Numerical Analysis of Mixed Convection Heat Transfer (Forced & Free) of Viscoelastic Fluid in a Square Channel for Laminar and Fully Developed Flow

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Abstract: - Heat transfer analysis of Viscoelastic fluids in noncircular channel has complication because of nonlinear behavior of fluid consistency equation and channel geometry.
In this paper a weak secondary flow is shown using generalized model CEF and numerical solution. Numerical solution is based on the Artificial Compressibility (Chorin method) and using staggered mesh. The effect of secondary flow on the forced and natural heat convection is studied. Effect of fast secondary flows at forced and free convection was assimilated. Quantities of static pressure and temperature distributions were obtained. Computer program was used for 3-D CFD technique.

Key-Words: Forced and Free convection – Viscoelastic fluid – square channel – Secondary flows

1 Introduction

Viscoelastic fluids are classified at Non-Newtonian fluids group. They have applications in many different industries such as petrochemistry, foodstuffs, polymer and etc [1]. They have both viscous and elastic property and normal stresses is not equal that these cause to phenomenon such as Rod-climbing, Die-swell, Hole pressure error, Recoil and etc [2,3]. Usually, apparent viscosity of these fluids is more than Newtonian fluids viscosity, so laminar solution of these fluids in channels is of interest. The secondary flows were seen by Dodson et. al [4] at the theory-empirical research and using CEF model. Hartnett and Kostic analysis heat transfer to Non-Newtonian fluids in rectangular ducts with power low model [5]. Rohsenow and Hartnett [6] presented the forced convection heat transfer solution in a channel by use of power law model. Talebi [7] show these flows by using aforementioned model and ADI method. Numerical solution of free convection is not investigated yet and a new subject to research [8]. The geometry characteristics and coordinates is shown in Fig.1.

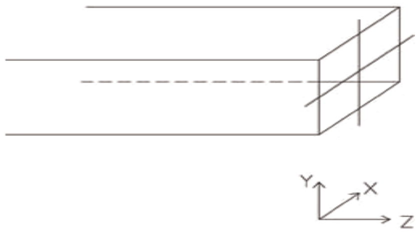

Figure 1: The channel geometry and coordinates.

Behavior of these fluids is expressed by linear, quasilinear and nonlinear models. Because fully develop flow and exist of secondary flow in channel, we use CEF nonlinear model [4].
The generalized model CEF is used for characteristic equation of viscoelastic fluid according to equation (1).

$$\tau_{ik} = 2\eta(q)\ e_{ik}^{(1)} - N_1(q)\ e_{ik}^{(2)} + 4\left[N_1(q) + N_2(q)\right]\ e_{ij}^{(1)} e_{jk}^{(1)} \quad (1)$$

Where $e_{ik}^{(n)}$ is the nth rate of strain tensor [4]. This model establishes for along the line flow and is applicable with a good approximation for weak secondary flows [4].The rheological functions η, N_1, N_2 specify as below and their definition depends on the fluid type.

$$\eta(q) = \eta_0 \frac{1+\sigma_2 q^2}{1+\sigma_1 q^2} \quad (2)$$

$$q=\sqrt{\frac{1}{2}.I_2}=\sqrt{\frac{1}{2}d_{ij}d_{ji}}=\frac{1}{2}\sqrt{\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2+2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right)\right]} \tag{3}$$

$$N_1(q)=C_{N1}\left[\eta\frac{(\lambda_1-\lambda_2)+(\lambda_1\sigma_2-\lambda_2\sigma_1)q^2}{1+\sigma_1 q^2}\right] \tag{4}$$

$$N_2(q)=C_{N21}-C_{N22}\eta(q) \tag{5}$$

$$e_{ik}^{(1)}=\frac{1}{2}\left(\frac{\partial v_k}{\partial x_i}+\frac{\partial v_i}{\partial x_k}\right) \tag{6}$$

$$e_{ik}^{(2)}=\frac{1}{2}\left(v_m\frac{\partial^2 v_k}{\partial x_i\partial x_m}+v_m\frac{\partial^2 v_i}{\partial x_k\partial x_m}+\frac{\partial v_m}{\partial x_i}\frac{\partial v_k}{\partial x_m}+2\frac{\partial v_m}{\partial x_i}\frac{\partial v_m}{\partial x_k}+\frac{\partial v_i}{\partial x_m}\frac{\partial v_m}{\partial x_k}\right) \tag{7}$$

