Damping analysis of an advanced sandwich composite structure

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Abstract: - The aim of the paper is to determine the most important features of damping in the case of an advanced sandwich composite structure starting from the dampings, dynamic Young moduli and Poisson ratio for every lamina. The structure features two carbon/epoxy skins reinforced with a twill weave fabric and an expanded polystyrene (EPS) core. At the damping analysis of fiber reinforced composite materials, a so called concept of complex moduli will be used in which the elastic constants will be replaced through their viscoelastic correspondences. The mechanical modeling is based on the correspondence principle of linear viscoelastic theory. Testing scheme allows specimens to be put in one side fixed connection and subjected at bending oscillations in normal conditions: 23°C, 50% relative air humidity. Dampings, rigidities and compliances of the composite structure are computed.

Key-Words: - Polymer matrix composites, Sandwich composite structure, Expanded polystyrene core, Carbon/epoxy skins, Twill weave fabric, Damping analysis, Linear viscoelastic theory, Complex dynamic moduli.

1 Introduction

Polymer matrix composites have been used increasingly in applications in aeronautics, in transportations, in automotive industry, in machine-tools construction, robotics, etc., where high dynamic loaded parts are needed. To avoid dangerous oscillating loadings, the designer of a fiber reinforced composite structure has the possibility to choose the materials couples, fibers orientation and plies succession, to improve significantly the damping of the respective structure.

The aim of the paper is to determine the most important features of damping in the case of an advanced sandwich composite structure starting from the dampings, dynamic Young moduli and Poisson ratio $\upsilon_{\parallel\perp}$ for every lamina. The mechanical modeling is based on the correspondence principle of linear viscoelastic theory, see for instance the papers [1], [2], [3].

2 Problem Formulation

In technique, the damping is usually defined as the decrease of oscillations, in which the mechanical energy contained in the system is converted into heat. This dissipation process which occurs at the interior of materials is called material's damping.

When a composite material is subjected to a sinusoidal varying stress in which the strain is also sinusoidal, the angular frequency is retarded in phase by an angle δ, retardation which takes place due to viscoelastic behavior of the matrix. The introduction of material's damping in the conditions of elastic deformations, occurs under the assumption of harmonic stresses and strains.

If we choose the abscissa as the time axis where the strain reaches its maximum, the strain and stress can be written as a function of time:

$$\varepsilon = \varepsilon_0 \cos \omega t , \tag{1}$$

$$\sigma = \sigma_0 \cos\left(\omega t + \delta\right). \tag{2}$$

In the analysis of harmonic systems is more convenient to write the stress function as a complex quantity σ^* which presents a real and an imaginary part [4]:

$$\sigma^* = \sigma_0' \cos \omega t + i \sigma_0'' \sin \omega t , \tag{3}$$

where σ_0' and σ_0'' can be expressed as following:

$$\sigma_0' = \sigma_0 \cos \delta , \tag{4}$$

$$\sigma_0'' = \sigma_0 \sin \delta . \tag{5}$$

Representing the ratios of stresses σ_0' and σ_0'' to ε_0, a dynamic or "storage" Young modulus and a "loss" modulus can be defined:

$$E' = \frac{\sigma_0'}{\varepsilon_0}, \quad (6)$$

$$E'' = \frac{\sigma_0''}{\varepsilon_0}. \quad (7)$$

According to equations (4) and (5), the ratio between the loss Young modulus and the dynamic modulus defines the material's damping:

$$\frac{E''}{E'} = \frac{\sigma_0''}{\sigma_0'} = tan\delta = d . \quad (8)$$

It is also convenient to express the harmonic stress and strain in the form of an exponential function:

$$\sigma^* = \sigma_0^* \cdot e^{i\omega t}, \quad (9)$$

$$\varepsilon^* = \varepsilon_0^* \cdot e^{i\omega t}. \quad (10)$$

Now, the complex Young modulus can be written as following:

$$E^* = \frac{\sigma^*}{\varepsilon^*}. \quad (11)$$

Taking into account the assumptions of linear viscoelasticity theory, the following material's law can be defined [4]:

$$\sigma^* = E^* \cdot \varepsilon^* = \left(E' + i \cdot E''\right) \cdot \varepsilon^* = E'\left(1 + i \cdot d\right) \cdot \varepsilon^*. \quad (12)$$

