THEORETICAL RESEARCHES REGARDING GENERATING VERY HIGH PRESSURE PULSATORY JETS OF IN ORDER TO ENHANCE PROCESSING PERFORMANCES AND EFFICIENCY OF CONCENTRATED WATER JETS

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Abstract: - This article presents a theoretical substantiation for pulsatory jets of very high pressure, specific physical phenomenons which are the base of producing pulsatory jets of very high pressure and their mathematical relations from a qualitative point of view. In the end the conclusions are presented, according to the theoretical study which was made, regarding to expected results in the particular case of pulsatory jets, relations between their parameters and those of continous jets.

Key-Words: High pressures, Water jets, Pulsatory jets

1 Introduction

Related to the undulatory phenomenons, the water flow through high pressure pipes is characterized by:

- the transmission environment of the waves is opened, communicating with the exterior at one end;
- the propagation direction is axial and unidimensional;
- the generator movement creates pressure pulses, by compressing the liquid.

These considerations lead to the conclusion that the waves formed inside the pressure pipes are progressive compression waves (unlike the stationary waves which are specific to the closed environments), like sound waves which convey by air. Because the conveying direction is axial, the waves are longitudinal, the equations which describing them being expressed in one dimension.

2 Problem Formulation

The progressive waves are determined by external forces with a periodical variation in time (harmonical), which convey through elastic environments of various types: string with continous mass distribution, many serial springs, a fluid with elastical properties.

The external force is in contact with the elastic environment at one end, the other end being free and its length being big, at the limit of inifinit.

Characteristic to this kind of waves is the fact that the deformation is conveyed through elastic environment, from the generator, with a certain phase velocity, points situated at different distances from this, oscillating in phase with the point which is in contact with the generator after a certain period of time, when the pulse reaches the specific point.

The wave function specific for progressive waves has the form given by the expression:

$$\psi(z,t) = A\cos(\omega t - kz) \qquad (1)$$

where:-t-time(s):

-z- displacement on the z axis (m);
-A- the oscillation amplitude (m);
-ω-pulsation (rad/s);
-k-wave number (m^{-1}).

The dispersion law, for a system of coupled oscilators, has an expression similar to the stationary waves expression, given by the following expression:

$$\omega^2 = \left(\frac{g}{l}\right) + \left(\frac{4k}{M}\right) \cdot \sin^2\left(\frac{k \cdot a}{2}\right) \qquad (2)$$

where:

-g-gravity acceleration (m/s^2);
-l-oscilator lenght (m);
-k-elasticity constant of the medium (N/m);
-M- the oscilating system mass (Kg);
-a- the distance between two oscilating

masses (m)
It is observed from relation (2) that there are two limit frequences, which establish the behaviour of oscilating systems.

Inferior cutting frequence is the limit which separates the sinusoidal oscilations by the exponential oscilations, specific to the systems which oscilate with frequences lower then those. Its expression is:

$$\omega_{\min}^2 = \frac{g}{l} = \omega_0^2 \qquad (3)$$

Superior cutting frequence is the limit which separates, at the superior part, the sinusoidal oscilations from the exponential oscilations. Its expression is:

$$\omega_{\max}^2 = \left(\frac{g}{l}\right) + \left(\frac{4k}{M}\right) \qquad (4)$$

Phase velocity exppresses the fact that the points displaced at the distance *z*, one from each other, arrive to oscillate in phase after a time *t*, which is necessary for the wave to travel the distance between them. Depending on the system which oscilates, this has various expressions:

$$v_\varphi = \frac{\omega}{k} \quad \text{(m/s)} \qquad (5)$$

$$v_\varphi = \lambda \cdot v \quad \text{(m/s)} \qquad (6)$$

$$v_\varphi = \frac{\lambda}{T} \quad \text{(m/s)} \qquad (7)$$

$$v_\varphi = \sqrt{\frac{T_0}{\rho_0}} \quad \text{(m/s)} \qquad (8)$$

$$v_\varphi = \sqrt{\frac{k_L L}{\rho_0}} \quad \text{(m/s)} \qquad (9)$$

, where –λ- wave-lenght (m);
-υ-the frequence (Hz);
-T- the period (s);
-T_0-the stress from continous string (N/m^2);
-k_L-spring elasticity constant (n/m);
-ρ_0-liniar mass density (kg/m).

Environment impedance expresses its reaction force, when it is excited by an external force which in fact is an inertial force.

The proportionality constant of this force is called impedance and its expression is:

$$F_{react} = -Z \cdot \left(\frac{\partial \psi}{\partial t}\right) = -T_0\left(\frac{\partial \psi}{\partial z}\right) \quad \text{(N)} \qquad (10)$$

$$Z = \frac{T_0}{v_\varphi} \quad \text{(N/(m/s))} \qquad (11)$$

Transmitter delivery power is connected to the fact that the amortization force absorbs energy and radiates it along transmittive environment towards a receiver displaced at the other end (energy which is not losed).

