CFRP Reinforced Masonry Walls: analytical and numerical homogenised models

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Abstract: - This work proposes a study for CFRP (Carbon Fibre reinforced polymers) symmetric and not symmetric reinforced masonry walls subject to in plane and out of plane actions in the elastic range. Masonry, CFRP reinforcement and reinforced masonry have been identified with a standard elastic continuum by means of a homogenisation method in several perturbative parameters. A numerical 3D homogenised model has been compared with this analytical model. An extensive numerical analysis has been carried out to investigate the capacity of the homogenisation method – by comparison with other models existing in the literature [1], [2], [3], [4], [5], [6], to grasp the effect of CFRP reinforcement in the behaviour of the masonry.

Key-Words: - masonry, homogenisation, CFRP reinforced, in plane actions, out of plane actions

1 Introduction

This work proposes a study for CFRP (Carbon Fibre Reinforced Polymers) reinforced masonry walls subject to in-plane and out-of-plane actions in the elastic range. Even if composites laminates are nowadays widely used in some specific fields, like aerospace industry and automobile (where there is the requirement to optimise the mechanical properties of every component), in Civil Engineer the use of CFRP materials for structural strengthening is quite recent and innovative. In order to study the efficiency of this technique, some researches in this field have been carried out recently in US and Japan, especially for the reinforcement of concrete new structures ([7]).

However, the research towards the reinforcement techniques applied to masonry structures is still incomplete; in fact, some adding difficulties lies in the heterogeneous character of the masonry, since it is composed by blocks between mortar joints of lower stiffness are laid. On the other hand, the damages caused by some seismic events highlight that, even if masonry panels are correctly assembled, some inadequacies exist due to its almost total lack of strength to tensile stresses.

In order to re-establish the mechanical characteristics of damaged masonry panels or to increase the strength for a seismic-upgrading, the reinforcement with fibers seems to be very efficient in practice, also in comparison with more traditional strengthening techniques, like the injection of hydraulic grouts (which in the past was often considered the only remedy in most cases of masonry damaged) or the introduction of post tensioned steel ties. For these reasons, the study of the elastic homogenised moduli (both for the in- and out-of-plane case) of reinforced masonry is required for the evaluation of its overall behaviour under static loads. CFRP reinforced masonry has been identified with a standard elastic continuum by means of a homogenisation method in several perturbative parameters. The generic cross section comprising the masonry wall may be modelled as a thin plate formed by two or three layers. In this paper, two cases have been analysed: 1. the presence of two layers (the masonry and a single CFRP sheet disposed in tensile zone), which is the not symmetric strengthening case, particularly used in the technical applications; 2. the presence of three layers that refers to a plate composed by a middle layer (the masonry) and two additional external layers with the function of reinforcement. If the FRP sheets have the same thickness and the same mechanical characteristics, a symmetrical multi-layer plate is obtained. The 3D homogenised model adopted considers the masonry periodic in the middle plane. The size of the thickness is comparable with the dimension of the elementary periodic cell. The asymptotic model that has been developed allows the identification of the 3D solid with a 2D Love - Kirchhoff's plate, in which the anisotropy is connected with the arrangement of blocks. The obtained results give the values of homogenised plate constants –membranal and bending constants – in an analytical form [8], [9], [10], [11]. By integration along the thickness of the homogenised constants of masonry and CFRP sheets the membranal and flexural constants of reinforced masonry are obtained [12].

2 First step: masonry homogenisation

The results of a simplified solution-in an "analytical" form [10], [11] are reported. This analytical solution is based on a multi-parameter asymptotic approach.

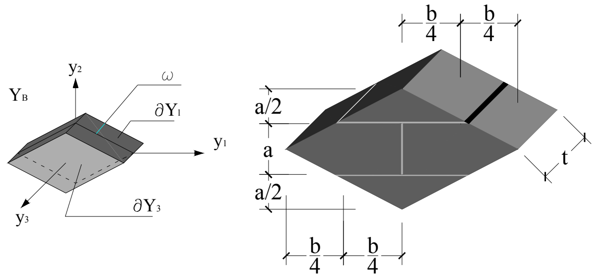


Figure 1: elementary cell with zero thickness of mortar joints (analytical model proposed by Cecchi and Sab ([10])).

Hence the following hypotheses have been adopted: the blocks are generally much stiffer than the mortar and mortar joints show a very small thickness if compared with the sizes of the blocks; this phenomenon is typical especially in the case of historical masonry. In this terms, a multi-parameters homogenisation study has been carried out to define the masonry constitutive function - when ε scale parameter (the ratio between the characteristic length of the cell and the characteristic length of the overall plate) tends to zero - by means of other two perturbative parameters: ξ the ratio between the Young's modulus of the block and the one of the mortar and φ the ratio between the thickness of the joints and the size of the characteristic module.

If the study is focused on the asymptotic case: $\xi \to 0$ and $\varphi \to 0$, the obtained asymptotic problem exhibits cohesive zero thickness interface between the blocks with possible jump of the displacement.

