A Dynamic Procedure with Propagation Delay Analysis for Performance Optimization in WDM Networks

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Abstract: **-** This paper presents a synchronous transmission WDMA protocol for passive star topology. The network architecture adopts the multi-channel control architecture (MCA) [2] to exchange control information. In our analysis, we exploit the effect of propagation delay latency to improve the system utilization by dynamically dividing the MCA into two groups of channels based on the knowledge of the stations status (free or backlogged). Also, the combination with the access algorithm introduced in [3] that avoids data channel collisions gives optimum results improving even more the performance. An analytic Morkovian model for finite population is developed considering the effect of receiver collisions to performance measures evaluation.

Key-Words:- Wavelength Division Multiplexing (WDM), Multi-channel Control Architecture (MCA), asymmetric access rights, propagation delay latency, receiver collisions.

1 Introduction

In WDMA protocols, a parameter that plays key role in the performance evaluation is the round trip propagation delay. In literature, few studies consider the effect of this parameter in their analysis. In [1] a passive star network with a centralized master/slave scheduler located at the hub is used and the effect of propagation delay is overcome by measuring the delays between the stations and the hub and assuming that delay when scheduling transmissions.

In this paper, the proposed network configuration adopts the Multi-channel Control Architecture (MCA) [2] that reduces the processing overhead of control information and efficiently improves the performance. We apply a similar to [3] "tell and wait" algorithm to access the control and data channels, given that the round trip propagation delay is longer than the packet transmission time. Also, the developed access method avoids data channel collisions improving even more the performance.

In our study, we extend the analysis of [3] exploiting the benefit of propagation delay as acknowledgement time to introduce a dynamic division of the MCA into two groups. In the first group the free stations compete to gain access, while in the second group the backlogged stations compete. The dynamic procedure of control channels separation and the stations asymmetric access rights for (re)transmission over the two groups of control channels achieve maximum MCA utilization. Also, a Markovian queueing model with finite population is analyzed considering receiver collision phenomenon [4]. So, the proposed access algorithm as well as the following analysis consist the innovation of our study managing performance optimization.

Our study is carried out as follows: Section 2 describes the network model and assumptions. Section 3 presents the Markovian analysis and the performance measures are derived. In Section 4 the protocol optimization is presented. In Section 5 numerical results and comments on them are given. Some conclusions are outlined in Section 6.

2 Network Model and Assumption

We consider a passive star network with v+N wavelengths $\lambda_{c1},..\lambda_{cv},\lambda_{d1},..\lambda_{dN}$ to serve a finite number M of stations, as Fig. 1 shows. The channels $\lambda_{c1},..\lambda_{cv}$ form the MCA while the remaining N channels $\lambda_{d1},..\lambda_{dN}$ form the data multi-channel system. The MCA is divided into two groups of channels, $v=v_f+v_b$. It means that v_f control channels are used by free stations, while v_b control channels are used by backlogged. The proposed model is described as $[CC]^v$-TT-$[FR]^v$-[TR]: there are v control channels and each station has a tunable transmitter tuned at $\lambda_{c1},..\lambda_{cv},\lambda_{d1},..\lambda_{dN}$. The outcoming traffic of a station is connected to an input of the passive star coupler. Each station uses v fixed tuned receivers one for each control channel and one tunable receiver to any of data channels $\lambda_{d1},..\lambda_{dN}$.

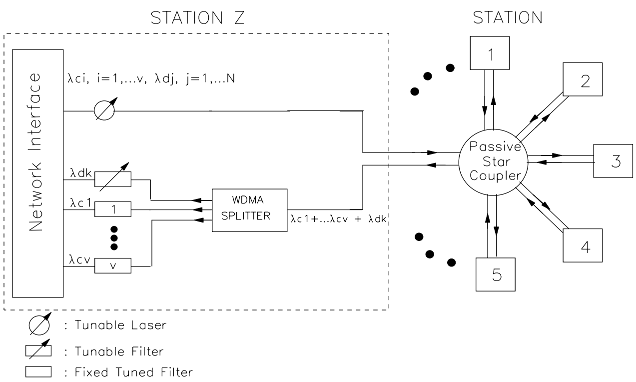

Fig. 1: Passive star multi-wavelength architecture.

