Numerical methods for determination the elastic stresses in rolling bearings

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Abstract: - The projection of the bearings elements especially the rolling bearings and the roller way is presented. It was studied the aspect in all rolling bearings with half-space method, finite elements (contact element), MathCAD programmes.

Key-Words: - Numerical methods, Bearings, Half-space method, Finite element method, Contact element, MathCAD programme, ANSYS programme.

1 Introduction

One of the best methods to determinate the stresses are the numerical methods. In this application we use same different numerical methods for determination the state of bearings stresses, very important for your projections.

2 Half-space method

We shall consider in this section a uniform pressure p applied to a region of the surface consisting of a straight-sided polygon, as shown in fig.1.a It is required to find the depression at a general point B (x,y) on the surface and the stress components at a subsurface point A(x,y), BH_1, BH_2, etc, are perpendiculars of lengths h_1, h_2, etc., onto the side of polygon DE, EF respectively. The loaded polygonal is then made up of the algebraic addition of eight right angle triangles:

$$\backslash EFG=[BEH_1+BEH_2+BFH_2+BFH_3]-[BDH_1+BDH_4+BGH_3+BGH_4] \quad (1)$$

A similar breakdown into rectangular triangles would have been possible if B had lain inside the polygon a typical triangular area is shown in fig. 1.b.

$$(\underline{u}_y)_B = \frac{1-\nu^2}{\pi E} p \int_0^{\phi_1} d\phi \int_0^{s_1} ds =$$

$$= \frac{1-\nu^2}{\pi E} p \int_0^{\phi_1} h \sec\phi \, d\phi = \frac{1-\nu^2}{\pi E} p \frac{h}{2} \ln\left(\frac{1+\sin\phi_1}{1-\sin\phi_1}\right). \quad (2)$$

The total displacement at B due to a uniform pressure on the polygonal region DEFG can then be found by combining the results of equations (2) for the eight constitutive triangles. The stress components at an interior point A(x,y,z) below B can be found by integration of the stress components due to a point force given by known equation but the procedure is tedious.

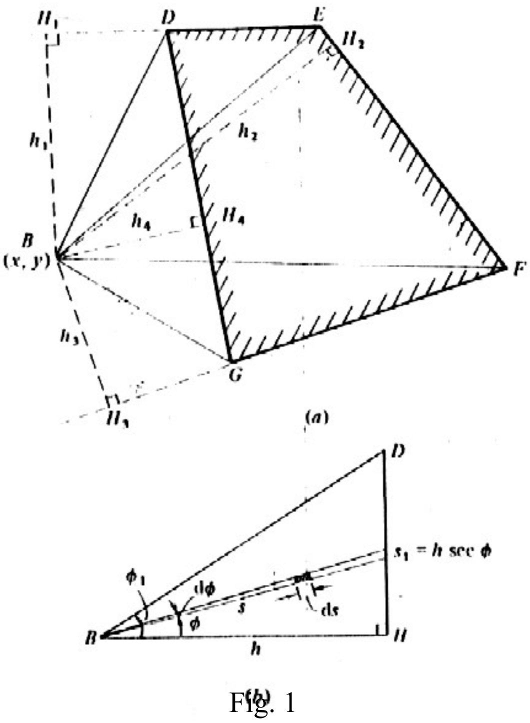

Fig. 1

The effect of a uniform pressure acting on a rectangular area 2a*2b has been analyzed in detail by Love (1929) . The deflection of a general point (x,y) on the surface is given by:

$$D = \frac{\pi E}{1-\nu^2}\frac{u_z}{\overline{p}} = (x+a)\ln\left[\frac{(y+b)+((x+a)+(x+a)^2)^{1/2}}{(y-b)+((y-b)+(x+a)^2)^{1/2}}\right]+$$

$$+(y+b)\ln\left[\frac{(x+a)+((y+b)^2+(x+a)^2)^{1/2}}{(x-a)+((y+b)^2+(x-a)^2)^{1/2}}\right]+$$