The constitutive function of the interface is a linear relation between the tractions on the block surfaces and the jump of the displacement field. Hence the following field problem may be formulated on the unit cell with zero thickness joints(Figure 1) in a variational form:

$$\max_{\substack{div_y\sigma=0,\sigma e_3=0 \text{ on } \partial Y_3^{\pm} \\ \sigma n \text{ antiperiodic}}} 2(E+\chi)\cdot\langle\sigma+y_3\sigma\rangle^* - \left\langle\sigma\cdot\left(a^B\right)^{-1}\sigma\right\rangle^* -$$
$$-\frac{1}{|S|}\int_\Sigma(\sigma n)\cdot K^{-1}(\sigma n)ds = \mathrm{E}\cdot\mathrm{A}^H\mathrm{E}+2\chi\cdot\mathrm{B}^{\mathrm{H}}\mathrm{E}+\chi\cdot\mathrm{D}^{\mathrm{H}}\chi =$$
$$\min_{u^{per}\ periodic}\left\langle\left(\mathrm{E}+y_3\chi+sym\left(grad_y\mathrm{u}^{per}\right)\right)\cdot\left[a^B\left(\mathrm{E}+y_3\chi+sym\left(grad_y\mathrm{u}^{per}\right)\right)\right]\right\rangle^*+$$
$$+\frac{1}{|S|}\int_\Sigma[u]\cdot K[u]ds \tag{1}$$

$\langle\cdot\rangle^*$ is the average operator on the module, **σ** is the Cauchy stress tensor, $\mathbf{u}^{per}$ is the displacement vector periodic on the module, **E** is the macroscopic in-plane strain tensor (membranal strain) that is a function of the displacement field $U_1(x_1, x_2)$, $U_2(x_1, x_2)$ and $U_3(x_1, x_2)$, hence $E_{\alpha\beta}=0.5(U_{\alpha,\beta}+U_{\beta,\alpha})$ with the greek index varying from 1 to 2 while the latin index *i,j*=1,2,3;χ is the out of plane strain tensor (plate curvature tensor) defined as : $\chi_{\alpha\beta}=-u_{3,\alpha\beta}$ and $\chi_{13}=0$. **a**(**y**) is the ω periodic constitutive function; $\mathbf{A}^H$, $\mathbf{B}^H$ and $\mathbf{D}^H$ are the constitutive asymptotic plate tensors; where Σ is the interface, [u] is the jump of displacement field at Σ and ***K*** is given by:

$$K_{ij}=\frac{1}{e}a^M_{ijkl}n_k n_l \tag{2}$$

Here *e* is the thickness of the actual joint and ***n*** is the normal to the interface. In the isotropic case, the above expression becomes:

$$\mathbf{K}=\frac{1}{e}\left(\mu^M\mathbf{I}+\left(\mu^{\mathrm{M}}+\lambda^M\right)(\mathbf{n}\otimes\mathbf{n})\right) \tag{3}$$

where μ^M and λ^M are the Lamé constants of the mortar [14]. Note that **K** tensor has a diagonal form in this case. Moreover, $\mathbf{K}=\mathbf{K}^h$ is relative to the horizontal interface and $\mathbf{K}=\mathbf{K}^v$ is relative to the vertical interface.

The asymptotic constitutive law of the plate becomes:

$$\mathbf{N}=\left\langle\sigma^H\right\rangle^*=\mathbf{A}^H\mathbf{E}+\mathbf{B}^H\chi \tag{4}$$
$$\mathbf{M}=\left\langle y_3\sigma^H\right\rangle^*={}^T\mathbf{B}^H\mathbf{E}+\mathbf{D}^H\chi \tag{5}$$

In this case the average operator $\langle\cdot\rangle^*$ is only relative to the Y^B area - the area of the blocks only – while in the standard homogenisation procedure is relative to the Y total area (blocks + mortar joints).

In the case of central symmetry of the unit cell, $\mathbf{B}^H=0$ and the asymptotic homogenised plate tensors can be equivalently defined in the following variational form:

$$\max_{\substack{div_y\sigma=0,\sigma e_3=0 \text{ on } \partial Y_3^{\pm} \\ \sigma n \text{ antiperiodic}}} 2E\langle\sigma\rangle^* - \left\langle \sigma \cdot \left(a^B\right)^{-1} \sigma \right\rangle^* - \frac{1}{|S|}\int_\Sigma (\sigma n)\cdot K^{-1}(\sigma n) ds =$$

$$= \mathrm{E}\cdot\mathrm{A}^H\mathrm{E} =$$

$$\min_{u^{per} \, periodic} \left\langle \left(\mathrm{E} + sym\left(grad_y \mathrm{u}^{per}\right)\right)\cdot\left[a^B\left(\mathrm{E} + sym\left(grad_y \mathrm{u}^{per}\right)\right)\right]\right\rangle^* +$$

$$+ \frac{1}{|S|}\int_\Sigma [u]\cdot K[u] ds \tag{6}$$

and:

$$\max_{\substack{div_y\sigma=0,\sigma e_3=0 \text{ on } \partial Y_3^{\pm} \\ \sigma n \text{ antiperiodic}}} 2\chi\cdot\langle y_3\sigma\rangle^* - \left\langle \sigma \cdot \left(a^B\right)^{-1} \sigma \right\rangle^* - \frac{1}{|S|}\int_\Sigma (\sigma n)\cdot K^{-1}(\sigma n) ds =$$

$$= \chi\cdot\mathrm{D}^H\chi =$$

$$\min_{u^{per} \, periodic} \left\langle \left(y_3\chi + sym\left(grad_y \mathrm{u}^{per}\right)\right)\cdot\left[a^B\left(y_3\chi + sym\left(grad_y \mathrm{u}^{per}\right)\right)\right]\right\rangle^* + \frac{1}{|S|}\int_\Sigma [u]\cdot K[u] ds \tag{7}$$