2 Governing Equations

First, the governing equation for forced convection case is presented. The index form of continuity, momentum and energy equations for incompressible and constant properties fluid and without free convection, source terms and viscous dissipation are as below:

$$\frac{\partial U_i}{\partial x_i}=0 \tag{8}$$

$$U_j\frac{\partial U_i}{\partial x_j}=\frac{1}{\rho}\left(-\frac{\partial P}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_j}\right) \tag{9}$$

$$U_i\frac{\partial T}{\partial x_i}=\frac{\mathrm{K}}{\rho C_P}\left(\frac{\partial^2 T}{\partial x_i^2}\right) \tag{10}$$

In the above equations, heat conduction in the main flow direction is neglected for steady and hydrodynamic developed flow. Except $\frac{\partial P}{\partial z}$ that has a constant value, $\frac{\partial}{\partial z}$ is equal to zero for other parameters.

Next, the governing equations for free convection are considered. In this case, continuity and energy equations do not change and the momentum equation changes as below:

$$U_j\frac{\partial U_i}{\partial x_j}=\frac{1}{\rho}\left(-\frac{\partial P}{\partial x_i}+\frac{\partial \tau_{ij}}{\partial x_j}\right)+F_i \tag{11}$$

It was assumed that $\frac{\partial \tau_{ij}}{\partial z}=0$ and body forces are:

$F_1=F_3=0$, $F_2=-g$. Density is a local function because it varies with temperature and so Reynolds number is a local function too as defined by these equations:

$$\rho=\frac{\rho_0}{1+\beta(T-T_0)} \tag{12}$$

$$\mathrm{Re}=\frac{\rho\,\overline{p}D_h^3}{\eta_0^2} \tag{13}$$

2.1 Boundary Conditions

Following boundary conditions are applied in governing equation solving:

1- Velocity is stated absolute and equal to the channel inlet velocity, and no slip condition on the wall is used. In addition, the wall temperature is constant and assumed to be 400k.
2- At the inlet, Dirichlet condition (constant value) is considered for velocity and temperature.
3- At the outlet, Neuman condition is considered (Fully developed conditions).

2.2 Flow Properties and Geometry

The viscoelastic fluid and geometry characteristics that are used in this research are:

1- Viscoelastic fluid properties:

$\rho=4756\ Kg/m^3, \eta_0=0.15\ N.S/m^2, \sigma_1=1.25\times10^{-4}\ s^2,$
$\sigma_2=6.25\times10^{-5}\ s^2, \lambda_1=0.01\ s, \lambda_2=0.005\ s, C_{N1}=10-20,$
$C_{N21}=0.0001-0.005\ N.s^2/m^2, C_{N22}=0.0015\ N.s^2/m^2$

2- Geometry properties:

$a=0.003\ m, D_h=0.003\ m$

3- Reynolds on the basis of appearance viscosity:

$\mathrm{Re}=5-30$

4- The entrance flow temperature=300k
5- Walls temperature=400k
6- Prandtl number=3
7- Channel length in z direction=12
8- Pressure=2103-4350 N/m^2

3 Numerical Solution

Numerical analysis steady flow is done by a quasi unsteady method. In this method after selection of proper initial conditions, the unsteady governing equations are solved until the unsteady solution converges to the steady solution [9]. The artificial compressibility is used in this research work. This method is usable for compressible Navier-stokes equation [10].

In this method, the continuity equation change as below by adding a time dependent pressure term.

$$\frac{\partial P}{\partial t}+C^2\frac{\partial U_i}{\partial x_i}=0 \tag{14}$$

The time dependent terms $\frac{\partial U_i}{\partial t},\frac{\partial P}{\partial t},\frac{\partial T}{\partial t}$ converge to zero, for the steady flows. Governing equations are discretized explicitly forward first order in time and central second order approximation in space (FTCS). Using a staggered mesh where the velocity components U and V are calculated at distance between $i+\frac{1}{2},j$ and $i,+j+\frac{1}{2}$ respectively [11]. The computer program that used for mentioned CFD technique was developed by authors.

4 Conclusion

Secondary flows streamlines in a quarter of cross section for different aspect ratio is shown in Fig.2, there are eight vortex flows in square cross section and two vortexes in the quarter cross section [12]. When the aspect ratio changes, four vortexes were deleted gradually and thus four vortexes remain. These remained vortexes will be weak gradually.

The secondary flows direction is opposite to the secondary flows that are in the turbulent Newtonian fluid flow ([2],[13]). The vortex direction can change with changing rheological constants when N_2 become positive as seen at Fig.3.