For the equation (12), Niederstadt has presented a special resonance method suitable for small amplitudes, where the specimen was subjected at bending- respective torsion oscillations [4]. According to the resonance diagram of a glass fiber reinforced lamina, the first three eigenfrequencies at bending, $f_{n,b}$, and first eigenfrequency at torsion, $f_{1,t}$, were determined.

To determine the dynamic Young modulus, E', and the dynamic shear modulus, G', the motion equations for bending, w (x,t), and torsion, θ (x,t) were analyzed. In the case of a rectangular cross section specimen with one side fixed connection, the following equations for bending are [4]:

$$E'\left(1 + i \cdot d_b\right) \cdot I_y \frac{\partial^4 w(x,t)}{\partial x^4} + \rho \cdot h \cdot \frac{\partial^2 w(x,t))}{\partial t^2} = 0, \quad (13)$$

$$E' = \frac{48\pi^2}{\left(\beta_n^2\right)^2} \cdot \frac{l^4}{h^2} \cdot \rho \cdot f_n^2, \quad (14)$$

$$d_b = \frac{\Delta f_b}{f_{nb}}, \quad (15)$$

with the eigenvalue equation:

$$1 + cosh\,\beta_n \cdot cos\,\beta_n = 0. \quad (16)$$

For torsion [4]:

$$G'\left(1 + i \cdot d_T\right) \cdot I_t \frac{\partial^2 \theta(x,t)}{\partial x^2} + r \cdot b \cdot h \frac{b^2 + h^2}{12} \cdot \frac{\partial^2 \theta(x,t)}{\partial t^2} = 0, \quad (17)$$

$$G' = \frac{4}{3 \cdot (2n-1)^2} \cdot \frac{1 + \left(\frac{b}{h}\right)^2}{x_1 \cdot \frac{b}{h}} \cdot l^2 \cdot r \cdot f_n^2, \quad (18)$$

$$d_T = \frac{\Delta f_T}{f_{nT}}. \quad (19)$$

The dampings d_b and d_T can be determined from the halve value domains Δf of the resonance peaks.

2.1 The sandwich composite structure

The sandwich structure taken into account to accomplish the damping analysis presents two carbon/epoxy skins reinforced with a 0,3 kg/m^2 twill weave fabric and an expanded polystyrene (EPS) 9 mm thick core with a density of 30 kg/m^3. The final thickness of the structure is 10 mm (fig. 1).

The carbon-fiber fabric used in this structure is a high rigidity one, that presents a so-called twill weave. The main feature of this weave is that the warp and the weft threads are crossed in a programmed order and frequency, to obtain a flat appearance (fig. 2). In order to accomplish the damping analysis, an equivalence model of the twill weave fabric is presented in fig. 3. The skins were impregnated under vacuum with epoxy resin and sticked to the core.

The data regarding the architecture of the sandwich structure are: structure's thickness: t_s = 10 mm; skins plies number: N = 2; thickness of each ply: $t'_{1...4}$ = 0,175 mm; skins thickness: t_{skin} = 0.35 mm; core thickness: h = 9 mm; fibers disposal angle of each ply: $\alpha_{1,\,3}$ = 90°, $\alpha_{2,\,4}$ = 0°; fibers volume fraction of each ply: $\varphi_{1...4}$ = 56%.

The data regarding the structure features: skins reinforcement: HM carbon fibers; fabric type: twill weave; fibers specific weight: 0,3 kg/m^2; matrix type: epoxy resin; core type: expanded polystyrene; core density: ρ_{core} = 30 kg/m^3; core Young's modulus: E_{core} = 30 MPa; core Poisson's ratio: υ_{core} = 0,35; core shear modulus: G_{core} = 11 MPa; fiber Young's modulus in longitudinal direction: $E_{F\parallel}$ =

540 GPa; fiber Young's modulus in transverse direction: $E_{F\perp}$ = 27 GPa; fiber Poisson's ratio: υ_F = 0,3; fiber shear modulus: G_F = 10,38 GPa; matrix Young's modulus: E_M = 3,5 GPa; matrix Poisson's ratio: υ_M = 0,34; matrix shear modulus: G_M= 1,42 GPa.