The incoming traffic to a station is splitted by an 1×(v+1) WDMA splitter. The fixed size control packet transmission time is the time unit and the data packet transmission time normalized in time units is L (data slot). The control packet consists of the transmitter and receiver address and the channel λ_k. The normalized round trip propagation time between any station to the star coupler hub and to any other station is R data slots. Both control and data channels use the same time reference (cycle). We define as cycle the time period that includes a time unit for control packet transmissions plus the normalized round trip propagation time R and the data packet transmission time L: cycle duration is C=1+(R+1)L time units. Time axis is divided into contiguous cycles of equal length and stations are synchronized for transmission during a cycle. At any point in time each station is able to transmit at a given channel λ_T and simultaneously receive at a channel λ_R. Finally, we assume negligible tuning times and very large tunable bandwidths. At the beginning of each cycle, all stations know the number of backlogged stations. This knowledge defines the optimum division of the MCA into the groups of v_f and v_b control channels to obtain the optimum control channels utilization.

Also, at the beginning of each cycle if a station has to send a data packet, first it chooses randomly a data channel for the transmission. It informs the other stations by sending a control packet choosing randomly one of the v_f or the v_b control channels depending on its state (free or backlogged). The control packets compete according to the Slotted Aloha scheme. The station continuously monitors the MCA with its fixed tuned receivers. The outcome of its control packet will be known R×L time units later (acknowledgement time). After the end of this time, the station is aware about all stations claims for transmission. If its control packet is successfully transmitted over the MCA and the same data channel is selected from other stations for data transmission, a collision avoidance algorithm is applied (let imagine many arbitration rules: the age of the packet, priority etc). Thus, only one among the competed the same channel stations gains access and starts transmission immediately while the others are getting backlogged [3]. Also, after the end of the same period the station knows the number of backlogged stations for the next cycle period. In this way, it knows the number of v_f and v_b control channels for the next cycle that provide the optimum throughput.

After data transmission, the receiver station waits R×L time units and adjusts its tunable receiver to the specified data channel. If more than one data packets have the same destination, one of them is correctly received and the others are aborted due to receiver collisions [4]. Stations that unsuccessfully (re)transmit either on control or data channels or due to receiver collisions are getting backlogged. Each station has a transmitter buffer with capacity of a data packet. If the buffer is empty the station is free, otherwise it is backlogged. A backlogged station is getting free at the end of a cycle if it retransmits without control channel collision, gains access over a data channel and is received without receiver collision. At each cycle, packets are generated independently following a geometric distribution with probability p. A backlogged station retransmits following a geometric distribution with probability p_1. If a backlogged station generates a new packet, the packet is lost.

3 Model Analysis

The system is described by a discrete time Markov chain. We denote the state of the system by X_t, t=1,2… where X_t=0,1…M is the number of backlogged stations at the beginning of a cycle. Let: H_t =The number of new control packets arrivals and A_t=The number of successfully (re)transmitted data packets over the N data channels during a cycle.

Also, let that during a cycle:
1) SB_k=The number of successfully retransmitted control packets over the v_b control channels, given that k backlogged stations retransmit, $0 \le SB_k \le \min(v_b,k)$. 2) SF_m=The number of successfully transmitted control packets over the v_f control channels, given that m free stations transmit, $0 \le SF_m \le \min(v_f,m)$. 3) A_n=The number of successfully (re)transmitted data packet over the N data channels, given that n successful (re)transmissions occurred over the v control channels, $SB_k + SF_m = n$ for every $SB_k + SF_m > 0$. 4) C_r=The number of correctly received packets at destination given that r successful

(re)transmissions occurred over the N data channels, $1 \le C_r \le A_n$ for every $A_n > 0$.

