averaging by the spatial averaging. The minimal scale is $1/32$, which is conditioned by the requirement that the difference problem considered should provide a good approximation of Eq.(1), Eq.(12). For solving Eq.(1), an iterative method combined with the Fourier transform and the sweep method is used [13]. According to the procedure of deriving the subgrid formulas, we have to numerically solve the complete problem and perform probability averaging over small-scale fluctuations to verify the formulas. As a result, we obtain a subgrid term, which can be compared to the theoretical expression. The probability averaging requires a multiple solution of the complete problem. We performed a more efficient verification, based on the power dependence of the velocity of labeled particles for a self-similar medium. We determine the corresponding mean values replacing the statistical averaging by the spatial averaging and calculate the same mean values using theoretical formulas. We also compare the results obtained to our theoretical formulas with the results obtained with "ordinary" perturbation theory. The ergodic hypothesis is verified. Effective conductivity and porosity should yield the true the velocity of the front (labeled particles) in the scale interval (l,L). The numerical verification of formula (11) has performed in [4]. Let w_i be the mean velocity of the front along Y-axis (the mean velocity along other axes is equal to zero):

$$w_k=-\left\langle\frac{\varepsilon_{\tau_k}(\mathbf{x})}{m_{\tau_k}(\mathbf{x})}\frac{\partial}{\partial}p(\mathbf{x},\tau_k)\right\rangle.$$

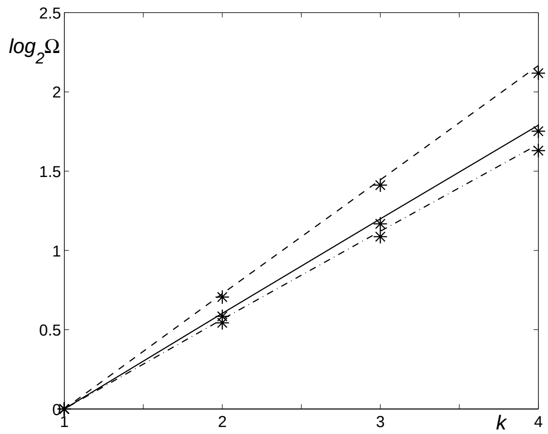

Fig.2 Lines 1, 2, 3 show the theoretical dependence for the correlation coefficient $\gamma=1,-1,0.5$, respectively. The results of numerical modeling are marked with asterisk.

The following formulas are compared with the results of numerical modeling:

$$\log_2\frac{w_\tau}{w_{\tau_L}}=\left(\langle\varphi\rangle-\frac{1}{6}\Phi_0^{\varphi\varphi}+\frac{2}{3}\Phi_0^{\varphi\chi}-\langle\chi\rangle-\frac{1}{2}\Phi_0^{\chi\chi}\right)\tau \quad (24)$$

The calculations results use the following parameters:

$\langle\varphi\rangle = 0.15,\ \Phi_0^{\varphi\varphi} = 0.3, m_0 = 0.2,\ \Phi_0^{\chi\chi} = 0.05,$

$\langle\chi\rangle = 0.7\ \ \Phi_0^{\varphi\chi} = \gamma\sqrt{0.015}.$

Fig.2 shows the dependences for different correlation coefficients. We plot the ordinate of a point $\Omega = \log_2\left(w_i / w_{-3}\right)$ and the abscissa indicates to the number of scales in (22). In Fig.3, we compare the theoretical results for w with the results of numerical simulation and results of "ordinary" perturbation theory.

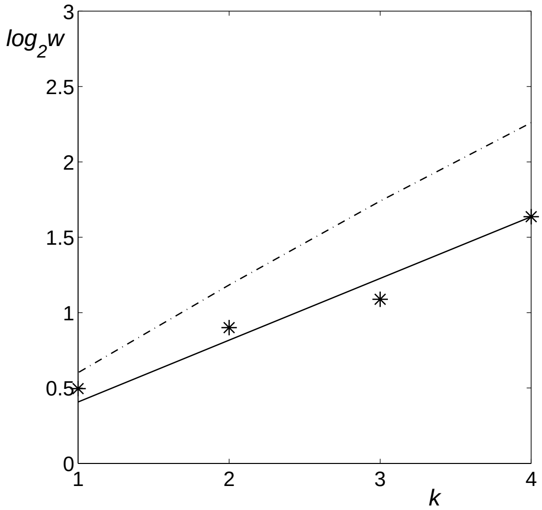

Fig.3 Lines 2, 3 show the results of "ordinary" perturbation theory and the theoretical results. The results of numerical modeling are marked with asterisk, $\langle\varphi\rangle = 0.3,\ \Phi_0^{\varphi\varphi} = 0.6, \gamma = 1.$

5 Conclusion

We have obtained the formulas enabling us to take into account the contribution of small-scale components to the calculation of mean characteristics of the fields. The conductivity and the porosity were simulated as extremely heterogeneous fields close to multifratals. The latter is attained if the scale l_0 in formulas (3), (13) tends to zero. Numerical verification is carried out for a medium, in which conductivity and porosity possess the self-similarity property. The power dependences on the scale for the effective porosity and conductivity have been calculated. The mean of filtration velocity possesses the dependence, as well as the mean velocity of the front. The formulas obtained are valid in the absence of self-similarity. In this case, the parameters of distributions in formulas (11), (20) depend on the scale. It is shown that the subgrid modeling makes possible, even with large variance of conductivity, to obtain good results. In the approach used, analysis is not beyond the scope of the differential equations apparatus and the theory of random fields. The subject of the investigations is parameters of mean values and correlation functions which can be measured

The work is supported by Russian Foundation for Basic Research, project No.04-05-64415, No.06-05-64149, Integration project No.75, SB RAS.

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