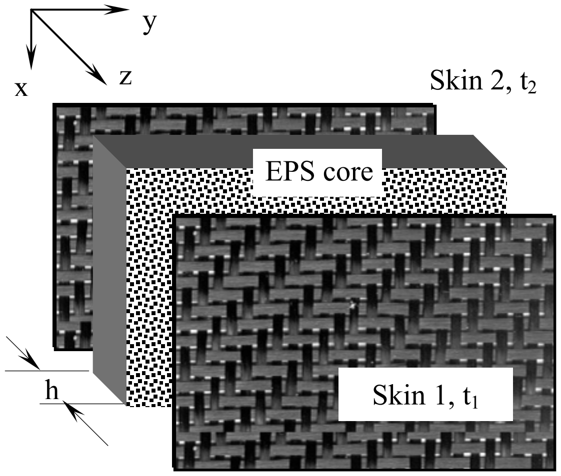

Fig. 1. The sandwich composite structure

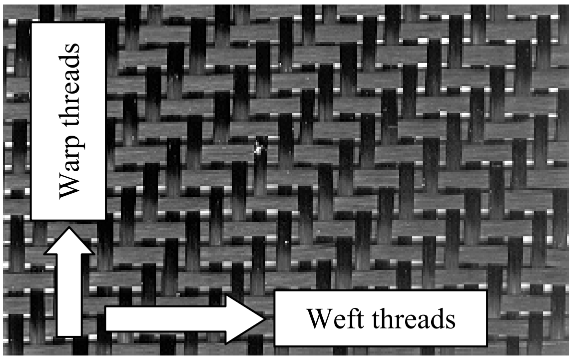

Fig. 2. The architecture of carbon/epoxy twill weave fabric skins

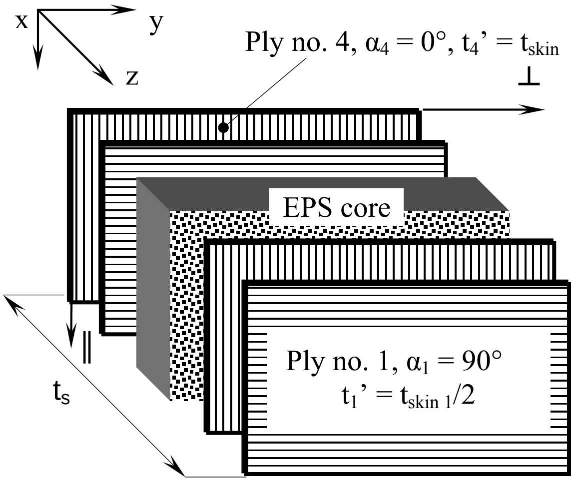

Fig. 3. The structure with an equivalence model of the twill weaves fabric skins

Regarding the dynamic behavior of the structure, we will consider the free, linear vibration of a mechanical system, which have a damped motion presented in fig. 4. Here, R is the force given by the damper, c represents the damping coefficient and k is the spring constant.

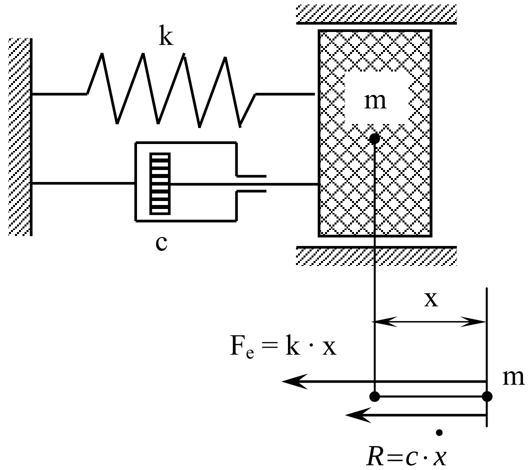

Fig. 4. Model of a free, linear, damped vibration [5]