The main flow velocity is stronger than vortex velocities. Fig.4 shows this velocity in the fully developed case.

The static pressure measurement for Non-Newtonian fluids is important because of Hole Pressure Error [2]. Numerical solution with this method helps to static pressure calculation as it is shown in Fig.5. There is maximum static pressure at the channel center because there aren't any secondary flows.

The decreasing of entrance length due to secondary flow is shown in Fig.6 in which temperature distribution is shown in three cross plan of channel.

The effect of Reynolds number on the overall Nusselt number was studied and results are depicted in Fig.7. Increasing the main flow velocity and being latitudinal flows to be cause increasing in the overall Nusselt number.

The vortex velocities increase with increasing the value of rheological constant C_{N21} [14], Fig.8 shows the aforementioned effect.

It is clear from Fig.9 that vortex velocity increment has no considerable effect on the overall Nusselt number.

Density gradient is the main cause of natural convection. Density gradients depends on β and ΔT. The temperature distribution for fully developed flow is shown in Fig.10 in half of square cross section. First ΔT is remained constant. Fig.11 shows the produced latitudinal vortexes by natural convection in half of the channel cross section with increasing the value of β. As it is shown the vortex velocity decreases when β decreases. At $\beta = 1\times10^{-6}$ the natural convection effect reduces so natural and forced convection has the same order. It seems that vortex flows velocities have no effect in forced convection but they can influence natural convection.

Change in β value and maximum of vortex velocities is shown in Fig.12.

Fig.13 shows the effect of β on the overall Nusselt number.

Finally β is remained constant and temperature difference is varied from 20 to 150. The results are shown in Fig.14 and Fig.15. It is clear that temperature difference don't have considerable effect on vortex velocity increasment.

Nomenclature

a	Length of square(m)
C_{Nij}	Rhelogical constants
C_p	Specific heat(J/kg.K)
D_h	Hydraulic diameter(m)
d_{ij}	Shear rate tensor
$e_{ik}^{(n)}$	The n^{th} rate of strain tensor
g	Acceleration due to gravity(m/s^2)
I_2	Second invariant of shear rate tensor
k	Thermal conductivity coefficient(W/K.m)
N_1	First normal stress difference($Pa.s^2$)
N_2	Second normal stress difference($Pa.s^2$)
Nu_m	Overall Nusselt number
P^*	Dimensionless static pressure
P	Pressure(Pa)
q	Shear rate(s^{-1})
Re	Reynolds number
T	Temperature(K)
u,v,w	Velocity components(m/s)
V_{max}	Maximum of velocity(m/s)
x,y,z	Cartesian coordinate
X_i^*	Dimensionless coordinate
ADI	Alternating Direction Implicit
CEF	Criminale-Ericksen-Filbey

Greek Symbols

ΔT	Temperature difference
β	Volume expansion coefficient(1/K)
η	Apparent viscosity($N.s/m^2$)
η_0	Apparent viscosity at zero shear rate($N.s/m^2$)
λ_i,σ_i	Rhelogical constants
ρ	Density(kg/m^3)
τ_{ik}	Stress tensor