The homogenised membranal and bending constants are found when the blocks are isotropic linear elastic bodies connected by cohesive interfaces made with isotropic mortar. For running bond, through the variational formulation it is possible to build analytical upper and lower bounds to compare to FEM results. Following Cecchi and Sab [11] the bounds on $\mathbf{A}^H$ membranal asymptotic elastic constants and $\mathbf{D}^H$ flexural asymptotic elastic constants may be defined:

$$E\cdot\left(ta^H E\right) \le E\cdot\left(A^H E\right) \le \min\left[E\cdot\left(t\tilde{a}E\right), E\cdot\left(A^R E\right)\right] \tag{8}$$

and

$$\chi\cdot\left(\tfrac{t^3}{12}\boldsymbol{a}^H\chi\right) \le \chi\cdot\left(\boldsymbol{D}^H\chi\right) \le \chi\cdot\left(\boldsymbol{D}^R\chi\right) \tag{9}$$

$\boldsymbol{a}^H$ is the homogenised 2D elasticity tensor obtained with plane stress in the blocks and 2D restriction of $\mathbf{K}$ at the interface (= plane strain in the mortar).
$\tilde{\boldsymbol{a}}^H$ is the homogenised plane strain elasticity tensor (= plane strain in both blocks and mortar).

$$\left(\boldsymbol{A}^R\right)^{-1} = \left[\left(\boldsymbol{t}\boldsymbol{a}^{B*}\right)^{-1} + \left(\boldsymbol{A}^F\right)^{-1}\right] \tag{10}$$

where $\boldsymbol{a}^{B*}$ is the plane stress elasticity tensor of blocks and $\mathbf{A}^F$ is the homogenised membranal tensor for rigid blocks connected by elastic interfaces.
$\mathbf{D}^R$ is the homogenised out of plane tensor:

$$\left(\boldsymbol{D}^R\right)^{-1} = \left[\left(\tfrac{t^3}{12}\boldsymbol{a}^{B*}\right)^{-1} + \left(\boldsymbol{D}^F\right)^{-1}\right] \tag{11}$$

where D^F is the homogenised out of plane tensor for rigid blocks connected by elastic interfaces.
Also the bound on $\mathbf{B}^H$ homogenised tensor may be built as:

$$2\chi\cdot\left(\tfrac{t^2}{2}\boldsymbol{a}^H\boldsymbol{E}\right) \le 2\chi\cdot\left(\boldsymbol{B}^H\boldsymbol{E}\right) \le 2\chi\cdot\left(\tfrac{t^2}{2}\tilde{\boldsymbol{a}}^H\boldsymbol{E}\right) \tag{12}$$

For the masonry, explicit formulas - membrane and flexural moduli - have been already obtained by Cecchi and Sab ([11]).

3 Second step: CFRP reinforced masonry homogenisation

A model for masonry reinforced with CFRP sheets is here performed. The generic cross section comprising the masonry wall may be modelled as a thin plate formed by two or three layers: the masonry and one or two external CFRP sheets. The R.E.V. is periodic in the y_1 and y_2 axes (the periodicity directions of the masonry), which represent respectively the horizontal and vertical directions in the middle plane of the wall. Along y_3 axis -orthogonal to the middle surface of the wall - the constitutive function is piecewise constant and every layer is assumed linear elastic and orthotropic. The $A_G(y_3)$ constitutive function is defined as follows:

$$\mathbf{A}_G\left(y_3\right) = \begin{cases} \mathbf{A}^{MS} & for\ y_3 \in masonry \\ \mathbf{A}^{CFRP} & for\ y_3 \in CFRP \end{cases} \tag{13}$$

The constitutive homogenised functions for the masonry walls strengthened with CFRP sheets have been obtained in two steps. In the first step the CFRP sheets and the masonry have been homogenised separately. The procedure for the masonry homogenisation has been already explained in the previous Section.

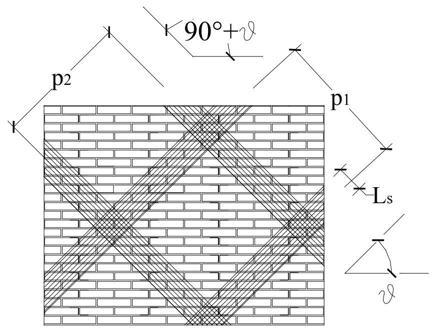


Figure 2: application of fiber strips on masonry panels (step: p_1 and p_2, width: L_s) along mutually orthogonal directions.

The CFRP has been considered make up of unidirectional fibres immersed in a polymeric matrix, with a mutually orthogonal disposition (Figure 2), hence the constitutive function is orthotropic. Here the longitudinal axis of the fibre has been considered coincident with the y_1 local axis of the composite layer. In the second step, the homogenisation of reinforced masonry has been obtained by integrating along the thickness of the wall the constitutive function of masonry and CFRP.

In the model the sheet has been substituted with a continuous equivalent orthotropic material (Figure 3).

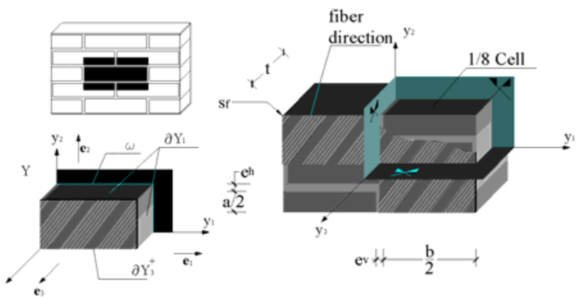


Figure 3: elementary cell used in FEM homogenisation and its geometrical characteristics.

