Concept Maps/Graphs/Trees/Vines In Education

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Abstract: - In this paper we provide a history and use of concept maps and discuss the use of such maps in education, referring to the various stages of the instructional process. We also define concept graphs and trees, which provide educational researchers access to the rich tools and methods of graph theory. For concept graphs and trees various optimality criterion have been introduced and an algorithm for constructing such graphs/trees has been provided. Updating concept graphs by using the Bayesian approach has been discussed and an example for a introductory statistics course is given. Later in the paper concept trees are extended to concept vines. An algorithm for the concept vines is also provided. Instructional technology tools that make use of concept maps/ graphs/trees/vines are needed to make them accessible in education.

Key-Words: - Concept maps, graphs, trees, vines, instructional process, Bayesian approach, instructional technology

1 Introduction, Motivation, and History

Cognitive maps, also referred to as concept maps, mind maps, knowledge maps, cause maps and graphic organizers have been used to explain cognitive structures in the brain and to make visible individuals' understanding of particular concepts. Two types of cognitive mapping studies are reported in the literature, those that are involved primarily with schemata and those that involve mapping perceptions held in the mind concerning space and place. Geographers, for example are interested in how mental images are constructed, how these images relate to real spaces, and how people differ in their evaluation of places [15] [24]. On the other hand, cognitive maps involving schemata in the form of ideas and perceptions are used for identifying expert knowledge, organizing ideas or plans, and evaluating individuals' understanding of concepts.

Cognitive mapping-spatial and information in the form of ideas and concepts involve schemata, the precepts or information that is coded and stored in the brain for later spatial ability and abstract thinking. Cognitive structures or coding systems are expanded as new information is mapped onto existing maps [22]. Assimilation occurs as new information is added to existing information; accommodation occurs when previous information is altered due to changes in the cognitive map [2] [16]. Overall, mental maps or schemata help to organize data, making it understandable and retrievable when needed to respond to stimuli [24]. Use of concept maps, specifically concept graphs/trees/vines as tools for educational curriculum design and instruction is the focus of this paper.

1.1 Concepts and Concept Maps

Concepts are general ideas or understandings that are the result of specific occurrences. Regularity in objects, events, or experiences over time is understood, labeled, and shared. Individuals come to understand concepts through meaningful learning experiences. For young children this is often through recognition of pattern and discovery learning as mental structures are built. For others learning is mediated through language, along with modeling, concrete experiences, and reception learning [2] [13] [17] [25].

Concept maps are tools which allow users to represent knowledge by illustrating relationships among concepts through use of nodes and links [23], nodes being the ideas, links being propositions. These maps may take the form of static, dynamic, interactive, or multi-layered. A variety of instructional technology tools allow users to make use sophisticated levels of concept mapping. Novak & Canas explain that these maps include "concepts, usually enclosed in circles or boxes of some type, and relationships between concepts indicated by a connecting line linking two concepts. Words on the line, referred to as linking words or linking phrases, specify the relationship between the two concepts" [13]. Propositions are statements about two or more concepts connected with linking words or phrases to form a meaningful statement. Maps acting as mental models, are used to model the cognitive structures and reflect cognitive processes of respondents [8]. Williams describes concept maps as a "direct method of looking at an individual's knowledge within a particular domain and at the fluency and efficiency with which knowledge can be used" [27].

1.2 History and Uses

Maps have been used over time to note and find location and to explain or put into context important information. Reiter reports the Chinese word t'u, often translated as picture, drawing, chart and map is found in the literature as early as the 3rd century B.C. While maps are a specific kind of t'u serving "topographical, administrative, and military purposes" [20], t'u also referred to pictures or charts and explanations, each being important in explaining information contained in the other. Throughout history pictures, diagrams, and flow charts have been used to explain ideas and organize plans.

Mind maps or mental maps have been researched by geographers to understand the cognitive-behavioral processes that link real world places and spaces with representation held in the mind and to understand attention as a factor in construction of mental maps [15]. In psychology cognitive maps have been investigated to understand behavior in relation to experiences, thoughts, and feelings. In the area of management, management organization cognition (MOC) research makes use of causemaps or directed graphs to understand cognitive structures that influence the decision making process at individual and organizational levels, though these processes are not observable. Causemaps are used not to *study* cognition, but rather as a tool for representation and analytic tasks. Causemaps "help capture-for overt observation and analysis-covert aspects of individual and social thinking" [8].