We consider the following probabilities:

1) $\Pr[SB_k=n]$ of n successes from k retransmissions over the v_b control channels during a cycle [5]:

$$\Pr[SB_k = n] = \frac{(-1)^n v_b! k!}{(v_b)^k n!} \sum_{j=n}^{\min(v_b,k)} \frac{(-1)^j (v_b - j)^{(k-j)}}{(j-n)!(v_b - j)!(k-j)!} \quad (1)$$

and $0 \le n \le \min(v_b, k)$.

2) $\Pr[SF_m=s]$ of s successes from m transmissions over the v_f control channels during a cycle [5]:

$$\Pr[SF_m = s] = \frac{(-1)^s v_f! m!}{(v_f)^m s!} \sum_{j=s}^{\min(v_f,m)} \frac{(-1)^j (v_f - j)^{(m-j)}}{(j-s)!(v_f - j)!(m-j)!} \quad (2)$$

and $0 \le s \le \min(v_f, m)$.

3) $\Pr[A_n = r]$ of r successful transmissions over the N data channels given that n successful (re)transmissions occurred over the v control channels during a cycle [3]:

$$\Pr[A_n = r] = \binom{N}{r} \sum_{i=0}^{r} (-1)^i \binom{r}{i} \left(\frac{r-i}{N}\right)^n \quad (3)$$

and $1 \le r \le \min(N,n)$ for every $n \ge 1$.

4) $\Pr[C_r = u]$ of u correctly received data packets at destination given that r successful (re)transmissions occurred over the N data channels during a cycle [4]:

$$\Pr[C_r = u] = \binom{M}{u} \sum_{i=0}^{u} (-1)^i \binom{u}{i} \left(\frac{u-i}{M}\right)^r \quad (4)$$

and $1 \le u \le \min(r,M)$ for every $r \ge 1$.

We define the following function: $\Phi(x,y,z,w,s,r)=$
$=\Pr[SB_x = y]\,\Pr[SF_z = w]\,\Pr[A_{y+w} = s]\,\Pr[C_s = r]$ (5)
where: x and z is the number of (re)transmissions on the v_b and v_f control channels respectively, y and w is the number of successes on the v_b and v_f control channels respectively, s is the number of successes on the N channels and r is the number of correctly received packets at destination, during a cycle.

We define the conditional probability q_{in} that i out of n backlogged stations attempt to retransmit with probability p_1 during the cycle: $q_{i,n} = \mathrm{bin}(n,i,p_1)$ (6)

Similar, we define the conditional probability Q_{in} that i out of (M-n) free stations attempt to transmit with probability p during a cycle: $Q_{i,n}=\mathrm{bin}(M-n,i,p)$ (7)

where: $$\mathrm{bin}(i,j,p) = \binom{i}{j} p^j (1-p)^{i-j}, \qquad i \ge j \quad (8)$$

The Markov chain X_t is homogeneous, aperiodic and irreducible. The one step transition probabilities $P_{ij}=(X_{t+1}=j \mid X_t=i)$ are:

Case A: $j < i - N$ then: $P_{ij} = 0$ (9)

Case B: $j = i - N$ then: $P_{ij}=$ (10)

$$Q_{0i}\left(\sum_{n=N}^{\min(i,v_b)} q_{ni} \Phi(n,n,0,0,N,N) + \sum_{n=N+2}^{i} q_{ni} \sum_{y=N}^{\min(n-2,v_b-1)} \Phi(n,y,0,0,N,N) \right)$$