By assuming as a reference co-ordinates system the one identified by the orthotropic principal axes of masonry, the CFRP sheet constitutive function may be referred to these axes. Two cases are possible:

1-the CFRP sheet orthotropic principal axes are coincident with masonry ones, hence the CFRP constitutive function is *especially orthotropic*;

2- the CFRP sheet orthotropic principal axes are not coincident with masonry ones, hence the CFRP constitutive function is *generally orthotropic*.

In the second case the $\mathbf{A}^{CFRP}$ constitutive tensor is expressed in a form identical to the one of an anisotropic material, but only four of the total six elastic constants that appear in the tensor are independent. In fact, the linear transformation to represent the orthotropic constitutive constants in a co-ordinate system with axes not coincident with the principal axes of the material is the following:

$$\breve{A}^{CFRP} = T^{-1} \cdot A^{CFRP} T^{-T} \qquad (14)$$

where T is the transformation tensor.

The relations between the constants of the $\breve{\mathbf{A}}^{CFRP}$ and the $\mathbf{A}^{CFRP}$ tensors are:

$$\breve{A}_{1111}{}^{CFRP} = A_{1111}{}^{CFRP} m^4 + 2\left(A_{1122}{}^{CFRP} + 2A_{1212}{}^{CFRP}\right) m^2 n^2 + A_{2222}{}^{CFRP} n^4$$
$$\breve{A}_{2222}{}^{CFRP} = A_{1111}{}^{CFRP} n^4 + 2\left(A_{1122}{}^{CFRP} + 2A_{1212}{}^{CFRP}\right) m^2 n^2 + A_{2222}{}^{CFRP} m^4$$
$$\breve{A}_{1122}{}^{CFRP} = \left(A_{1111}{}^{CFRP} + A_{2222}{}^{CFRP} - 4A_{1212}{}^{CFRP}\right) m^2 n^2 + A_{1122}{}^{CFRP}\left[m^4 + n^4\right]$$
$$\breve{A}_{1212}{}^{CFRP} = \left(A_{1111}{}^{CFRP} + A_{2222}{}^{CFRP} - 2A_{1122}{}^{CFRP}\right) m^2 n^2 + A_{1212}{}^{CFRP}\left[m^2 - n^2\right]^2$$
$$\breve{A}_{1112} = A_{1111} m^3 n - A_{2222} m n^3 + \left(A_{1122} + 2A_{1212}\right)\left[mn^3 - m^3 n\right]$$
$$\breve{A}_{2212} = A_{1111} m n^3 - A_{2222} m^3 n - \left(A_{1122} + 2A_{1212}\right)\left[mn^3 - m^3 n\right]$$
$$m = \cos\theta \qquad n = \sin\theta$$

The $\mathbf{A}_{1111}{}^{CFRP}$ constant is considered, as already mentioned, corresponding to the longitudinal fibers development.

Some meaningful cases may be pointed out:

Masonry reinforced with a single CFRP sheet: Hence the following constitutive membranal and flexural functions are obtained:

$$A_{Gijkl} = \frac{1}{t+s}\left(tA_{ijkl}^{MS} + s\breve{A}_{ijkl}^{CFRP}\right) \qquad (15)$$

$$B_{Gijkl} = \frac{1}{t+s}\left(-\frac{st}{2}A_{ijkl}^{MS} + \frac{st}{2}\breve{A}_{ijkl}^{CFRP}\right) \qquad (16)$$

$$D_{Gijkl} = \frac{1}{12(t+s)}\left(t^3 A_{ijkl}^{MS} + 3st^2 A_{ijkl}^{MS} + s^3 \breve{A}_{ijkl}^{CFRP} + 3ts^2 \breve{A}_{ijkl}^{CFRP}\right) \qquad (17)$$

where t is the thickness of the masonry walls and s is the thickness of the CFRP layer.

Middle layer of masonry reinforced with two external layers of CFRP with the same thickness and the same mechanical characteristics, but with different principal axes directions:

$$A_{Gijkl} = \frac{1}{t+2s}\left(tA_{ijkl}^{MS} + s\left(\breve{A}_{ijkl}^{CFRP\,\sup} + \breve{A}_{ijkl}^{CFRP\,\inf}\right)\right) \qquad (18)$$

$$B_{Gijkl} = \frac{1}{t+2s}\left(t\breve{A}_{ijkl}^{CFRP\,\sup} - t\breve{A}_{ijkl}^{CFRP\,\inf}\right)\frac{t+s}{2} \qquad (19)$$

$$D_{Gijkl} = \frac{1}{12(t+2s)}\left(A_{ijkl}^{MS} t^3 + \left(\breve{A}_{ijkl}^{CFRP\inf} + \breve{A}_{ijkl}^{CFRP\sup}\right)\left(3st^2 + 6ts^2 + 4s^3\right)\right) \qquad (20)$$

where t is the thickness of the masonry walls and s is the thickness of each CFRP layer. $\mathbf{A}^{CFRPinf}$ and $\mathbf{A}^{CFRPsup}$ are relative to the constants of the two CFRP layers.

Middle layer of masonry reinforced with two external CFRP layers with the same thickness, the same mechanical characteristics and coincident principal axes directions.