2 Conceptual Maps in Education

In education, concept maps have been identified as good representational tools for cognitive developmental change. David Ausubel and later Joseph Novak and colleagues have done extensive research in the use of concept maps [2] [11] [12] [13]. Ausbuel's work involves use of concept maps in the form of advance organizers for eliciting prior knowledge and organizing learning processes for students. Novak's work, a twelve-year long longitudinal study, began at Cornell University with investigations of children's science learning. Novak's work based on constructivist learning theory and Ausubel's assimilation theory involves use of concept maps as assessment tools. Both Ausubel and Novak suggest that good maps represent hierarchical structure and ability of respondents to identify cross-links between concepts.

In education concept maps may be used at different stages in the instructional process. Conceptual maps are used to organize knowledge prior to instruction [12]. Concept maps used in curriculum mapping help develop an overall framework of the curriculum or course, identify important concepts, determine prerequisite knowledge, and construct a course outline.

Used within the instructional process, concept maps serve as scaffolds to organize and structure knowledge [2] [3] [9], to elicit prior knowledge-a component of meaningful learning [13], to bridge the gap between what students already know and new knowledge [23], and to motivate critical thinking about a topic. Creating concept maps by brainstorming or representing what students already know about a given topic is done as a large group or as individuals. Advance organizers in the form of partially completed maps are used to help students gather information from texts and lectures, as they search for the key concepts and principles [12]. Advance organizers may also be concept maps used to initiate instruction by providing a picture or map of the topic for students. Used in this way, concept maps supply big picture and detailed information, providing important context for students who are field dependent and field independent students. These maps serve to organize information, demonstrating for students where a line of discussion will lead. Use of concept maps as advance organizers [2] provides clarity for students but also involves disequilibrium, a search for knowledge, and a return to equilibrium. Similarly, concept maps are an effective tool in helping students synthesize knowledge construction in online environments, as reported by Ortegano-Layne & Gunawardena, in which concept maps were used as a tool for organization (management and preservation) of information [14]. Trepagnier also reports "students achieve satisfaction (equilibrium) by determining relationships among pieces, some of which are obvious, but others of which are more obscure, requiring considerable thought and imagination" [23]. Lin notes that learners understand interrelationships among basic concepts due to topics and subordinate levels represented on concept maps [9]. Hyperlinks embedded within concept maps allow learners to choose paths that support further investigation of concepts based on need, interest, and time.

2.1 Use in Assessment

Conceptual maps are also used for assessment. The map is assumed to make visible students' thinking, though it may be a representative sample and may not include tacit knowledge [27]. Careful scoring of concept maps provides important information useful for making informed teaching decisions. Novak & Musonda suggest providing students with a list of relevant concepts (always context dependent) and assigning numerical scores to concepts, propositions, and correct links [11]. "The values selected are based on assimilation learning theory, where derivative subsumption (e.g., new examples of the same concept) is viewed as more easily achieved than correlative subsumption (e.g., acquisition of alternative but closely related concepts" [11]. Williams (1988) also reports use of concept maps for assessment as instruments of conceptual understanding of function in mathematics education. Williams lists valid propositions, levels of hierarchy, and cross-links ad categories for scoring concept maps [26]. Sim-Knight, Upchurch, Pendergrass, Meressi, Fortier, Tchimev, VonderHeide, & Page conclude that concept maps may be used to effectively assess students' understanding of content *and* provide information for course and curricular improvement [21].

2.2 Use in Design and Instruction

Conceptual graph analysis, a form of cognitive task analysis is also used to represent cognitive structure, in this case, the structure of an expert's thinking. Originally developed to elicit and represent detailed knowledge from computer science experts it was later adapted for use in instructional design [6]. While instructional design is concerned with improving instructional organization and methods, for the purposes of optimizing learning), conceptual graph analysis (CGA) uses question probes organized by node type to elicit tacit expert knowledge [18] [19]. The authors of this paper (ours) suggest concept graphs may be used to organize an instructor's expert conceptual knowledge for curriculum design in preparation for instruction. Such design incorporates knowledge of instructional elements including concepts to be taught, sequencing of concepts to support connections between concepts organized around course objectives, and careful monitoring of student feedback and progress. Conceptual graph analysis may also be used for redesign of curriculum plans. Such redesign would follow elicitation of students' prior knowledge and use of student assessment within the instructional process. It is believed use of concept graphs may support efficient, efficacious teaching and learning.