$$+(x-a)\,ln\left[\frac{(y-b)+\left((y-b)^2+(x-a)^2\right)^{1/2}}{(y+b)+\left((y+b)^2+(x-a)^2\right)^{1/2}}\right]+$$

$$+(y-b)\,ln\left[\frac{(x-a)+\left((y-b)^2+(x-a)^2\right)^{1/2}}{(x+a)+\left((y-b)^2+(x+a)^2\right)^{1/2}}\right]. \qquad (3)$$

Expressions have been found by Love (1929) from which the stress components at a general point in the solid can be found. Love comments on the fact that the components of shear stress τ_{xy} have a theoretically infinite value at the corner of the rectangle. Elsewhere all stress components are finite. On the surface at the centre of the rectangle:

$$[\sigma_x]_0=-p\{2\nu+(2/\pi)(1-2\nu)\tan^{-1}(b/a)\},$$
$$[\sigma_y]_0=-p\{2\nu+(2/\pi)(1-2\nu)\tan^{-1}(a/b)\}, \qquad (4)$$
$$[\sigma_z]_0=-p.$$

These results are useful when a uniform loaded rectangle is used as a ‘boundary elements’ in the numerical solution of more general contact problems. The elastic deformation in a point (x,y) make by the uniform distributed pressure from the rectangular surface (2a*2b) will be fig.(2):

$$\delta'=\frac{p}{\pi E}\int_{-a}^{a}\int_{-b}^{b}\frac{dxdy}{\left[(y-y_1)^2+(x-x_1)^2\right]^{1/2}}. \qquad (5)$$

By integrating the equation (5) it results:

$$\delta=\frac{pD}{\pi E'}, \qquad (6)$$

where the displacement D is calculated by the formula (2).

The expression δ represent the elastic deformation in the point (x,y) make by the uniform pressure p, distribute from the rectangular surface (2a*2b). If the contact surface is divided in a number of rectangular equal surface ,the total deformation in point (x,y) make by contribution of the diverse uniform rectangular surface load, in the contact surface made by numerical evaluation. The total deformation make by the uniform load from the rectangular surface in the inside of the cone is:

$$\delta_i=\frac{1}{\pi E}\sum_{j=1}^{n}p_jD_{i,j}. \qquad (7)$$

The results obtained by this method utilized for the contact of cylindrical bearings N2256 is giving in application.

3 Finite element method

The compression of a cylinder in contact “nonconformist” with two surfaces, who are in opposition at the extremity of roles, can be analyses satisfactory (fig.3). A compression force on the unity of length we give a hertz distribution of pressure in O_1 equal with:

$$p=\frac{2P}{\pi a_1}\left(1-\frac{x^2}{a^2}\right)^{\frac{1}{2}}, \qquad (8)$$

and:

$$a_1^2=4PR/\pi E_1^*, \qquad (9)$$

where E_1^* is the Young modulus.

The stresses in A are given by the contribution:
- the stresses given by the hertz distribution in O_1
- the stress given by the pressure in O_2, may be considered as for a concentrate force P
- the biaxial stress given by the equation:

$$\sigma_1-\sigma_2=P/\pi R. \qquad (10)$$

Assembly the three contributions, we obtain:

$$\sigma_x=\frac{P}{\pi}\left[\frac{1}{R}-\frac{2(a_1^2+2z^2)}{a_1^{2(}(a_1^2+z^2)^{\frac{1}{2}}}+\frac{4z}{a_1^2}\right], \qquad (11)$$

$$\sigma_z=\frac{P}{\pi}\left[\frac{1}{R}-\frac{2}{2R-z}-\frac{2}{(a_1^2+z^2)^{\frac{1}{2}}}\right]. \qquad (12)$$

