Complex Dielectric Constant Computation from Three Dimensional Simulations of Random Media Using The Finite Difference Time Domain Method

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Abstract: - Several mixing models are available in order to determine the effective dielectric constant of materials made up of different constituents. Most of these models have regions of validity that depend on the fractional volume of inclusions and the operating frequency. In this paper we compare results from the Claussius Mossotti mixing law to results obtained from a three dimensional Finite Difference Time Domain model of a random medium such as sea ice. The comparison shows that the FDTD model agrees with the mixing law for low fractional volume (less than 10 %) and low frequency less than 5 GHz.

Key-Words: - random media, dielectric constant, FDTD, mixing law, sea ice, Monte Carlo.

1 Introduction

Natural media which consist of a host material containing a random distribution of inclusions are very common. Examples include soil and sea ice. When studying scattering of electromagnetic waves from such media it is necessary to obtain an equivalent or average value of the dielectric constant of the medium.

Several theoretical mixing models are available for obtaining equivalent dielectric constants of random media[4]. These models have limitations in terms of validity for certain values of inclusion fractional volumes and frequency.

Numerical simulations are often used to determine the effective dielectric constant of random media outside the regions of validity of mixing formulas [1,2,3,4,5,6,7]. Mixing formulas in turn can provide a validation for numerical methods in the region where these formulas are applicable. In order to show a validation of a Finite Difference Time Domain (FDTD) [8] model for sea ice scattering, and to study the results from the mixing formulas outside their regions of validity, the effective dielectric constant of an inhomogeneous medium obtained from the FDTD model is compared with analytical results from the dielectric mixing formulas. For our comparison we take the example of sea ice and consider the brine inclusions in sea ice as spherical particles which are randomly distributed in a pure ice background.

2 Problem Formulation

The brine inclusions are generated in the FDTD model as follows:

a- Form the FDTD mesh

b- For each cell in the FDTD domain determine if a brine pocket is present through the use of a random number generator with a threshold determined by the fractional volume of brine in the sea ice sample. The threshold is determined as follows. Given the fractional volume of brine f_{vb} and the size of the brine pocket v_i we can determine the number n of brine pockets in the sea ice sample;

$$n = \frac{V * f_{vb}}{v_i} \quad (1)$$

where V is the volume of the FDTD domain. Let the number of FDTD cells be N, then the random number generator threshold t is given the presence of one brine pocket per FDTD cell:

$$t = \frac{n}{N} \quad (2)$$

c- For each cell that contains a brine pocket we compute the effective dielectric constant using the Claussius Mossotti mixing law. Since the volume of the brine inclusion is smaller than that of the FDTD cell, the fractional volume of the FDTD cells that contain brine inclusions(f_{v-FDTD}) is related to that of the brine inclusions(f_{vb}) by:

$$f_{v-FDTD} = f_{vb} \frac{v_{FDTD}}{v_b} \quad (3)$$

where v_b and v_{FDTD} are the volumes of the brine inclusion and the FDTD cell respectively.

The result of this procedure is an FDTD domain containing cells with two different values of dielectric constant and randomly distributed. Representing the brine pockets, assumed to have spherical shapes, with cubical FDTD cells having a larger volume and lower dielectric constant is an approximation that is necessary to avoid discretizing the small brine cells. Such discretization can cause the problem to have a much larger number of unknowns. Given the frequency range considered here and the size of the FDTD cells, this approximation is a reasonable one. The validity of this approximation is tested by comparison with analytical models and sea ice measurements. Figure 2 shows the real part dielectric constant distribution for the FDTD cells for a realization where the fractional volume of brine is 10 percent. The background dielectric constant is set to 3.2-j0.0, that of pure ice. We can visualize the propagation of the field in such a medium by running the code and plotting the magnitude of the field at several time instants. This is shown in Figure 3 where the magnitude of the field in the x-direction is plotted in dB along a cut through the center of the domain in the x-z plane. The four different plots in Figure 3 show the field magnitude after 80, 160, 320 and 800 time steps.

In order to get the effective dielectric of the inhomogeneous medium modeled with FDTD we perform Monte Carlo simulations for a slab of random medium backed by a PEC. Taking the average of the scattered fields from the different realizations, we obtain the coherent reflected field from the medium since the incoherent scattered fields average to zero due to their random nature. After obtaining the coherent reflected field, we compare it to the fields obtained from a series of FDTD simulations of homogeneous media backed by a PEC of the same size as the inhomogeneous medium and having different dielectric constant values (Figure 1).

2.1 Monte Carlo Simulations

Monte Carlo simulations were performed using machines at the MHPCC. The fractional volume of brine inclusions in the sea ice medium was varied from 2.5 percent to 40 percent. For each value of fractional volume 8 realizations were generated and the reflected field at normal incidence computed. In order to reduce the number of homogeneous media simulations, we use interpolation between adjacent points.

3 Problem Solution

Since the range of fractional volumes studied here extends beyond the region of validity of the De Loor formula, we use the Claussius Mossotti formula also known as the Maxwell-Garnett mixing formula to obtain the effective dielectric constant for the inhomogeneous media. The scatterers are assumed to be spherical and having a volume equal to that of the FDTD cells. The imaginary part of the dielectric constant used in the mixing formula is frequency dependent since the conductivity used in the FDTD model is constant with frequency. The radius of the brine inclusions is 0.5 mm. For each FDTD cell that contains a brine inclusion, an effective dielectric constant for the cell is computed using the mixing law.