A multi-layer plate constructed in this way is symmetrical, hence the chosen axes are principal axes of inertia and $\mathbf{B}^H$ is zero.

$$A_{Gijkl} = \frac{1}{t+2s}\left(tA_{ijkl}^{MS} + 2s\breve{A}_{ijkl}^{CFRP}\right) \qquad (21)$$

$$B_{Gijkl} = 0 \qquad (22)$$

$$D_{Gijkl} = \frac{1}{12(t+2s)}\left(t^3 A_{ijkl}^{MS} + \breve{A}_{ijkl}^{CFRP}\left(8s^3 + 12ts^2 + 6st^2\right)\right) \qquad (23)$$

4 FEM strategy

The aim of this Section is to illustrate a numerical approach for the evaluation of the membranal and flexural homogenised moduli of masonry reinforced with CFRP sheets. For the sake of simplicity, only the running bond pattern is considered.

The technology of reinforcement varies frequently in practice, nevertheless the reinforcement is generally executed applying strips (density: 300-500 g/m^2) 100 mm-200 mm width with a fixed step (usually 0.5-1 m) along two orthogonal directions. In order to improve masonry mechanical characteristics when in-plane loads act, fiber directions not always coincide with principal axes of masonry but are disposed (more conveniently) diagonally in the so-called "$\pm 45°$ disposition".

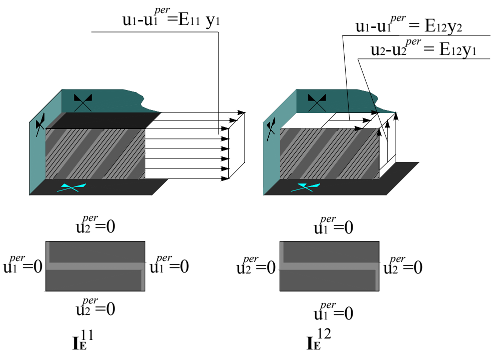


Figure 4: applied mixed boundary conditions on 1/8 cell for symmetrically disposed reinforcement and in-plane case.

The periodicity of masonry pattern do not coincide necessarily with the periodicity of CFRP strips; however, in order to conserve the periodicity of the masonry, a computationally suitable approximation can be introduced supposing the reinforcement homogeneously disposed on the surface. Consequently, the homogenisation problem for the CFRP reinforced masonry case can be treated in analogy with the not-reinforced one. Moreover, the same elementary cell used for the masonry without reinforcement can be utilised (Figure 3).
For the problem at hand, a strain-periodic displacement field, up to a rigid body motion, may always be written as follows [13]:

$$\begin{aligned} u_1(\mathbf{y}) &= E_{11}y_1 + E_{12}y_2 + y_3(\chi_{11}y_1 + \chi_{12}y_2) + u_1^{per} \\ u_2(\mathbf{y}) &= E_{12}y_1 + E_{22}y_2 + y_3(\chi_{12}y_1 + \chi_{22}y_2) + u_2^{per} \\ u_3(\mathbf{y}) &= -\left(\chi_{11}\frac{y_1^2}{2} + \chi_{22}\frac{y_2^2}{2} + \chi_{12}y_1y_2\right) + u_3^{per} \end{aligned} \tag{24}$$

where E_{ij} and χ_{ij} are respectively the *ij* components of the macroscopic in-plane and out-of-plane strain tensor and $\mathbf{u}^{per}$ is a field periodic on *Y*. In order to find the membranal and flexural homogenised moduli, two cases have to be analysed:

- *In-plane case (Membranal moduli)*

In this case, χ=0 is assumed and only the macroscopic strain tensor E is assigned. Imposing suitable boundary conditions on ∂Y_l, the elastostatic problem can be formulated only on Y; the boundary conditions are imposed using both the periodicity of u^{per} and the anti-periodicity of $\boldsymbol{\sigma}\,\mathbf{n}$ (Figure 4).
Moreover, the symmetry of the cell makes it possible to simplify the numerical model and permits to mesh respectively 1/8 of the elementary cell for the symmetrical reinforcement and ¼ for the not symmetrical one.

It is worth noting that the homogenised A_{ijhk} membranal moduli and the B_{ijhk} coupling moduli (that are $\neq 0$ only for the not symmetrical case) can be found by applying separately three different "elementary" in-plane strain tensors defined as follows:

$$\left(\mathbf{I}_E^{hk}\right)_{ij} = \frac{1}{2}\left(\delta_{ih}\delta_{jk} + \delta_{ik}\delta_{jh}\right) \tag{25}$$

where δ_{ih} is the Krönecker symbol .

If the macroscopic elementary in-plane strain tensor $\mathbf{I}_{hk}$ is applied, the homogenised A_{ijhk} and B_{ijhk} moduli can be obtained by evaluating numerically the following integrals:

$$A_{ijhk} = \frac{2}{\hat{S}}\int \sigma_{ij}dV \qquad B_{ijhk} = \frac{2}{\hat{S}}\int \sigma_{ij}y_3dV \tag{26}$$

Where: $\hat{S} = 1/4\omega$.