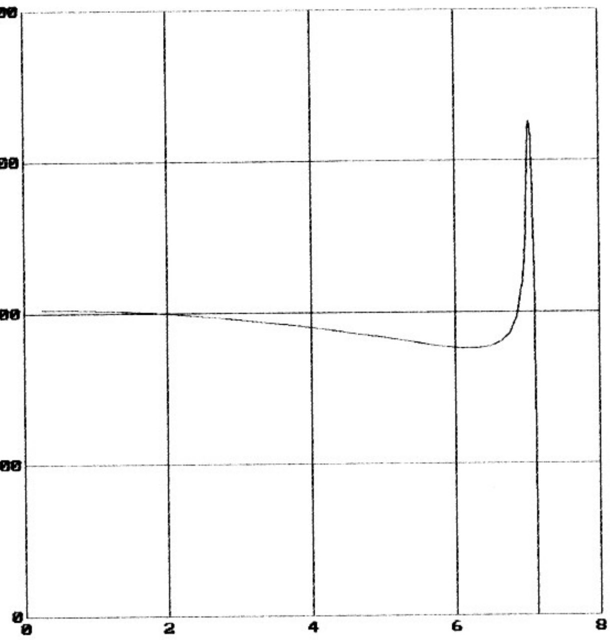

Fig. 2

The real cylinders are finite length and the important deviations at the Hertz theory appear to their end.

3.1 The description of the construction solution

With the finite element program ANSYS use plane elements (triangular, rectangular and contact elements 48 we realized in the case of a cylindrical roles one other profile, a Lundberg modified profile.

3.2 The advantage of the proposed solution

In fig.3 and fig.4 we can observe the distribution of stresses in two types of roles – cylindrical right roles fig. 3 and cylindrical roles with Lundberg modified profile. We can also observe that the stresses at the end of the roles are large small in the second case.

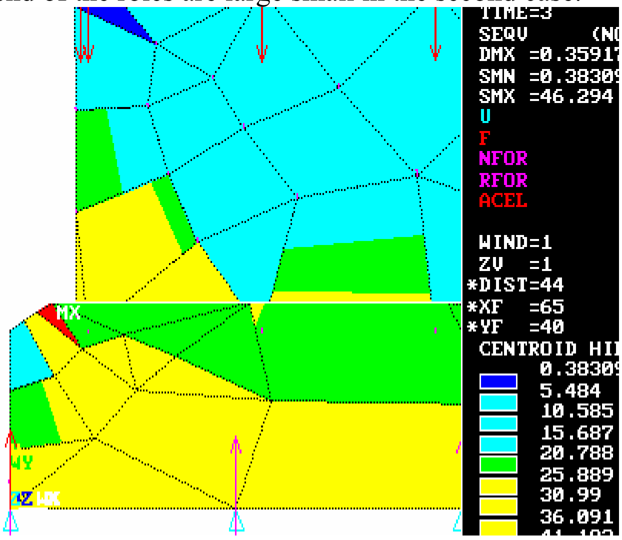

Fig. 3

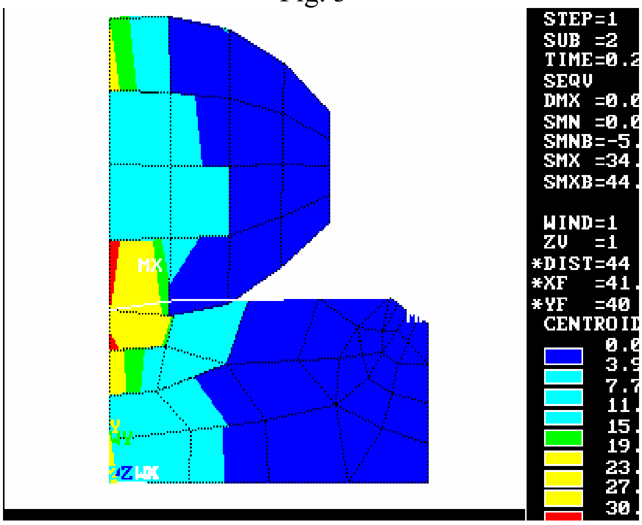

Fig. 4

4 Hertzian constants computer assisted process design using MathCAD

For the understanding the hertzian models, it was study first the constituent equations for the vertical displacement u_z. The hypothesis I is associate to establishment the path in a median elastic plane dependent by the curves of the conjugated surface and the elastics contacts of the two surface cylinders the account of contact verifying the consigns equations (fig.5).