Subscripts

0	Component Property at inlet flow temperature
in	Inlet flow
w	Wall

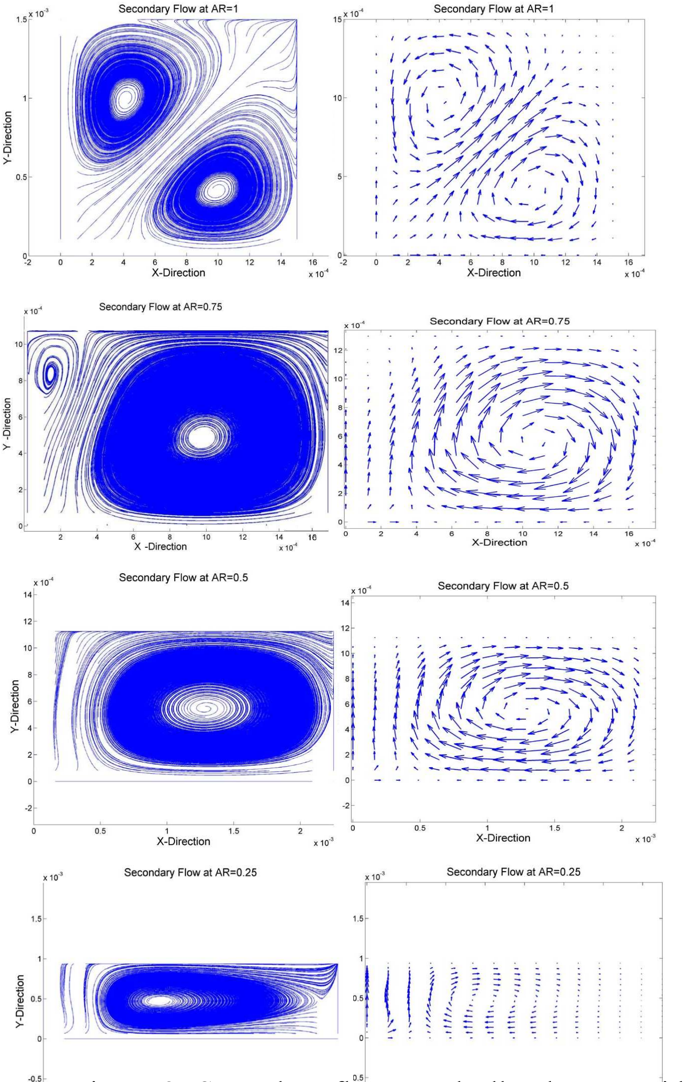

Figure 2: Secondary flows gradually changes with change in the longitude to latitude ratio.

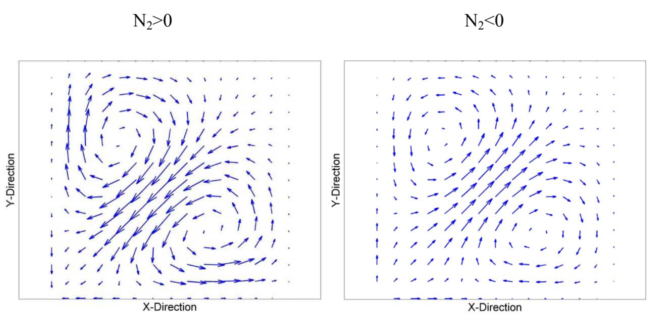

Figure 3: Change of the secondary flows direction with N_2.

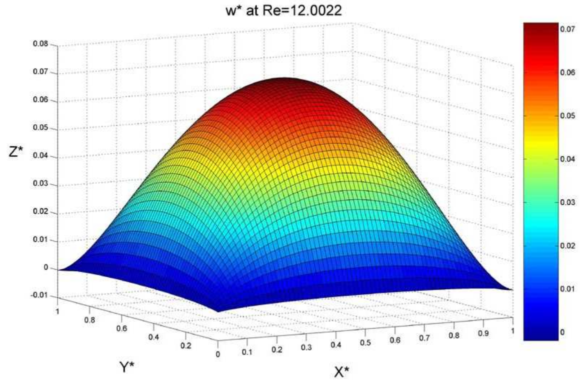

Figure 4: Non dimensional annual velocity.

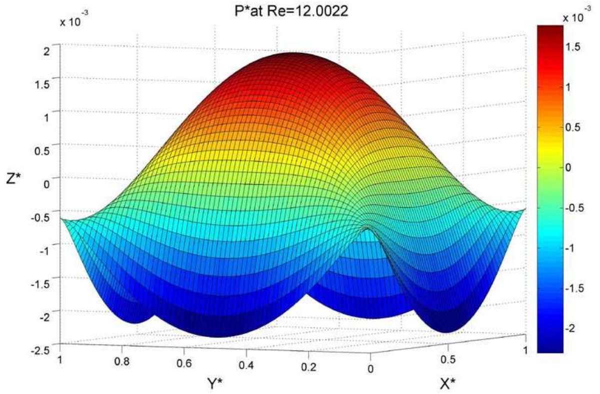

Figure 5: Static pressure contours in flow direction.