Figures 4 and 5 show a comparison between results obtained from the FDTD model and the Claussius-Mossotti(CM) formula. The De Loor mixing formula [4] is only valid for low fractional volume $\leq$ 10 %. The Polder Van Santen mixing formula [4] for the general case was found to produce unrealistic values for the bulk dielectric constant and hence was not used here. For the low fractional volume case, where the mixing law results apply, we see a very good agreement between FDTD and mixing law for all frequencies. As the fractional volume increases the FDTD results become noticeably larger than the mixing law results for the conductivity; however, the real part results still show good agreement. The increase in the FDTD results is due to the scattering loss due to the higher fractional volume. Scattering losses are not taken into account in the mixing law. The scattering component increases with frequency since scattering is inversely proportional to the wavelength to the fourth power.

4 Conclusion

These results show that the FDTD model for sea ice is valid as evidenced by the low fractional volume results at low frequencies where the agreement with the mixing law is very good. Also this comparison shows that caution should be exercised when using mixing laws with high fractional volumes of scatterers and high frequencies, in this case fractional volumes greater than 10 percent and frequencies above 5 GHz. The imaginary part of the dielectric constant increases with frequency and fractional volume of brine, while the real part increases with fractional volume of brine but does not show a strong frequency dependence.

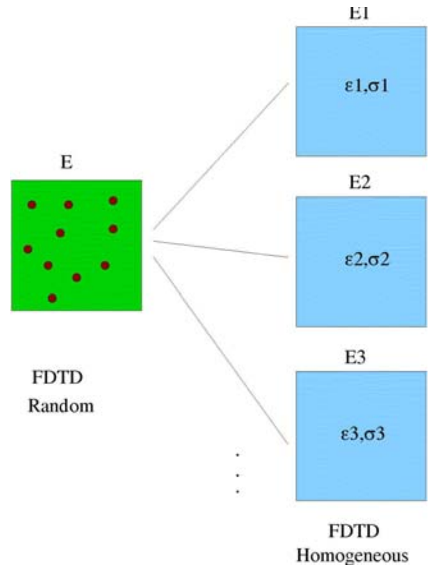

Figure 1: Matching of simulations to homogeneous media.

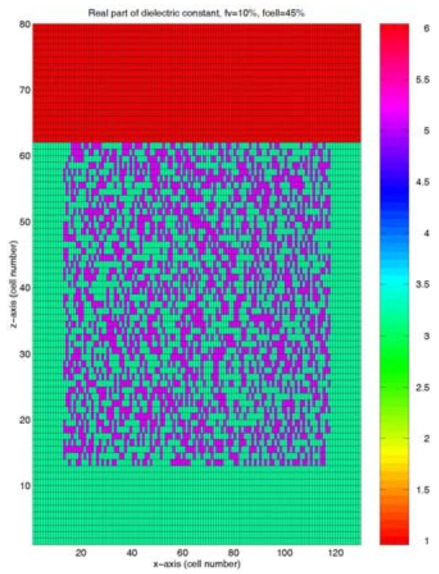

Figure 2: A two-dimensional cut through the FDTD domain.

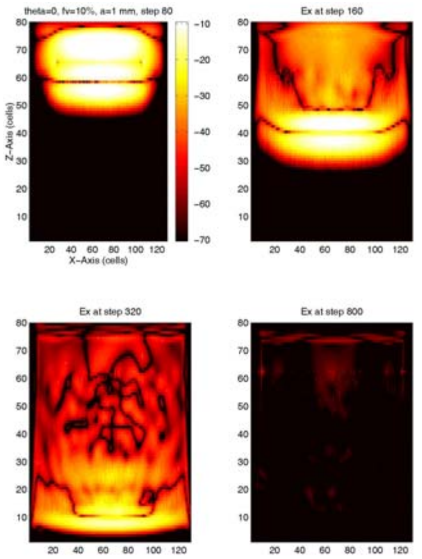

Figure 3: Wave propagation in the FDTD domain at several time instants

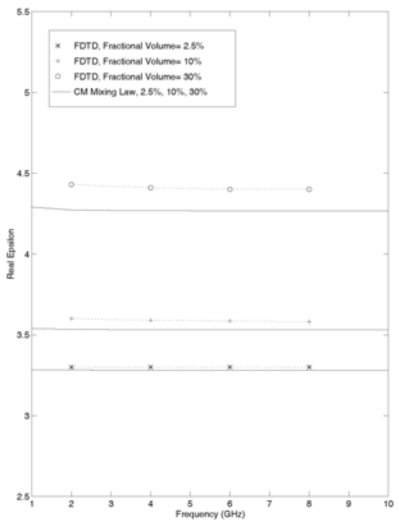

Figure 4: Comparison of FDTD results for the real part of the dielectric constant with the CM mixing law

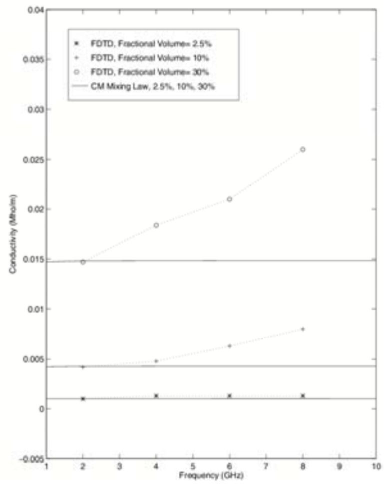

Figure 5: Comparison of FDTD results for the imaginary part of the dielectric constant with the CM mixing law.

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