3 From Concept Maps to Graphs/Trees

Well-developed definitions, tools, methods and approaches of the graph theory can be applied to concept maps for use in education. **Concept graphs** are directed graphs with annotated nodes and edges, in which the concepts (ideas/topics) are contained in the nodes of the graph, and the edges serve to represent relationships between them. If the relationships between the concepts do not lead to a cycle, then the concept graph is called a **concept tree**. Concept trees describe a linear, hierarchical learning process. On the other hand cycles, reinforce the previous knowledge and initiate a new perspective for the learner.

Given a concept graph one can create subgraphs that are trees, which will be referred as **spanning concept trees**. Under the assumption that two concepts cannot be introduced at the same time special trees called **paths** become worthy of study. In application to the instructional process one needs to move from concept graphs to spanning trees and then the paths.

3.1 Optimality Criterion

It can be easily noticed that for a given family of concepts, graphs, trees, spanning trees, paths, and vines are not unique. To be able to form "optimal" ones, one needs to create a criterion. In graph theory, this has been achieved by specifying a "distance, $d_G(u,v)$" measure between two nodes. At various stages of the instructional process a different measure can be critical. Some examples of such distance measures in education are the following: proportion of the knowledge that needs to be retained to move from one concept to other, a measure of association between the students' performances on two concepts, and amount of time required to cover the subsequent concept after covering the preceding concept.

Therefore, our criterion will be based on $D(G)=\sum_{\substack{\text{all pairs}\\ \text{of concepts}}} d_G(u,v)$, which is known as the Wiener index of the graph [25].

3.2 An Algorithm for Instructional Process

First of all we will make distinction between primary, secondary, and preliminary concepts. Primary concepts will stem from the main learning objectives of the course. Secondary concepts will feed the primary ones. Preliminary concepts are the ones that learners bring to the course. They are part of students' prior knowledge developed in prerequisite courses and/or previous experiences.

Step 1. Create a concept graph for the primary concepts.

Step 2. Consider all possible spanning concept trees and select the one with the minimum Wiener distance.

Step 3. For each of the primary concepts create a concept graph.

Step 4. Consider all possible spanning concept trees and select the one with the minimum Wiener distance.

Step 5. Consider all of the possible paths and select the one with the minimum Wiener distance. At this step we construct an order for the contents, that is, an ordering such that all ancestors of content i appear before i in the ordering. A content order begins with a source/basic content and ends with a sink/target/outcome content. In most cases the content order is not unique, a criterion is needed to select the "optimal" one, such as the Wiener distance.

Step 6. Update the Wiener distances based on the course evaluation/assessment process.

Step 7. Repeat 1-5.

3.3 Updating the Concept Graphs

The dynamic and iterative nature of the instructional process requires constant updating of the concept graphs. Integration of prior knowledge and conversion to posterior knowledge based on the evaluation/assessment process are the key elements of instruction and learning. In this section we will look at a way of updating edge properties. We will consider the correlation between the students understanding or performance between two concepts. Let ρ_{ij} be such a correlation between concepts *i* and *j*. we are going to use the Bayesian approach to update edge properties. When instructors design a course, they will possess a prior state of knowledge, which will be reflected in the prior distribution, $\pi(\rho_{ij})$. Given the data from the course evaluation/assessment process, one can obtain the posterior distribution, $\pi(\rho_{ij} \mid data)$, by using the Bayes' theorem as follows:

$$\pi(\rho_{ij} \mid data) = \frac{f(data \mid \rho_{ij})\pi(\rho_{ij})}{f(data)} \qquad (1)$$

The term $f(data \mid \rho_{ij})$ will tell us how likely one is to observe student understanding/performance data given the instructors prior knowledge. The term $1/f(data)$ should be viewed as a factor that makes the total probability equal to 1 when we consider all possible values of the understanding/performance measure and it is often referred to as the normalizing constant. Therefore, Bayes' theorem can be written as

Posterior Probability $\propto$
Likelihood $\times$ *Prior Probability.*

Since the prior distribution has been formed from previous experiences, they can be "weighted" with the previous number of students the instructor had.