According to the model, the fundamental equation of dynamics for a rigid body can be expressed as following [5]:

$$m \cdot \ddot{x} = -k \cdot x - c \cdot \dot{x} . \tag{20}$$

Equation (20) can be written under the form:

$$\ddot{x} + \frac{c}{m} \cdot \dot{x} + \frac{k}{m} \cdot x = 0 , \tag{21}$$

or:

$$\ddot{x} + 2\alpha \cdot \dot{x} + p^2 \cdot x = 0 , \tag{22}$$

with the notations:

$$\frac{c}{m} = 2\alpha; \quad \frac{k}{m} = p^2 . \tag{23}$$

The differential equation (22) is linear, homogeneous with constant coefficients. The characteristic equation:

$$r^2 + 2\alpha r + p^2 = 0 , \tag{24}$$

presents the roots:

$$r = -\alpha \pm \sqrt{\alpha^2 - p^2} . \tag{25}$$

In the case that α < p, the roots are complex. With the notation $\alpha^2 - p^2 = -\beta^2$, the general solution of the differential equation (22) can be under the form [5]:

$$x = e^{-\alpha t}\left(C_1 \cos\beta t + C_2 \sin\beta t\right). \quad (26)$$

Since the expression from brackets can be put under the form a·cos(βt − φ), the equation (26) can be written in the following manner:

$$x = a \cdot e^{-\alpha t} \cos(\beta t - \varphi). \quad (27)$$

Equation (27) represents a vibration modulated in amplitude and the motion is shown in fig. 5.

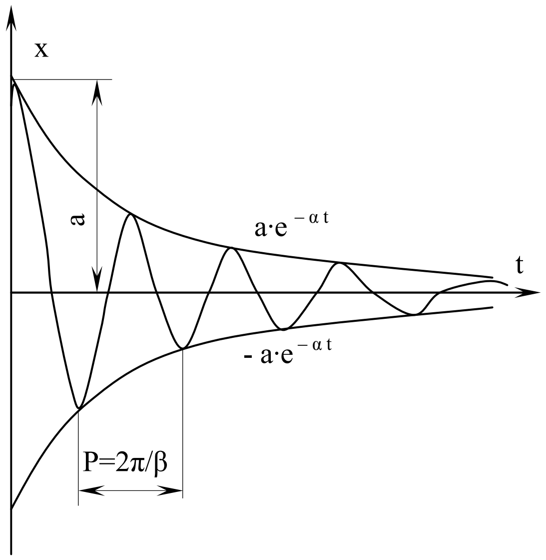

Fig. 5. Diagram of a vibration modulated in amplitude [5]

3 Problem Solution

In the followings, we will consider exclusive linear damping mechanisms, linear elastic behavior of the reinforcement and marked linear viscoelasticity of the matrix [6].

We consider that the specimen is put in one side fixed connection and subjected at bending oscillations (normal conditions: 23°C, 50% relative air humidity), see the scheme presented in fig. 6.

We consider also that the composite lamina behaves as a homogeneous continuum with anisotropic, linear viscoelastic properties.

Generally, at the damping analysis of fiber reinforced composite materials, a so called concept of complex moduli will be used in which the elastic constants will be replaced through their viscoelastic correspondences [7].