Case C: $i - N < j < i$ then: $P_{ij} =$

$$= \begin{cases} \displaystyle\sum_{n=i-j}^{\min(i,v_b)} q_{ni} \sum_{m=0}^{\min(M-i,N-i+j,v_f)} Q_{mi} \sum_{l=m+i-j}^{\min(m+n,N)} \Phi(n,n,m,m,l,m+i-j) + \\ \displaystyle\sum_{n=i-j+2}^{\min(i,v_b)} q_{ni} \sum_{m=2}^{\min(M-i,N-i+j)} Q_{mi} \sum_{x=\max(0,m+i-j-n)}^{\min(m-2,v_f-1)} \sum_{l=m+i-j}^{\min(x+n,N)} \Phi(n,n,m,x,l,m+i-j) + \\ \displaystyle\sum_{n=i-j+2}^{i} q_{ni} \sum_{m=0}^{\min(M-i,N-i+j,v_f)} Q_{mi} \sum_{y=i-j}^{\min(n-2,v_b-1)} \sum_{l=m+i-j}^{\min(y+m,N)} \Phi(n,y,m,m,l,m+i-j) + \\ \displaystyle\sum_{n=i-j+4}^{i} q_{ni} \sum_{m=2}^{\min(M-i,N-i+j)} Q_{mi} \sum_{y=i-j}^{\min(n-2,v_b-1)} \sum_{x=\max(0,m+i-j-y)}^{\min(m-2,v_f-1)} \sum_{l=m+i-j}^{\min(x+y,N)} \Phi(n,y,m,x,l,m+i-j) \end{cases} \quad (11)$$

Case D: $j = i$ then: $P_{ij} =$

$$= \begin{cases} \displaystyle q_{0i} Q_{0i} \Phi(0,0,0,0,0,0) + Q_{0i} \sum_{n=2}^{i} q_{ni} \Phi(n,0,0,0,0,0) + \\ \displaystyle\sum_{n=0}^{\min(i,v_b)} q_{ni} \sum_{m=1}^{\min(M-i,N,v_f)} Q_{mi} \sum_{l=m}^{\min(n+m,N)} \Phi(n,n,m,m,l,m) + \\ \displaystyle\sum_{n=2}^{\min(i,v_b)} q_{ni} \sum_{m=2}^{\min(M-i,N)} Q_{mi} \sum_{x=\max(0,m-n)}^{\min(m-2,v_f-1)} \sum_{l=m}^{\min(x+n,N)} \Phi(n,n,m,x,l,m) + \\ \displaystyle\sum_{n=2}^{i} q_{ni} \sum_{m=1}^{\min(M-i,N,v_f)} Q_{mi} \sum_{y=0}^{\min(n-2,v_b-1)} \sum_{l=m}^{\min(m+y,N)} \Phi(n,y,m,m,l,m) + \\ \displaystyle\sum_{n=4}^{i} q_{ni} \sum_{m=2}^{\min(M-i,N)} Q_{mi} \sum_{y=0}^{\min(n-2,v_b-1)} \sum_{x=\max(0,m-y)}^{\min(m-2,v_f-1)} \sum_{l=m}^{\min(x+y,N)} \Phi(n,y,m,x,l,m) \end{cases} \quad (12)$$

Case E: $j > i$ then: $P_{ij} =$

$$= \begin{cases} \displaystyle Q_{j-i,i} \sum_{n=0,n\ne 1}^{i} q_{ni} \Phi(n,0,j-i,0,0,0) + \\ \displaystyle\sum_{n=0}^{\min(i,v_b)} q_{ni} \sum_{m=j-i+1}^{\min(M-i,v_f)} Q_{mi} \sum_{l=m+i-j}^{\min(n+m,N)} \Phi(n,n,m,m,l,m+i-j) + \\ \displaystyle\sum_{n=0}^{\min(i,v_b)} q_{ni} \sum_{m=j-i+1}^{\min(M-i,N-i+j)} Q_{mi} \sum_{x=\max(0,m+i-j-n)}^{\min(m-2,v_f-1)} \sum_{l=m+i-j}^{\min(n+x,N)} \Phi(n,n,m,x,l,m+i-j) + \\ \displaystyle\sum_{n=2}^{i} q_{ni} \sum_{m=j-i+1}^{\min(M-i,v_f)} Q_{mi} \sum_{y=0}^{\min(n-2,v_b-1)} \sum_{l=m+i-j}^{\min(m+y,N)} \Phi(n,y,m,m,l,m+i-j) + \\ \displaystyle\sum_{n=2}^{i} q_{ni} \sum_{m=j-i+1}^{\min(M-i,N-i+j)} Q_{mi} \sum_{y=0}^{\min(n-2,v_b-1)} \sum_{x=\max(0,m+i-j-y)}^{\min(m-2,v_f-1)} \sum_{l=m+i-j}^{\min(x+y,N)} \Phi(n,y,m,x,l,m+i-j) \end{cases} \quad (13)$$