Under the assumption of bivariate normality it can be shown that the posterior estimate of the correlation is

$$\hat{\rho}_{ij} = \tanh\left\{\frac{n_{prior}\tanh^{-1} r_{prior} + n_{data}\tanh^{-1} r_{data}}{n_{prior} + n_{data}}\right\}. \quad (2)$$

Consider the following example. An instructor had 120 students in the past. Based on this prior knowledge base, the instructor believes that the correlation of performances between two concepts is 0.70. The instructor teaches the course to 26 students and the observed correlation turns out to be 0.27. Then the posterior estimate of the correlation will be:

$$\hat{\rho}_{ij} = \tanh\left\{\frac{120\tanh^{-1} 0.70 + 26\tanh^{-1} 0.27}{120 + 26}\right\} = 0.642 \quad (3)$$

Another issue that to be addressed is the fact that data that will come from student learning outcome measures between two concepts will be conditional on the concept path followed. Therefore, one needs to transform partial correlations, that are the correlations of two variables while controlling for (or after removing the effect of) a third or more other variables, to pairwise correlations. The partial correlation between two concepts *i* and *j* given that the concepts *k, l, m* and *n* covered previously will be represented by $\rho_{ij.klmn}$ for the population and $r_{ij.klmn}$ for the sample. For instance, $r_{12.34}$ is the sample correlation of learning outcome measures on concepts 1 and 2, controlling for (or after removing the effect of) learning outcome measures on concepts 3 and 4. The educator compares the controlled correlation (ex., $r_{12.34}$) with the original correlation (ex., r_{12}) and if there is no difference, the inference is that the control measures on the outcomes have no effect. If the partial correlation approaches 0, the inference is that the original correlation is spurious, i.e. there is no direct causal link between the two original variables because the control variables are either (1) common anteceding causes, or (2) intervening variables. This translation of correlation will be achieved by using the following formulas [1]:

$$\rho_{ij.k} = \frac{\rho_{ij} - \rho_{ik}\rho_{jk}}{\sqrt{\left(1-\rho_{ik}^2\right)\left(1-\rho_{jk}^2\right)}}, \quad (4)$$

$$\rho_{ij} = \rho_{ij.k}\sqrt{\left(1-\rho_{ik}^2\right)\left(1-\rho_{jk}^2\right)} + \rho_{ik}\rho_{jk}. \quad (5)$$

In general:

$$\rho_{ij.q+1,\ldots,p} = \frac{\rho_{ij.q+2,\ldots,p} - \rho_{i,q+1.q+2\ldots,p}\,\rho_{j,q+1.q+2\ldots,p}}{\sqrt{\left(1-\rho_{i,q+1.q+2\ldots,p}^2\right)\left(1-\rho_{j,q+1.q+2\ldots,p}^2\right)}}. \quad (6)$$

3.4 An Example

A simple example from an introduction to statistics course may clarify these ideas. The primary concepts of the course are; producing data, descriptive statistical analysis, probability models, inferential statistical analysis, and statistical models such as regression, analysis of variance and analysis of contingency tables. Fig.1 provides the connections between these primary concepts. The edge properties are the prior correlations. Fig. 2 & 3 are the two alternative concept paths and associated partial correlations based on the priors that have been set by the instructor.

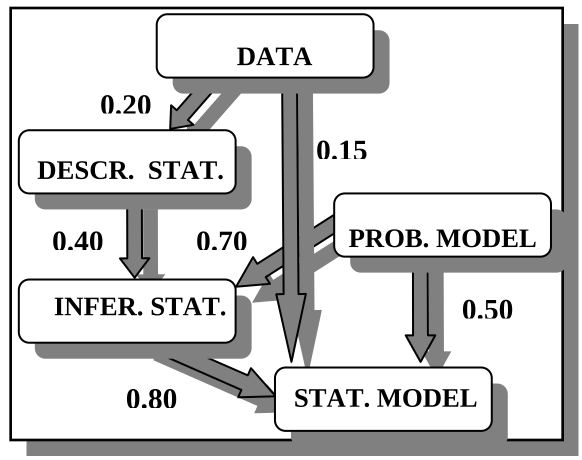

Fig. 1 Primary concept graph and prior correlations for an introduction to statistics course.