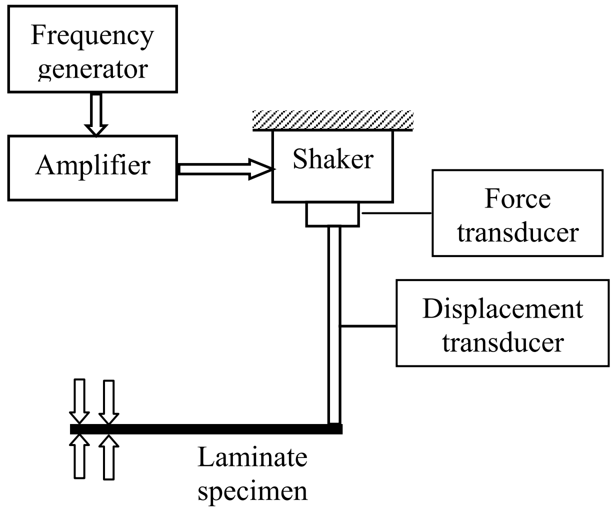

Fig. 6. Testing scheme of the sandwich composite structure

3.1 Micromechanics of lamina's damping

A lamina reinforced with continuous, parallel fibers embedded in matrix is considered. To describe the viscoelastic features of an orthotropic lamina, two coordinates axes systems will be considered (fig. 7): the global coordinates system (x-y-z) and the local coordinates system (‖ - ⊥ - z).

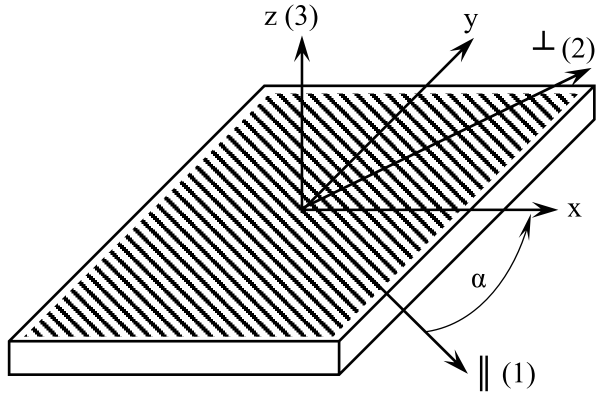

Fig. 7. Defining the coordinates axes of a lamina

For the analysis of micromechanical lamina behavior, the prism model described by Tsai [9] has been used. So, the dynamic modulus along the fibers direction can be computed from the mixture rule as following:

$$E'_{II} = E'_{F\,II} \cdot \varphi + E'_M \cdot (1 - \varphi). \quad (28)$$

Perpendicular to fibers direction, the dynamic modulus presented by Niederstadt [4], as a function of fibers- and matrix dynamic moduli as well as the fibers- and matrix dampings, can be used:

$$E'_{\perp} = -\frac{E'_{F\perp} \cdot E'_M \cdot \{d^2_{F\perp} \cdot E'_{F\perp} \cdot (\varphi-1) -}{d^2_{F\perp} \cdot E'^2_{F\perp} (\varphi-1)^2 -} \ldots$$

$$\ldots \frac{-\left[d^2_M \cdot E'_M \cdot \varphi +\right.}{-2 \cdot d_{F\perp} \cdot d_M \cdot E'_{F\perp} \cdot E'_M \cdot \varphi \cdot (\varphi-1) +} \ldots$$

$$\ldots \frac{+ E'_M \cdot \varphi -}{+ d^2_M \cdot E'^2_M \cdot \varphi^2 + E'^2_{F\perp} \cdot (\varphi-1)^2 -} \ldots$$

$$\ldots \frac{\left. - E'_{F\perp} \cdot (\varphi-1)\right]\}}{-2 \cdot E'_{F\perp} \cdot E'_M \cdot \varphi \cdot (\varphi-1) + E'^2_M \cdot \varphi^2} . \qquad (29)$$

For the damping of unidirectional reinforced lamina, the computing relations given by Saravanos and Chamis [10], starting from the cylinder model presented by Tsai [9], can be used:

$$d_{II} = \frac{d_{F\,II} \cdot E'_{F\,II} \cdot \varphi + d_M \cdot E'_M \cdot (1-\varphi)}{E'_{II}}, \qquad (30)$$

$$d_{\perp} = d_{F\perp} \cdot \sqrt{\varphi} \cdot \frac{E'_{\perp}}{E'_{F\perp}} + d_M \cdot \left(1-\sqrt{\varphi}\right) \cdot \frac{E'_{\perp}}{E'_M}, \qquad (31)$$

$$d_{\#} = d_{F\#} \cdot \sqrt{\varphi} \cdot \frac{G'_{\#}}{G'_{F\#}} + d_M \cdot \left(1-\sqrt{\varphi}\right) \cdot \frac{G'_{\#}}{G'_M}, \qquad (32)$$

The index F describes the fibers, index M is used for matrix, φ represents the fibers volume fraction and υ_M is the Poisson ratio for matrix.