- *Out-of-plane case (Plate moduli)*

For the out-of-plane case, **E**=**0** is assumed and only the macroscopic curvature tensor χ is assigned. Similar remarks done for the in-plane case for the formulation of the elastostatic problem can be repeated in this case. If, for instance, the strain tensor $\mathbf{I}_\chi^{11}$ is applied, the homogenisation problem can be solved through the so-called "displacement method" ([15]) using a standard finite element program, considering only 1/8 or ¼ (for the not symmetrical case only) of the elementary cell and imposing specific boundary conditions. As a matter of fact, in this case, (24) can be simplified as follows:

$$\begin{aligned} u_1(\mathbf{y}) &= y_3\chi_{11}y_1 + u_1^{per} \\ u_2(\mathbf{y}) &= u_2^{per} \\ u_3(\mathbf{y}) &= -\chi_{11}\frac{y_1^2}{2} + u_3^{per} \end{aligned} \tag{27}$$

Taking into account the periodicity and symmetry (or anti-symmetry) conditions, it is possible to show that the mixed boundary conditions reported in Figure 5 have to be applied.
Homogenised flexural and coupling moduli can be found evaluating numerically the following integrals:

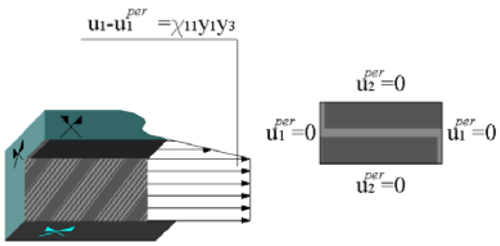
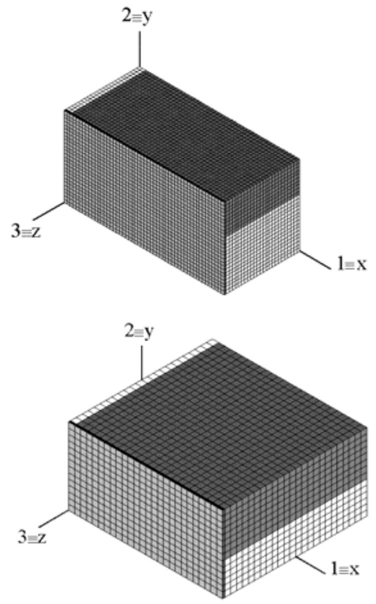


Figure 5: boundary conditions applied for I_χ^{11} case ("displacement method").

$$B_{ijhk} = \frac{2}{\hat{S}}\int \sigma_{ij} dV \qquad D_{ijhk} = \frac{2}{\hat{S}}\int \sigma_{ij} y_3 dV \tag{28}$$

5 Comparison between different homogenisation procedures

The aim of this section is to assess the accuracy of the results provided by the two-steps homogenisation reported in Section 2 and 3 in comparison with the one-step numerical procedure reported in Section 4. In the first step, the homogenised out-of-plane moduli for unreinforced masonry are evaluated using both some analytical formulas presented in technical literature ([10], [17], [16]) and a standard 3D F.E. procedure ([13]); in the second step, the homogenisation of reinforced masonry is obtained by Eq.from (15) to (23).
The reference solution is obtained by the numerical homogenisation procedure described in Section 4. Meshes –a and –b of Figure 6 have been used respectively for the symmetrical and the not symmetrical cases. Two typologies of reinforcement have been analyzed: the first is the symmetrical and bi-directional "$\pm 45°$ disposition", - particularly effective for shear loads -, the second is the not-symmetrical (bi-directional) "$90°/0°$ reinforcement", - utilized for out-of-plane loaded panels -.
As a meaningful example, let us assume the dimensions of the blocks 250 mm x120 mm x55 mm (brick UNI 5628/65) and the thickness of the mortar joints 10 mm. This is a typical structure of bearing and not-bearing walls with running bond texture. The ratio between mortar thickness and block length is 1/25; as for the in-plane case, a significant error for the solutions obtained using the interface models is expected, because of the assumption of zero thickness of mortar joints. The mechanical characteristics of blocks and mortar joints are shown in Table I. Many numerical simulations have been performed progressively reducing E_m Young modulus, in order to check the influence of reinforcement on the overall mechanical characteristics of the wall also for deteriorated mechanical properties of the joints. In fact, the reinforcement with high strength materials is employed, as a matter of fact, both for re-establishing original mechanical characteristics of damaged panels and for improving the mechanical characteristics of the brickwork.



a

b

Fig 6 -a Mesh utilized for symmetrical reinforcement (42224 nodes, 33800 8-noded brick elements). –b Mesh utilized for not-symmetrical reinforcement (10530 nodes, 9100 8-noded brick elements).

Table I	Blocks	Mortar
E: Young modulus [MPa]	11000	2200
ν : Poisson ratio [-]	0.2	0.25

The mechanical properties of reinforcement (fiber + epoxy resin) are reported in Table II.
It is worth mentioning here that, even if the equivalent orthotropic homogenised moduli for the CFRP (which is itself a composite material) should be derived experimentally or by the application of rigorous homogenisation techniques, it has been shown that quite reliable results can be obtained applying the approximated rule of moistures.
Moreover, it has been shown experimentally [19] that some differences exist on the homogenised FRP moduli due to the different fabrication techniques (Bag Molding, Resin Transfer Molding, Electric beam irradiation, etc.).

Table II	Fiber	Matrix3.0 –Epoxy PRM
Young modulus [GPa]	230	314
Volumetric fraction [%]	33.3	66.6

On the other hand, it has to be underlined that the mechanical characteristics of blocks and mortar can sensibly vary inside a panel and can be determined imprecisely. For this reasons, a simplified approach for the evaluation of CFRP moduli may be adopted in what is following.

- ***A simple 1D flexion test***

The F.E. model of Figure 7 is used for simulating a CFRP reinforced masonry column subjected to uniaxial bending. The model is constituted by 3 bricks of dimensions axbxt= 55mm x250mm x120 mm, 2 mortar joints with thickness 10 mm and a unidirectional vertical CFRP strip 50 mm wide. Mechanical properties of the constituent materials are assumed in agreement with Table I and Table II. 8750 brick elements and 10192 nodes are used for the simulation in Strand 7.01 ® and a bending moment is applied on the top through the introduction of a rigid support. It is worth noting that a similar test has been carried out both experimentally and numerically in [21] for the evaluation of the strength of eccentrically loaded reinforced masonry specimens.