Selecting the “better” concept path will depend on the instructor’s teaching philosophy (since all possible concept paths were not considered, the term “better” is used instead of optimal. One may aim to make the connections between the primary concepts weaker or stronger. In this case our objective will be to maximize the strength of the connections. Since the Wiener distance for alternative A is higher than B, we will view Fig. 2 as a “better” path to follow.

Now we will discuss how these prior probabilities may be updated given the course evaluation/ assessment process results. For simplicity, we are going to assume that each one of the five primary concepts are evaluated with an exam, and prior/posterior correlations are based on 120 past and 26 present students, respectively. The prior, observed, and posterior correlations are given in Table. 1 and the resulting concept paths are given in

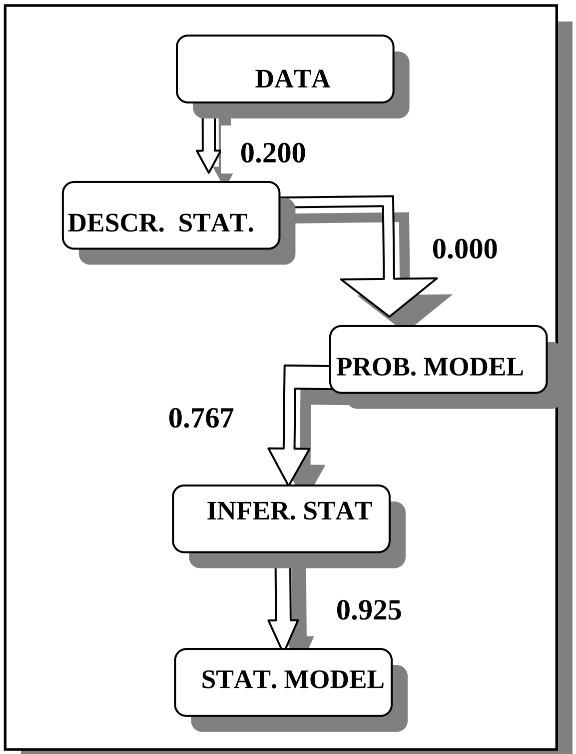

Fig. 2 Primary concept path and associated partial correlations on prior correlations (alternative A).

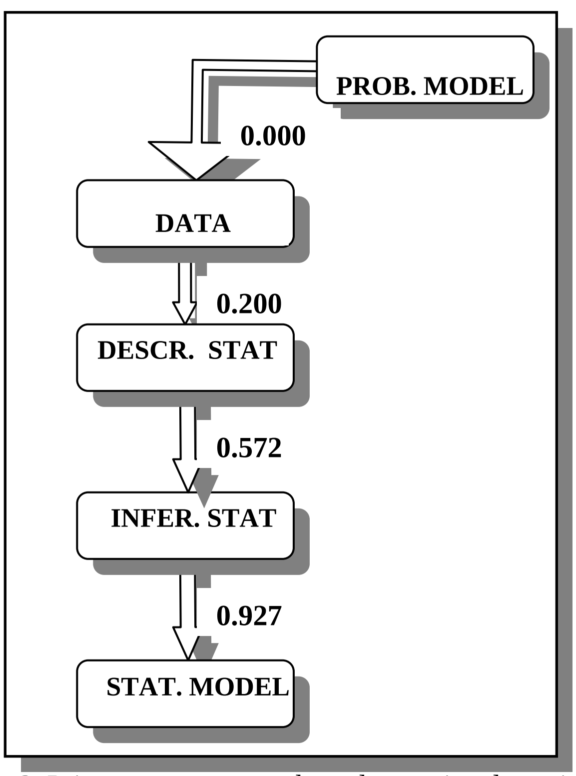

Fig. 3 Primary concept path and associated partial correlations on prior correlations (alternative B).

Fig. 4 & 5. Even if the "better" path is still alternative A, there is a substantial change in the Wiener distance of alternative B. This may indicate that further collection of data, may make path B "better".