3.2 Dampings, rigidities and compliances of an orthotropic lamina

The viscoelastic material's law according to the concept of complex moduli, for an orthotropic lamina, can be written as following:

$$\begin{bmatrix} \varepsilon^*_{II} \\ \varepsilon^*_{\perp} \\ \gamma^*_{\#} \end{bmatrix} = \begin{bmatrix} c^*_{II} & c^*_{II\perp} & 0 \\ c^*_{\perp II} & c^*_{\perp} & 0 \\ 0 & 0 & c^*_{\#} \end{bmatrix} \cdot \begin{bmatrix} \sigma^*_{II} \\ \sigma^*_{\perp} \\ \tau^*_{\#} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{E^*_{II}} & \frac{-\upsilon_{II\perp}}{E^*_{\perp}} & 0 \\ \frac{-\upsilon_{\perp II}}{E^*_{II}} & \frac{1}{E^*_{\perp}} & 0 \\ 0 & 0 & \frac{1}{G^*_{\#}} \end{bmatrix} \cdot \begin{bmatrix} \sigma^*_{II} \\ \sigma^*_{\perp} \\ \tau^*_{\#} \end{bmatrix}, \qquad (33)$$

Expressing the complex stresses as a function of complex strains, we obtain:

$$\begin{bmatrix} \sigma^*_{II} \\ \sigma^*_{\perp} \\ \tau^*_{\#} \end{bmatrix} = \begin{bmatrix} r^*_{II} & r^*_{II\perp} & 0 \\ r^*_{\perp II} & r^*_{\perp} & 0 \\ 0 & 0 & r^*_{\#} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon^*_{II} \\ \varepsilon^*_{\perp} \\ \gamma^*_{\#} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{E^*_{II}}{1-\upsilon^2_{II\perp} \cdot \frac{E^*_{\perp}}{E^*_{II}}} & \frac{\upsilon_{II\perp} \cdot E^*_{\perp}}{1-\upsilon^2_{II\perp} \cdot \frac{E^*_{\perp}}{E^*_{II}}} & 0 \\ \frac{\upsilon_{II\perp} \cdot E^*_{\perp}}{1-\upsilon^2_{II\perp} \cdot \frac{E^*_{\perp}}{E^*_{II}}} & \frac{E^*_{\perp}}{1-\upsilon^2_{II\perp} \cdot \frac{E^*_{\perp}}{E^*_{II}}} & 0 \\ 0 & 0 & G^*_{\#} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon^*_{II} \\ \varepsilon^*_{\perp} \\ \gamma^*_{\#} \end{bmatrix}, \qquad (34)$$

For the fiber reinforced polymer matrix composites, assuming that the dampings $d^2 << 1$, the complex compliances and rigidities for an unidirectional reinforced lamina are:

$$c^*_{ij} = c'_{ij} + i \cdot c''_{ij} = c'_{ij} \cdot \left(1 + i \cdot d_{c_{ij}}\right), \qquad (35)$$

$$r^*_{ij} = r'_{ij} + i \cdot r''_{ij} = r'_{ij} \cdot \left(1 + i \cdot d_{r_{ij}}\right). \qquad (36)$$