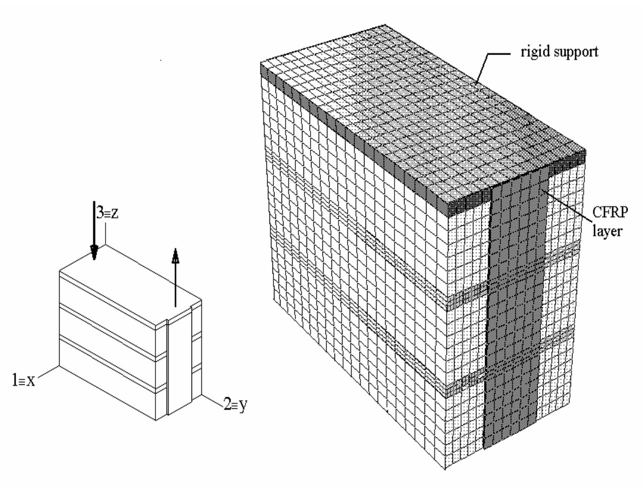
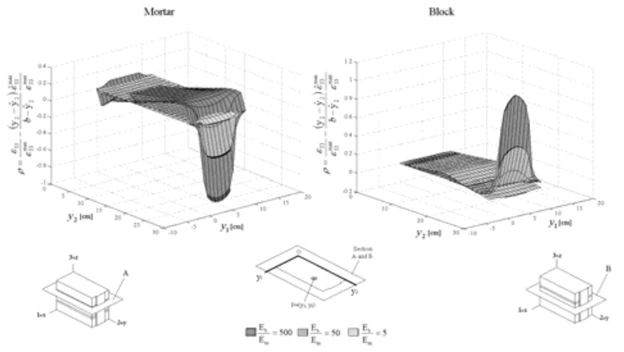


Fig. 7 Numerical 1D bending test (8750 brick elements, 10192 nodes, 30215 equations).

The aim of the elastic analysis here proposed is to show that the linearity of vertical strains along the thickness is almost respected only for low values of the ratios, while it vanishes when mortar Young modulus progressively decreases.

With regard to this aspect, in Figure 8-a the vertical strain distribution inside the mortar joint is shown for 3 different values of the E_b/E_m ratio (5, 50, 500), whereas in Figure 8-b the same component of the strain tensor evaluated in the middle of the central block is reported. The distributions are normalized in the following way:



a b

Fig. 8 Trend of normalized vertical strains in the mortar joint (-a) and in the central block (-b) for different E_b/E_m ratios.

$$\rho = \frac{\varepsilon_{33}}{\varepsilon_{33}^{\max}} - \frac{(y_2 - \hat{y}_2)}{b - \hat{y}_2}\frac{\hat{\varepsilon}_{33}^{\max}}{\varepsilon_{33}^{\max}} \tag{29}$$

where: $\hat{\varepsilon}_{33}^{\max}$ is the maximum absolute value of the vertical strain in the section for the different E_b/E_m ratios analyzed (5, 50, 500); $\varepsilon_{33}^{\max}$ is the maximum absolute value of the vertical strain in the section for E_b/E_m =500; $\hat{y}_2$ is the neutral axis from the beam theory; ε_{33} is the vertical strain numerically evaluated in correspondence of point $P = (y_1; y_2)$, Figure 8.

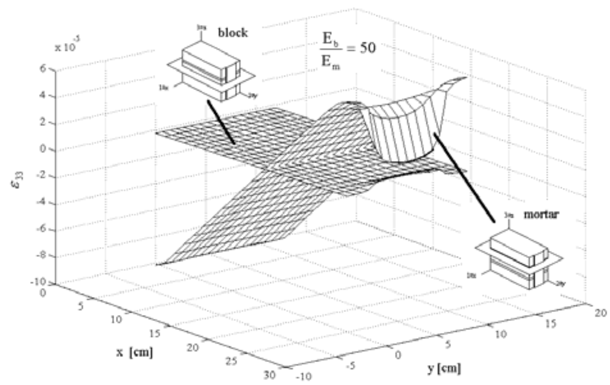


Fig. 9 Trend of vertical strains in the mortar joint and in the central block for $E_b/E_m = 50$

Finally, in Figure 9 a comparison between the vertical strain inside the joint and in the middle of the block is shown for E_b/E_m = 50. As one may expect, the strain is mainly concentrated on the joint and significantly different from the classical linear distribution; on the contrary, the distribution in the brick may be considered, at least for practical purposes, almost linear.

6 Structural comparisons

The case of simply supported square panel symmetrically reinforced subjected to uniform pressure is investigated. The panel size are: L=3100 mm and thickness t+2s=122 mm; the bricks dimensions are 250 mm x120 mm x 55 mm and the thickness of the mortar joints is assumed to be 10 mm; finally, the CFRP reinforcement is assumed disposed symmetrical and diagonal. The panel is studied by means of a full 3D F.E. analysis. The mesh utilized is shown in Figure 19 (40470 8-noded brick elements, 48384 nodes); only ¼ of the plate is considered for symmetry.