The power, which is radiated by a transmitter, is equal with the composition between the force and the velocity which are exercised over the transmittive environment in the contact point (z=0). It has the expressions:

$$P(t) = -F_{react} \cdot \left(\frac{\partial \psi}{\partial t}\right) = Z\left(\frac{\partial \psi}{\partial t}\right)^2 \quad \text{(W)} \qquad (12)$$

or, considerring the second equality from (10) expression:

$$P(t) = T_0\left(\frac{\partial \psi}{\partial z}\right) \cdot \left(\frac{\partial \psi}{\partial t}\right) \quad \text{(W)} \qquad (13)$$

The relation between the temporal derivative and the spatial derivative of wave function is:

$$\left(\frac{\partial \psi}{\partial z}\right) = -\left(\frac{l}{v_\varphi}\right) \cdot \left(\frac{\partial \psi}{\partial t}\right) \qquad (14)$$

Replacing (14) in (13) is obtained the following relation for power:

$$P(t) = \left(\frac{l}{Z}\right) \cdot \left[-T_0\left(\frac{\partial \psi}{\partial z}\right)\right]^2 \qquad (15)$$

2.1 Characteristic measures for progressive waves from pressure pipes

The characteristic measures determination is made in analogy with sound waves in air.

Phase velocity is determined according to the theoretical model, which was used by Newton to determine the phase velocity of the sound waves in air.

We start from the phase velocity expression (1.9).

The “elasticity constant” of medium can be determined considerring that the force which acts the transmitter, in a liquid under pressure, has the expression:

$$dF = A \cdot dp = A \cdot \left(\frac{dp}{dV}\right) \cdot AdL = A^2 \cdot \left(\frac{dp}{dV}\right) \cdot dL \quad (16)$$

, where: -A-the contact surface area between the transmitter and medium;

-p-the induced suprapressure in the wave medium;

-V-the liquid volume subject to the suprapressure;

-L-the volum element length.

Comparing the (16) expression with the elastic force expression ($df=-k_L dL$), it is observed that the expression of the "elasticity constant" for liquid becomes:

$$k_L = -A^2 \left(\frac{dp}{dV}\right)^2 \quad (17)$$

Replacing the expression (17), in (16), and the liniar mass density expression in function the density ($\rho_0= \rho \cdot A$), for the phase velocity results:

$$v_\varphi^2 = -\left(\frac{V}{\rho}\right) \cdot \left(\frac{dp}{dV}\right) \quad (18)$$

The pressure variation expression in function the volume variation is made from the liquids' state equation.

$$\varepsilon = \left(\frac{1}{\beta}\right) = -V\left(\frac{dp}{dV}\right) \quad (19)$$

Considerring the (19) relation, the phase velocity expression of pressure waves in the installation's pipes is:

$$v_\varphi = \sqrt{\frac{\varepsilon}{\rho}} = \sqrt{\frac{1}{\beta \cdot \rho}} \quad (20)$$

The impedance, considerring the (20),has the following expression:

$$Z = \sqrt{\varepsilon \cdot \rho} = \sqrt{\frac{\rho}{\beta}} \quad (21)$$

The power, on a semiperiod, using (12) expression, is:

$$< P >= Z \cdot A^2 \cdot \omega^2 = A^2 \cdot \omega^2 \cdot \sqrt{\varepsilon \cdot \rho} \quad (22)$$

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3 Problem Solution

3.1 Pulsatory jets

The pulsatory jet will be formed by a special device, which is attached to the work head of the high pressure equipment.

Regarding the oscilatory movements, it means that through the elastic environment (the elastic string with mass continous distribution, more serial springs, a fluid) simultaneously are transmitted two oscilatory movements (waves), which interact: one is representing the continous jet and, the other, the exterior movement, which is introduced by device.

The purpose is to obtain a resultant wave which is energetical superior to each of the two components. This can be done when the two movements have their frequences as close as possible regarding the size order (equal at limit), and their amplitudes as well.

The pulsatory jets can be modeled, theoreticaly, like being the result of two harmonic, progressive waves interaction, in an elastic environment, excited by a generator with a complex motion, which is the vectorial sum of the other two. Those waves have as a characteristic the appearance of some beats (amplitudes sum) with high energetical values, similar to the resonance phenomenon from the maintained stationary waves case.

The motion characteristics are:

- compressing progressive and liniar waves;
- waves which are modeled in amplitude;
- the two waves frequences have proximate values (equal at limit);
- the two waves amplitudes must be as close as possible.

3.2 High pressure pulsatory jets equations

It will be studied the case of a wave which is modeled in amplitude by laping two sinusoidal progressive waves.

Being given two sinusoidal progressive waves, with their pulsations -ω_1, ω_2- and amplitudes $-A_1$, A_2, the resultant progressive wave has the expression:

$$\psi(z,t) = A_1 \cos(\omega_1 t - k_1 z) + A_2 \cos(\omega_2 t - k_2 z) \quad (23)$$

For obtaining an oscilation almost harmonical and moduled in amplitude, is necessary that the two amplitudes to be equal ($A_1=A_2=A$).