$$(z_0 + u_0) + (z_1 + u_1) = h, \quad h=h_0+h_1. \qquad (13)$$

The external point of the contact, verifiable the inequation

$$(z_0 + u_0) + (z_1 + u_1) < h\,. \qquad (14)$$

Take by P_z (x,y) ,the distribution of the contact pressure we have:

$$u_i{'}\frac{1}{\pi E_i^*}\iint \frac{P_z(\xi,\eta)}{r} d\xi d\eta\,,$$

$$E^*_i = \frac{E}{1-\nu^2}\,,\ \mathrm{I}=0..n, \qquad (15)$$

where: E_i , ν_I is the Young and Poisson coefficients of this two materials.

4.1 The description of the construction solution

For construct an imagine of the sliding in the hertz plain I am stimulated one of two sphere by the plane structural complex by beam elements, for 7 radial level and twenty one angular (266 elements, 21*7=147 nodes). To fix to structure embed for the contour 0,…,20,41,62,146 radial sliding for the 21,42,…,126 nodes. The impose reshuffle of force is corresponding of flatten in the profile plane (fig.5).

5 Conclusions

The numerical methods are one of the best methods to determinations the stresses in the rolls and rolling ways. It is very important to know, because the projects of the profile of rolls are very important for determinations of the stage of stresses.

It rested that the slides for the contact plane and at the same time is making be determinates the pressure of contact distribution (fig.6).

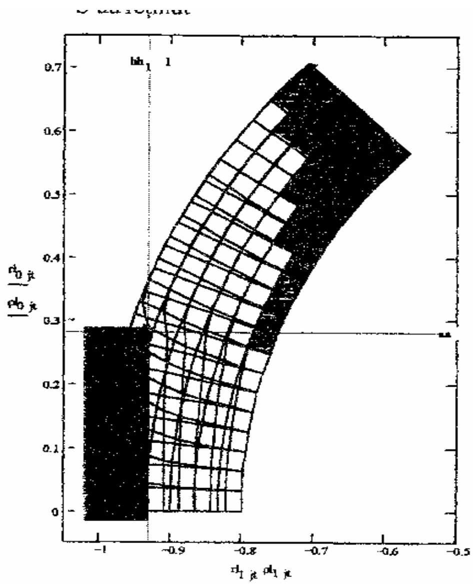

Fig. 5

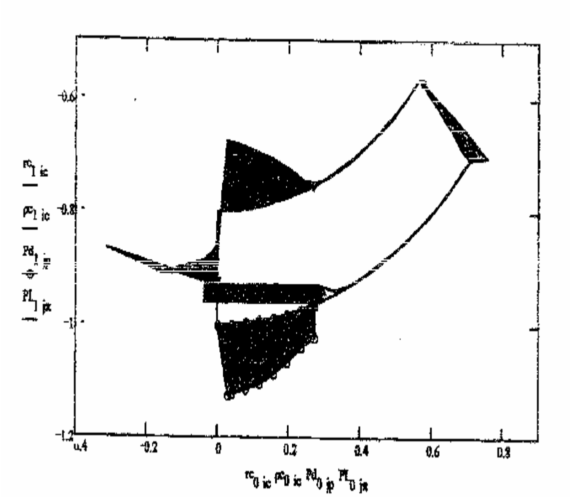

Fig. 6

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