CONCETS	**Data**	**Desc. Stat.**	**Prob. Model**	**Infer. Stat.**	**Stat. Model**
Data	1.000 1.000 1.000	0.200 0.800 0.347	0.000 0.700 0.153	0.000 0.350 0.065	0.150 0.400 0.197
Desc. Stat.	*Prior* *Obs.* *Post.*	1.000 1.000 1.000	0.000 0.900 0.256	0.400 0.650 0.451	0.000 0.300 0.055
Prob. Model		*Prior* *Obs.* *Post.*	1.000 1.000 1.000	0.700 0.950 0.778	0.500 0.950 0.651
Infer. Stat.			*Prior* *Obs.* *Post.*	1.000 1.000 1.000	0.800 0.800 0.800
Stat. Model				*Prior* *Obs.* *Post.*	1.000 1.000 1.000

Table. 1 Prior, observed, and posterior correlations

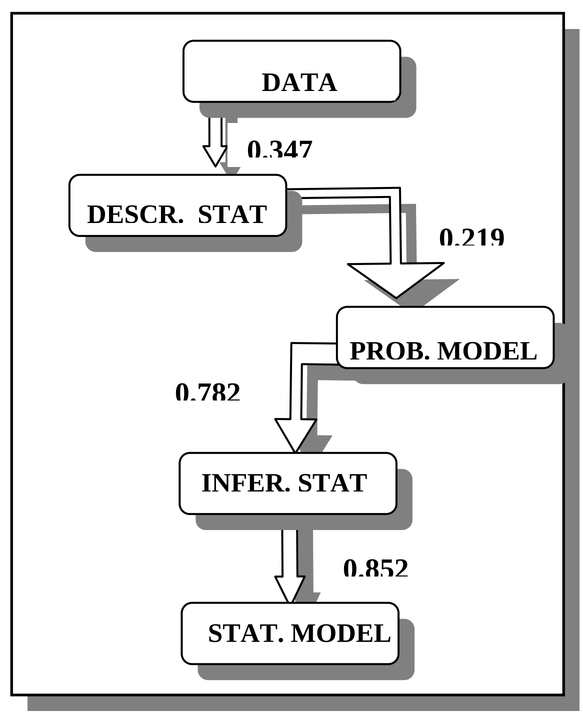

Fig. 4 Primary concept path and associated partial correlations on posterior correlations (alternative A).

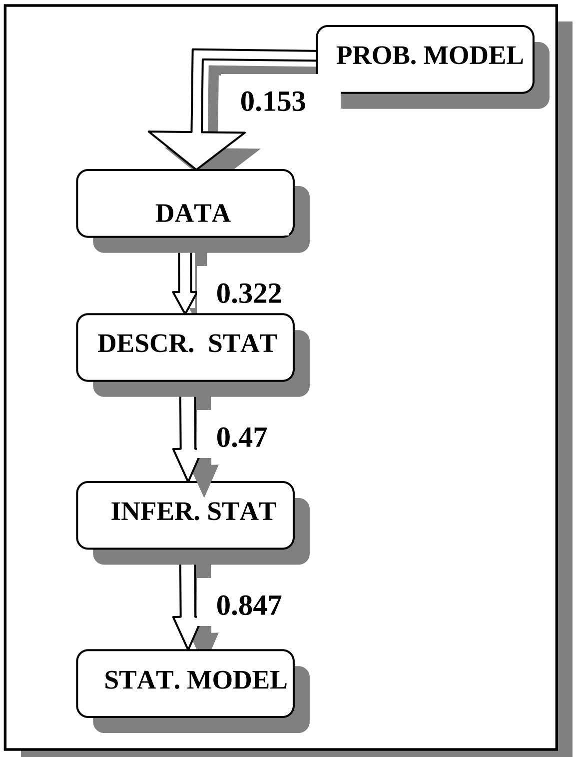

Fig. 5 Primary concept path and associated partial correlations on posterior correlations (alternative B).

4 From Concept Trees to Vines

Vines are another from of concept graphs that serve as effective tool for modeling concepts. The **concept vine** is a nested concept tree where edges of each concept tree become the concepts of another concept tree [7]. Concept vines are useful when the integrated knowledge of pre-concepts (primary and secondary) are required.