There are introduced the following measures:

- the modulation amplitude, which is a time and position function:

$$A_{mod}(z,t) = 2A\cos(\omega_{mod} t - k_{mod} z) \quad (24)$$

- the modulation pulsation:

$$\omega_{mod} = \frac{\omega_1 - \omega_2}{2} \quad (25)$$

- the medium pulsation:

$$\omega_m = \frac{\omega_1 + \omega_2}{2} \quad (26)$$

- the modulation wave number:

$$k_{mod} = \frac{k_1 - k_2}{2} \quad (27)$$

- the medium wave number:

$$k_m = \frac{k_1 + k_2}{2} \quad (28)$$

Utilising the realtions (24, …, 27) the modulated wave function relation (23) can be writen this way:

$$\psi(z,t) = A_{mod}(z,t) \cdot \cos(\omega_m t - k_m z) \quad (29)$$

(24) and (28) describe the aproximatively sinusoidal progressive waves modelated in amplitude. It is observed that the modulation pulsation ω_{mod} has values much lower than the medium.pulsation ω_m, this being a direct consequence of the fact that the two amplitudes have close values.

The resulted wave measures are similar to the singular progressive wave, with specific differences. We have these measures:

Modulation velocity is the velocity which a top of an amplitude is moving. Its realtion is:

$$v_{mod} = \frac{\omega_{mod}}{k_{mod}} = \frac{\omega_1 - \omega_2}{k_1 - k_2} \quad (30)$$

Group velocity is the real movement velocity of the signal. It is resulted from Taylor sery dezvoltation for the (29) relation. Taken into account that, for these waves, the dependence between pulsation and wave number is not liniar (a dispersion relation which differs from the medium dispersion relations for the component waves), we have the relation:

$$v_g = \frac{d\omega}{dk} \quad (31)$$

,where: ω=ω(k) – dispersion law for the modulated progressive wave.

Power transmitted by the wave

The power expression, mediated for a semiperiod, can be written, similary with the one of a sinusoidal progressive wave in which it is introduced, for pulsation the medium value and for amplitude its value given by (4.2) mediated in time. It is considered that the environment impedance (Z) is a measure independent from its exciting module, utilising, for this, (2.6). We have the relation:

$$< P >=< A_{mod}^2 > \omega_{mod}^2 \sqrt{\varepsilon \rho_0} = 4 A^2 \omega_m^2 \sqrt{\varepsilon \rho_0} \quad (32)$$

Comparation with continous jets

Comparing (31) with the power radiated from a transversal progresssive wave (32), it is observed that the composed wave has beatings that can reach values four times higher than the singular wave case. The transmission frequency of the beatings is given by the frequency od modulation ω_{mod}, as the semidiference between the two components frequency.

4 Conclusion

The pulsatory jets can be modelled, theoreticly, being the result of the interaction of two progressive waves, harmonics, in an elastical environment excited by a generator with a complex movement which is the vectorial sum of the other two. The modulated wave function relation is:

$$\psi(z,t) = A_{mod}(z,t) \cdot \cos(\omega_m t - k_m z) \quad (33)$$

,where the modulating amplitude has the expression

$$A_{mod}(z,t) = 2A\cos(\omega_{mod} t - k_{mod} z) \quad (34)$$

The specific measures of the modulated wave are the followings:

- group velocity (real signal propagation velocity): $v_g = \frac{d\omega}{dk}$, where ω=ω(k) – dispersion law for the modulated progressive wave.
- power, mediated over a semiperiod, is $< P >=< A_{mod}^2 > \omega_{mod}^2 \sqrt{\varepsilon \rho_0} = 4 A^2 \omega_m^2 \sqrt{\varepsilon \rho_0}$

Refering to the nature of the pulsatory jets, it can be said that they are the result of some exterior harmonical forces over a system under pressure, that they are complex waves, modulated and in amplitude, obtained by merging two waves: the one coresponding to the pressure generator (pump) –the delivering wave- and the one given by the external exciting force –modulating wave.

Comparing the power expression

of the pulsatory jet with the one of continous jet, results that the pulsatory jet has components which are four times bigger.

To obtain some results superior to the carrier wave (continous jet) the exciting force should satisfy the following requires:

- the two waves frequence should have the same magnitude order and the amplitudes must have equal sizes;
- the device design must ensure a pulsatory jet, not interrupt, it requireing the existence of an accumulation chamber for the debit, which is provided by pomp during the pressure/debit pulse generateing;
- then, the cumulate debit must be delivered to the system;
- the comppresion system course length must be as small as possible.

In this conditions, water jets, with the energetical efficience bigger then the one of continous jets, are obtained and those allow accomplishing of some qualitative superior and with a bigger productivity cuttings, when a low continous pressure is maintained, at 200MPa. The jet power increasing, introducing the pulsatory component, is between 20...80%.

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