Since the Markov chain X_t, t=1,2… is ergodic, the steady state probabilities are given by solving the system of the following linear equations:

$$\boldsymbol{\pi} = \boldsymbol{\pi}\ \boldsymbol{P} \quad (14) \qquad \text{and} \qquad \sum_{i=0}^{M} \pi_i = 1 \quad (15)$$

where $\boldsymbol{P}$ is the transition matrix with elements the probabilities P_{ij} and $\boldsymbol{\pi}$ is a row vector with elements the steady state probabilities π_i. The conditional throughput $S_{rc}(i)$ is the expected value of the output rate during a cycle given that the number of backlogged stations at the beginning of the cycle is i, i.e. $S_{rc}(i)=E[A_t \mid X_t=i]$. It is given by: $S_{rc}(i) =$

$$\left\{\begin{aligned}
&\sum_{k=1}^{N} k \Bigg(\sum_{n=\max(k,M-i)-(M-i)}^{\min(i,v_b)} q_{ni} \sum_{\substack{m=0\\ m+n\geq k}}^{\min(M-i,v_f)} Q_{mi} \sum_{l=k}^{\min(n+m,N)} \Phi(n,n,m,m,l,k) + \\
&\sum_{n=\max(k,M-i)-(M-i)}^{\min(i,v_b)} q_{ni} \sum_{\substack{m=0\\ m+n\geq k}}^{M-i} Q_{mi} \sum_{\substack{x=0\\ x\geq k-n}}^{\min(m-2,v_f-1)} \sum_{l=k}^{\min(n+x,N)} \Phi(n,n,m,x,l,k) + \\
&\sum_{n=\max(k,M-i)-(M-i)}^{i} q_{ni} \sum_{\substack{m=0\\ m+n\geq k}}^{\min(M-i,v_f)} Q_{mi} \sum_{\substack{y=0\\ y\geq k-m}}^{\min(n-2,v_b-1)} \sum_{l=k}^{\min(m+y,N)} \Phi(n,y,m,m,l,k) + \\
&\sum_{n=\max(k,M-i)-(M-i)}^{i} q_{ni} \sum_{\substack{m=0\\ m+n\geq k}}^{M-i} Q_{mi} \sum_{x=0}^{\min(m-2,v_f-1)} \sum_{\substack{y=0\\ y\geq k-x}}^{\min(n-2,v_b-1)} \sum_{l=k}^{\min(x+y,N)} \Phi(n,y,m,x,l,k) \Bigg)
\end{aligned}\right. \quad (16)$$

The steady state average throughput S_{rc} is:

$$S_{rc} = \frac{L}{C} E[S_{rc}(i)] = \frac{L}{C} \sum_{i=0}^{M} S_{rc}(i)\pi_i \quad (17)$$

The steady state average number B of backlogged stations is given by: $B = E[i] = \sum_{i=0}^{M} i\pi_i$ (18)

The conditional input rate $S_{in}(i)$ is the expected number of arrivals during a cycle, given that the number of backlogged stations at the beginning of a cycle is i. It is: $S_{in}(i) = E[H_t \mid X_t = i] = (M - i)p$ (19)

The steady state average input rate S_{in} is given by:

$$S_{in} = \sum_{i=0}^{M} p(M-i)\pi_i \quad (20)$$

The delay D is defined as the average number of time units that a packet has to wait until its successful transmission and is calculated by Little's formula:

$$D = \{1 + (R+1)L\} + \{1 + (R+1)L\} \frac{B}{S_{in}} \quad (21)$$

We define the throughput per data channel S_d in steady state as the number of the correctly received packets at destination per data channel during a cycle:

$$S_d = \frac{S_{rc}}{N} \quad (22)$$

4 Performance Optimization

In order to achieve the optimum performance, the following considerations are made:

The conditional throughput $S_{vb}(i)$ from the v_b control channels is [6]: $S_{vb}(i) = ir\left(1 - \frac{r}{v_b}\right)^{i-1}$ (23)

Similar, the conditional throughput $S_{vf}(i)$ from the v_f control channels is defined:

$$S_{vb}(i) = (M - i)p\left(1 - \frac{p}{v_f}\right)^{M-i-1} \quad (24)$$

Also, the conditional throughput $S_v(i)$ from the v control packets is defined:

$$S_v(i) = ir\left(1 - \frac{r}{v_b}\right)^{i-1} + (M-i)p\left(1 - \frac{p}{v_f}\right)^{M-i-1} \quad (25)$$

4.1 An approximate analysis

We assume that $SB_k(i)+SF_m(i) = n$ control packets are successfully transmitted on the v control channels during a cycle, given that the state of the system is i. Also, we assume that the transmitted data packets are uniformly distributed among the N data channels. Thus, the random distribution in N data channels gives N^n arrangements, each with probability N^{-n}. Let $P_{N0}(n,i)$ be the conditional probability that no one from the n data packets has selected the data channel Z for the transmission. Thus, the n data packets are transmitted over the remaining (N-1) data channels in $(N-1)^n$ different ways. Then, $P_{N0}(n,i)$ can be written:

$$P_{N0}(n,i) = \frac{1}{N^n}(N-1)^n = \left(1 - \frac{1}{N}\right)^n \quad (26)$$

In steady state it is:

$$E[SB_k(i) + SF_m(i) = n] = S_v(i) \quad (27)$$

Thus, in steady state (26) is written:

$$P_{N0}(i) = \left(1 - \frac{1}{N}\right)^{S_v(i)} \quad (28)$$

We define the conditional probability $P_N(i)$ that one data packet is transmitted over data channel Z during a cycle in steady state. Thus $P_N(i)$ implies that at least one data packet has selected channel Z and has won the data channel collision avoidance competition. It is: $P_N(i) = 1 - P_{N0}(i) = 1 - \left(1 - \frac{1}{N}\right)^{Sv(i)}$ (29)

Let $H_N(S_v(i))$ be a random variable representing the number of different data channels selected, given that $S_v(i)$ is the conditional output rate of successful (re)transmitted control packets over the v control channels, during a cycle in steady state. We define the conditional probability $\Pr[H_N(S_v(i)) = x]$ that x different data channels have been selected for transmissions during a cycle in steady state. It is:

$$\Pr[H_N(S_v(i)) = x] = \binom{N}{x} P_N(i)^x (1 - P_N(i))^{N-x} \quad (30)$$

The conditional throughput $S_N(i)$ from the N data channels is defined: $S_N(i) = E[\Pr[H_N(S_v(i)) = x]] =$

$$= \sum_{x=1}^{N} x \Pr[H_N(S_v(i)) = x] = NP_N(i) \quad (31)$$

Substituting (29) to (31), we get:

$$S_N(i) = N\left(1 - \left(1 - \frac{1}{N}\right)^{Sv(i)}\right) \quad (32)$$

We assume that A_n= s data packets are successfully transmitted over the N data channels during a cycle, given that the state of the system is i. We assume that the data packets are uniformly distributed among the M stations (for sake of simplicity of the analysis, we assume that a station may send packets to itself). Thus, the random distribution in M stations gives M^s arrangements each with probability M^{-s}. Let $P_{M0}(s,i)$ be the conditional probability that no one from the s successfully transmitted data packets has as destination the station X. Thus, the s data packets are destined to the remaining (M-1) stations in $(M-1)^s$ different ways. Then, $P_{M0}(s,i)$ can be written as [4]:

$$P_{M0}(s,i) = \frac{1}{M^n}(M-1)^s = \left(1 - \frac{1}{M}\right)^s \quad (33)$$

In steady state it is: $E[A_n = s] = S_N(i)$ (34)