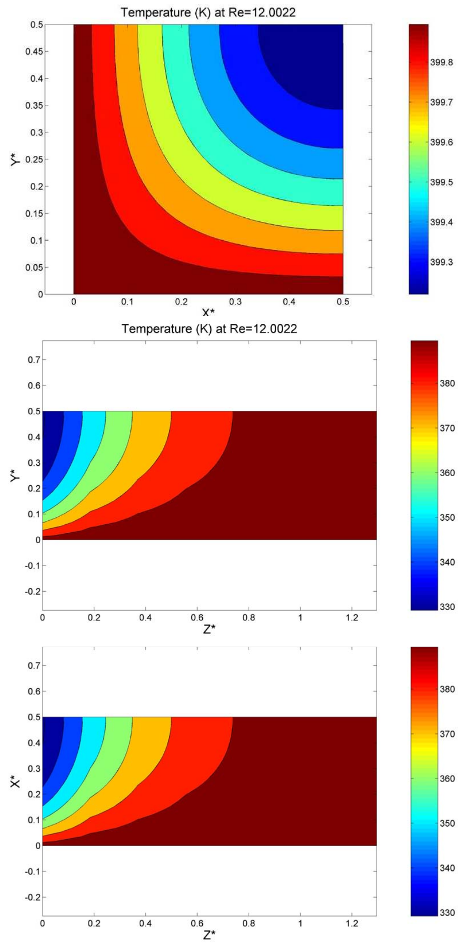

Figure 6: Temperature distribution at three planes of channel.

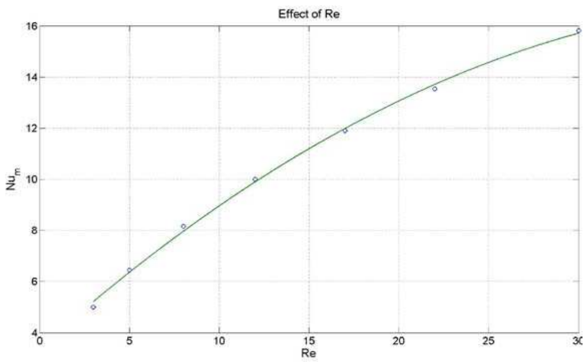

Figure 7: Effect of Reynolds number on the overall Nusselt number.

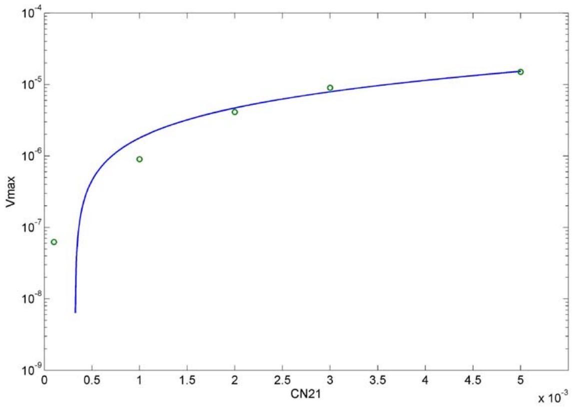

Figure 8: Effect of C_{N21} on the maximum velocity of vortexes.

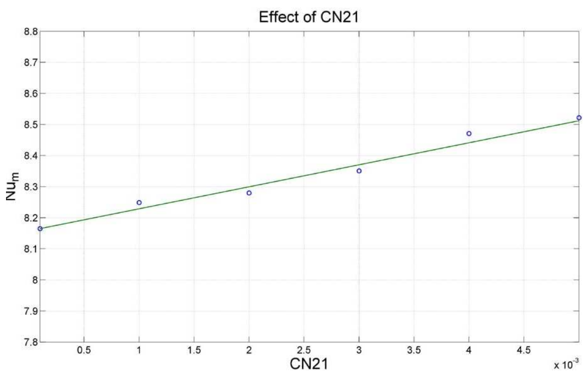

Figure 9: Effect of C_{N21} on the overall Nusselt number.

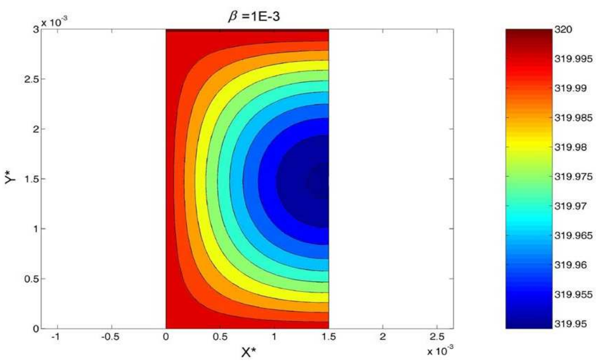

Figure 10: Temperature distribution for natural convection.