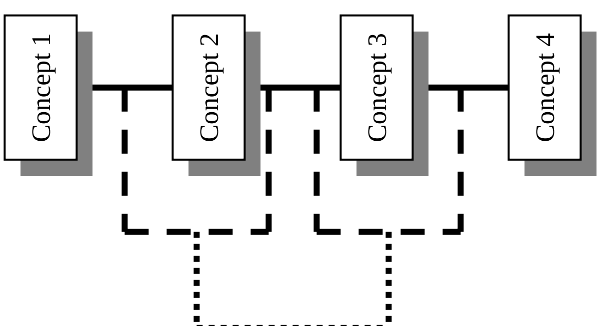

Fig. 6 An example of concept vine.

Fig. 6 shows a concept vine on 4 concepts. The three nested concept trees are distinguished by the line style of the edges; tree 1 has solid lines, tree 2 has dashed lines, etc. The conditioned (before |) and conditioning (after |) sets associated with each edge are determined as follows: the concepts reachable from a given edge are called the constraint content set of that edge. When two edges are joined by an edge of the next content tree, the intersection of the respective constraint content sets are the conditioning contents, and the symmetric difference of the constraint content sets are the conditioned contents. Note that each pair of contents occurs once as conditioned variables.

The following is an algorithm for construction of concept vines.

Step 1. Content Order. We construct an order for the contents, that is, an ordering such that all ancestors of content i appear before i in the ordering. A content order begins with a source/basic content and ends with a sink/target/outcome content. In most of the cases the content order is not unique.

Step 2. Factorize Overall/Main Content. We first factorize the overall content following the content order. If the order is 1,2,...n,

Content(1, 2, …, n)
Content(1)
Content(2|1)
Content(3|1,2)
…
Content(n|1,2, …, n-1)

Next, we underscore those learning nodes in each learning condition (prerequisite) which are not necessary in the comprehension of the conditioned content. For each term, we order the conditioning contents, i.e. the contents right of the "|", such that the underscored contents (if any) appear right-most and the non-underscored contents left-most.

Step 3. Quantify Concept Vine for Content n. Suppose the last term looks like:

Content(n | n-1,n-3,...n-2, 3,2,1).

Construct the concept vine with the content nodes in the order in which they appear, starting with n (left) and ending with the last underscored node (if any). If the vine Content(n-1, n-3,...1) is given, the vine Content(n, n-1,...1) can be obtained by adding the edges:

(n, n-1), (n, n-3| n-1)....(n, 1 | n-1,...2).

For any underscored content node k, we have contents n and k unrelated given all non-underscored nodes or any subset of underscored's not including k.

For any non-underscored content node j, the joint relationship between contents n and j given non-underscored contents before j needs to be assessed. The conditioned contents (n,j) correspond to an arc in the graph. Write these conditional contents next to the corresponding arcs in the graph.

Step 4. Quantify Concept Vine for contents n-1, n-2 etc. Proceed as in step 3 for contents 1,2,...n-1. Notice that the order of these contents need not be the same as in the previous step. Continue until we reach the content vine for 1 and 2 or until the order doesn't change in smaller subvines. i.e, if for content 4 the vine is Content(4321) and for content 3 it is Content(321) then we can stop with content 4; or better, we can quantify the vine Content(321) as a subvine of Content(4321).

Step 5. Construct the Concept Vine. Use in the instructional process.

5 Conclusion

The research that we have presented is an outcome of interdisciplinary collaboration that includes education, graph theory, and statistics. Moving from concept maps to concept graphs/trees and concept trees to concept vines opens up various opportunities to improve all stages of the instructional process. The tools introduced in this paper have the potential to allow optimization of instruction based on the identified target student body, integration of students' prior knowledge and needs, integration of instructors' experience and expertise, and empowerment of instructors' decision making. One needs to weigh the time factor for developing such tools with the impact on students' learning that results from the use of these tools. We are in the process of expanding the implementation, and creating an extensive database in more than one field to address this issue. Our research indicates a need for further theoretical work on instructional design, statistics, and graph theory. We aim to present more results in these fields in our next paper. To provide access to the ideas introduced in this paper by the educators we aim to develop various instructional technology tools in the future.

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