Thus, in steady state (33) is written as:

$$P_{M0}(i) = \left(1 - \frac{1}{M}\right)^{S_N(i)} \quad (35)$$

We define the conditional probability $P_M(i)$ that one data packet with destination X is correctly received during a cycle in steady state. It is:

$$P_M(i) = 1 - P_{M0}(i) = 1 - \left(1 - \frac{1}{M}\right)^{S_N(i)} \quad (36)$$

Let $H_M(S_N(i))$ be a random variable representing the number of different stations selected as destination, given that $S_N(i)$ is the conditional output rate of successful (re)transmitted data packets over the N data channels during a cycle in steady state. We define the conditional probability $\Pr[H_M(S_N(i)) = y]$ that y different stations are selected as destination during a cycle in steady state. It is:

$$\Pr[H_M(S_N(i)) = y] = \binom{M}{y} P_M(i)^y (1 - P_M(i))^{M-y} \quad (37)$$

Finally, we define the conditional throughput $S_{RC}(i)$ at the destination as:

$S_{RC}(i) = E[\ \Pr[H_M(S_N(i) = y]\ ]= MP_M(i)$ (38)

Substituting (35) to (37), we get:

$$S_{RC}(i) = M\left(1 - \left(1 - \frac{1}{M}\right)^{S_N(i)}\right) \quad (39)$$

4.2 Optimum performance parameters

We explore the best capabilities of the protocol. For each cycle, the system state is denoted by i.

4.2.1 Optimum retransmission probability r_{opt}

The optimum retransmission probability r_{opt} is obtained by setting the first derivative of (39) with respect to r equal to zero: $\frac{\partial S_{RC}(i)}{\partial r} = 0 \Rightarrow \frac{\partial S_v(i)}{\partial r} = 0$ (40)

Since $0 \le r_{opt} \le 1$, we get: $r_{opt} = \frac{v_b}{i}$, $i > v$ (41)

Also: $r_{opt} = 0$, $i = 0$ (42) and $r_{opt} = 1$, $0 < i \le v_b$ (43)

4.2.2 Optimum rate v_f / v_b

The optimum rate v_f / v_b is obtained by setting the first derivative of (39) with respect to v_b equal to zero. It is: $\frac{\partial S_{RC}(i)}{\partial v_b} = 0 \Rightarrow \frac{\partial S_v(i)}{\partial v_b} = 0$ (44)

That is: $\frac{v_f}{v_b} = \frac{p}{r}\sqrt{\frac{(M-i)(M-i-1)}{i(i-1)}}$, $i \ne 0,1$ (45)

Also, we define: $v_b = 1$ and $v_f = v-1$, if i =1 (46)

and: $v_b = 0$ and $v_f = v$, if i =0 (47)

4.3.3 Optimum MCA division - v_{f_opt}, v_{b_opt}

In order to obtain the optimum division of the MCA into the groups of v_{f_opt}, v_{b_opt} ($v_{f\_opt} \le v$, $v_{b\_opt} \le v$) control channels, we assume that the backlogged stations retransmit with r_{opt}. In this case, the v_{f_opt} is given by substituting (45) to (41):

$$v_{f\_opt} = \text{Integer Part}\left(p\sqrt{\frac{(M-i)(M-i-1)}{i(i-1)}}\right),\ i \ne 0,1 \quad (48)$$

In all cases, it is: $v_{b\_opt} = v - v_{f\_opt}$ (49)

5 Numerical Results

The numerical results are evaluated for optimum parameters (r_{opt} v_{f_opt}, v_{b_opt}). Especially for each cycle calculations, the optimum number of v_{f_opt}, v_{b_opt} are determined by dynamically splitting the total number v of control channels into two groups according (48)-(49). In the figures we compare the performance of the proposed protocol with this of the system of [3] on which all stations have symmetric access rights to the MCA. In this way, we representatively illustrate the significant performance improvement achieved by the asymmetric access rights to the MCA of the proposed protocol. We assume that L=10 time units.