For $d^2 << 1$, according to equations (35) and (36), the dynamic compliances can be written in the form:

$$\left[C'\right] = \begin{bmatrix} c'_{II} & c'_{II\perp} & 0 \\ c'_{\perp II} & c'_{\perp} & 0 \\ 0 & 0 & c'_{\#} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{E'_{II}} & \frac{-\upsilon_{II\perp}}{E'_{\perp}} & 0 \\ \frac{-\upsilon_{\perp II}}{E'_{II}} & \frac{1}{E'_{\perp}} & 0 \\ 0 & 0 & \frac{1}{G'_{\#}} \end{bmatrix}, \qquad (37)$$

and the dynamic rigidities can be written under the following form [4]:

$$\left[R'\right] = \begin{bmatrix} r'_{II} & r'_{II\perp} & 0 \\ r'_{\perp II} & r'_{\perp} & 0 \\ 0 & 0 & r'_{\#} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{E'_{II}}{1-\upsilon^2_{II\perp} \cdot \frac{E'_{\perp}}{E'_{II}}} & \frac{\upsilon_{II\perp} \cdot E'_{\perp}}{1-\upsilon^2_{II\perp} \cdot \frac{E'_{\perp}}{E'_{II}}} & 0 \\ \frac{\upsilon_{II\perp} \cdot E'_{\perp}}{1-\upsilon^2_{II\perp} \cdot \frac{E'_{\perp}}{E'_{II}}} & \frac{E'_{\perp}}{1-\upsilon^2_{II\perp} \cdot \frac{E'_{\perp}}{E'_{II}}} & 0 \\ 0 & 0 & G'_{\#} \end{bmatrix}. \qquad (38)$$

According to Niederstadt, the dampings $d_{c_{ij}}$ and $d_{r_{ij}}$ are [4]:

$$d_{r_{II}} = d_{II} + \left(d_{\perp} - d_{II}\right) \cdot \frac{E'_{\perp} \cdot \upsilon^2_{II\perp}}{E'_{II} - E'_{\perp} \cdot \upsilon^2_{II\perp}}, \quad (42)$$

$$d_{r_{II\perp}} = d_{r_{\perp}} = d_{\perp} + \left(d_{\perp} - d_{II}\right) \cdot \frac{E'_{\perp} \cdot \upsilon^2_{II\perp}}{E'_{II} - E'_{\perp} \cdot \upsilon^2_{II\perp}}, \quad (43)$$

$$d_{r_{\#}} = d_{II\perp}, \quad (44)$$

$$d_{c_{II}} = -d_{II}, \quad (45)$$

$$d_{c_{II\perp}} = d_{c_{\perp}} = -d_{\perp}, \quad (46)$$

$$d_{c_{\#}} = -d_{II\perp}. \quad (47)$$

3.3 Input data

The input data taken into account in the damping analysis are presented in table 1.

Table 1. Input data

E'_M [GPa]	2,6
υ_M [-]	0,34
d_M [%]	1,4
$E'_{F\,II}$ [GPa]	226
$E'_{F\perp}$ [GPa]	16
$G'_{F\#}$ [GPa]	43
$d_{F\,II}$ [%]	0,13

3.4 Results

The results are presented in the following tables.

Table 2. Micromechanical calculus of the laminas damping

E'_{II} [GPa]	127,7
$E'_{\perp}$ [GPa]	5,89
d_{II} [%]	0,141
$d_{\perp}$ [%]	0,833
$d_{\#}$ [%]	1,929
G'_M [GPa]	0,97

Table 3. Compliances, rigidities, dampings

c'_{II} [GPa^{-1}]	0,00783
$c'_{II\perp}$ [GPa^{-1}]	- 0,04923
$c'_{\perp}$ [GPa^{-1}]	0,16977
$c'_{\#}$ [GPa^{-1}]	0,18939
d_{cII} [%]	0,141
$d_{cII\perp}$ [%]	- 0,833
$d_{c\#}$ [%]	- 1,929
r'_{II} [GPa]	128,19
$r'_{II\perp}$ [GPa]	1,71
$r'_{\perp}$ [GPa]	5,91
$r'_{\#}$ [GPa]	5,28
d_{rII} [%]	0,143
$d_{rII\perp}$ [%]	0,835
$d_{r\#}$ [%]	1,929

4 Conclusions

The dampings of unidirectional reinforced laminas are very different along and transverse to the fibers direction. The maximum value of the damping seems to be at 45° against the fibers direction.

In the future researches the whole sandwich structure will be experimentally tested to obtain more useful data for the damping analysis of this structure with many applications.

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