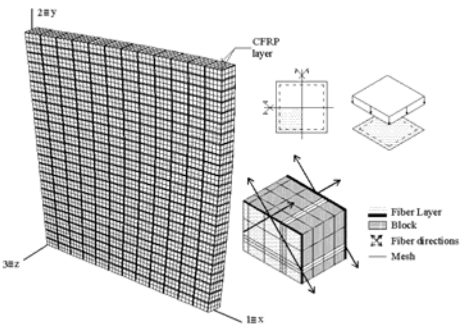


Fig 10 Deformed shape and detail of the mesh for the 3D heterogeneous model–symmetrical $\pm 45°$ disposition–.

The mechanical characteristics of blocks and mortar are summarized in Table I, while mechanical characteristics of reinforcement are summarized in Table II; several comparisons are performed progressively reducing the E_m Young modulus.

The middle point deflexions obtained using the heterogeneous 3D model are compared both with the solutions provided by the analytical models and with the F.E. homogenisation procedure previously described. It is worth noting that the displacement in the middle point of a Kirchhoff-Love simply supported orthotropic square plate subjected to uniform pressure is:

$$\delta_{max} = \frac{16pL^4}{\pi^6 D_{ort}} \sum_{n-odd} \sum_{m-odd} \frac{\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{m\pi}{2}\right)}{nm\left(n^4 + 2\left(\nu + \frac{2\beta(\alpha-\nu^2)}{\alpha}\right)n^2 m^2 + \alpha m^4\right)} \tag{30}$$

where: $D_{ort} = E(t+2s)^3/[12(1-\nu^2/\alpha)]$; $E_{11} = E$; $\alpha = E_{22}/E$; $\nu_{12} = \nu$; $\beta = G_{12}/E$; p is a constant pressure.

The flexural moduli that appear in Eq. (30) are obtained by the plate (or in-plane) homogenisation procedures previously described. In Figure 11, for instance, the horizontal strains in the middle of the plate obtained by means of the heterogeneous model of Figure 10 and by means of the F.E. one-step homogenised model are compared. It is worth noting that for the homogenised (orthotropic) plate, the strains ε_{11} are evaluated making use of the classical hypothesis of linear variation of the strain along the thickness, using the following expression for the curvature χ_{11} :

$$\chi_{11} = \frac{16pL^2}{\pi^4 D_{ort}} \sum_{n-odd} \sum_{m-odd} \frac{\sin\left(\frac{ny_1\pi}{L}\right)\sin\left(\frac{my_2\pi}{L}\right)}{\frac{m}{n}\left(n^4 + 2\left(\nu + \frac{2\beta(\alpha-\nu^2)}{\alpha}\right)n^2 m^2 + \alpha m^4\right)} \tag{31}$$

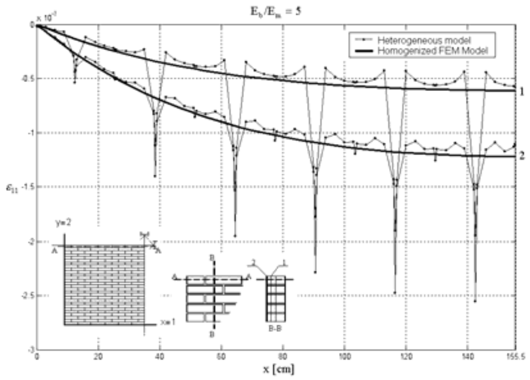


Fig 11 strains ε_{11} at H/2 in correspondence of blocks and vertical joints.

In order to validate the hypothesis of linearity of $\boldsymbol{\varepsilon}$, in Figure 12 the distributions along the thickness of the wall of strain components in different positions of the wall of Figure 10 are plotted for different E_b/E_m values of ratios (2, 5, 50, 500). As it is possible to note, for low values of E_b/E_m , the components of the strain tensor are almost linear, while a significant variation in strains is present near the CFRP reinforcement when mortar Young modulus decreases.

6 Conclusions

The obtained results show that the application of the homogenisation technique to CFRP reinforced masonry structures requires greater attention in comparison with both the unreinforced and the in-plane reinforced case, especially in presence of low values of the modulus. Nevertheless –due to the variability and uncertainty in the determination of the mechanical properties of the components (blocks and mortar)– less accurate but simpler multi-step approaches appear suitable for technical purposes, especially when mortar of good mechanical characteristics is present.

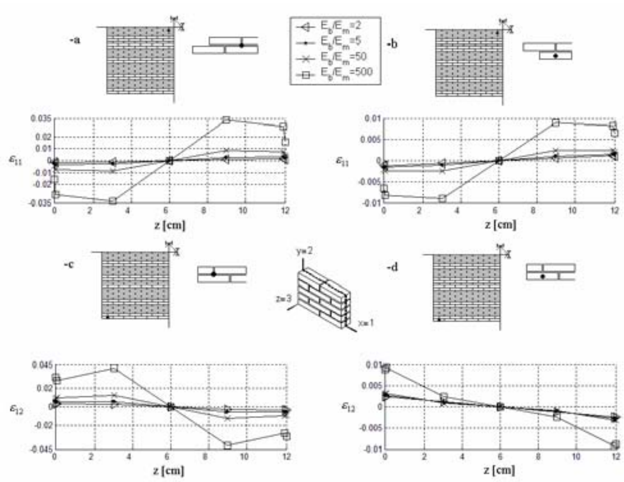


Fig 12 Strains along the thickness of the wall in some meaningful positions of the shear wall; -a center in the mortar joint, -b center in the block, -c corner in the mortar joint, -d corner in the block.

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