Fig. 2 shows the throughput per data channel S_d curves versus birth probability p for M=50 stations, R=5 data slots, v=30 control channels and r_{opt}, for N=10,15,20 data channels. It is shown that the proposed protocol with asymmetric access rights on the MCA essentially increases the S_d as compared to the symmetric access rights protocol of [3] for all N. This is due to the fact that considering optimum protocol parameters r_{opt}, v_{f_opt}, v_{b_opt}, the probability of a control channel collision decreases that consequently increases the probability of a data packet successful transmission on the data channels. This fact essentially increases the S_d as compared to the symmetric access rights protocol of [3]. This is observed for p=0.21 where S_d improves for N=10 at 17%, for N=15 at 18% and for N=20 at 18%.

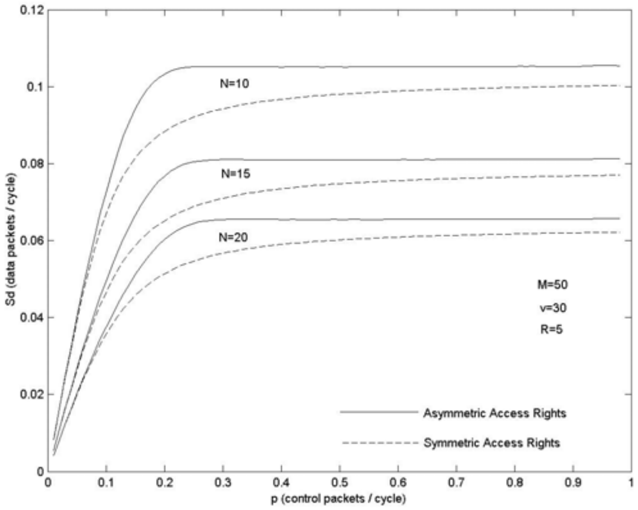

Fig. 2. S_d vs p, M=50, v=30, R=5, $r_{opt,}$ N=10,15,20.

The significant performance improvement is shown in Fig. 3 that presents the delay D curves versus S_d for M=50, R=5, v=30 and r_{opt}, for N=10,15,20.

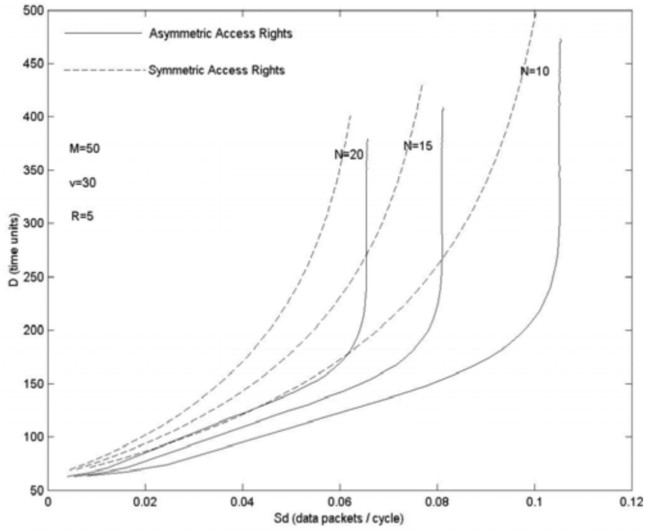

Fig. 3. D vs S_d, M=50, v=30, R=5, r_{opt}, N=10,15,20.

In fact, it is observed that the proposed protocol provides lower values of D while it reaches higher values of S_d, for all N. The explanation comes from the optimum division of the v control channels into the groups of v_{f_opt}, v_{b_opt} channels. In this case, the probability of a control channel collision decreases that causes essential decrease of the number of backlogged stations at this stage of transmission. This fact provides lower values of D while it guarantees the increase of S_d, as it is previously remarked.

6 Conclusion

In this paper we develop a Markovian model with receiver collision analysis for finite population in a synchronous transmission WDMA protocol based on the round trip propagation latency to avoid data channel collisions. Also, the division of the MCA into two groups of channels as well as the stations asymmetric access rights to the MCA consist the criterion to manage optimum performance efficiency. This is the main advantage of this study. As the figures show and the numerical results denote the new access algorithm achieves significant performance improvement compared with the protocol of [3] which in many cases reaches to 18%.

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