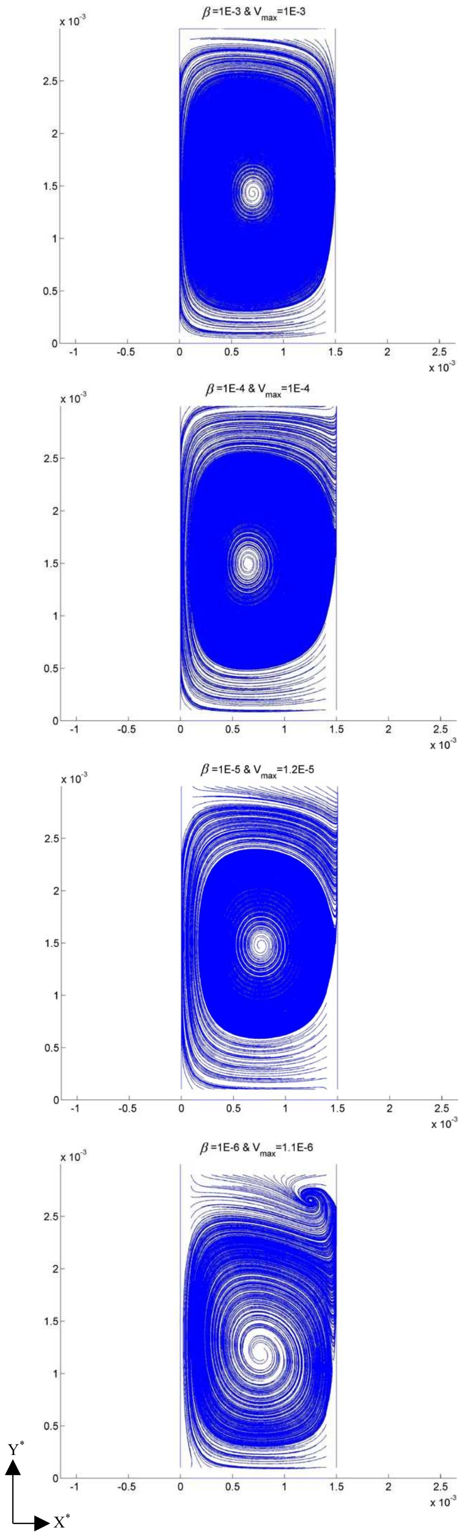

Figure 11: Vortexes streamlines and effect of β on natural convection vortexes.

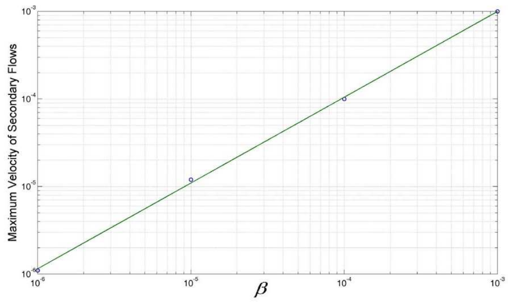

Figure 12: Effect of β on maximum velocity of vortex flows.

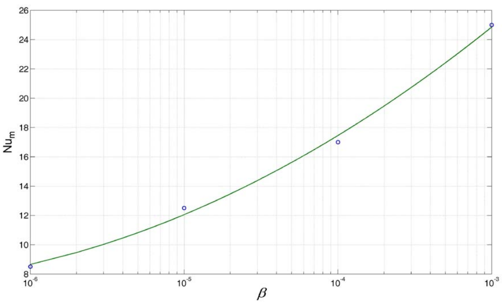

Figure 13: Effect of β on the overall Nusselt number.

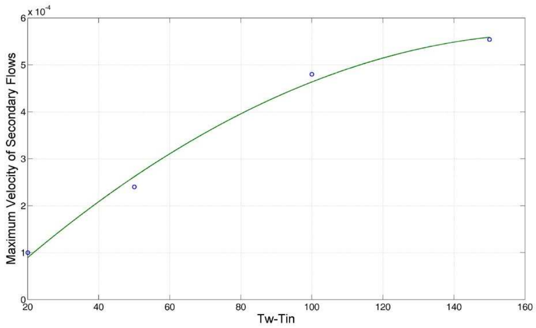

Figure 14: Effect of ΔT on maximum velocity of vortex flows.

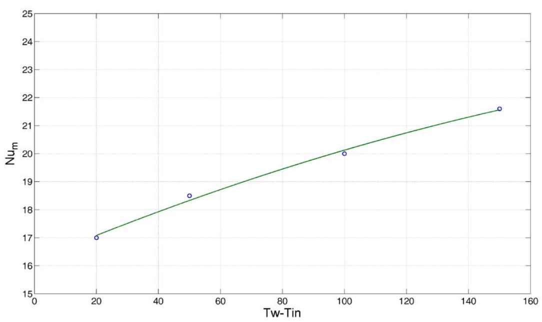

Figure 15: Effect of ΔT on the overall Nusselt number.

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