A New Space-Repetition Code Based on One Bit Feedback Compared to Alamouti Space-Time Code

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Abstract: - Alamouti space-time code for two transmit antennas is most attractive due to simplicity of its encoding and decoding algorithms. However, it does not provide any coding gain. This paper proposes a new space-repetition code for two transmit antennas and one receive antenna based on one bit feedback. This can provide a relatively significant coding gain while it's encoding and decoding complexity is less than Alamouti code. Although the orthogonal design theory does not satisfy, the new code achieves maximum transmit diversity. Even if the reverse channel is not reliable, the performance of the new code still remains better than that of the Alamouti code. All the above advantages are achieved while preserving spectral efficiency with respect to Alamouti code. The only price paid for these advantages is one bit feedback from the receiver to the transmitter which can be easily returned with the ACK signal to the transmitter. Energy computations are also presented and it is shown that the new code can save energy compared to Alamouti code.

Key-Words: - MIMO channels, space-time coding, antenna diversity, multipath propagation, channel modeling, channel estimation.

1 Introduction

Multiple-Input Multiple-Output (MIMO) systems can considerably increase wireless communication capacity by using multiple transmit and receive antennas. MIMO channel capacity grows approximately linearly with the minimum number of transmit and receive antennas [1],[2]. An effective and practical way to approach the capacity of MIMO channels is to employ space-time coding. Space-time coding is a coding technique designed for multiple antenna transmission. There are several space-time coding techniques that introduce correlation between signals transmitted from various antennas at various time periods [3].

Space-time block codes are attractive due to simplicity of their encoding and decoding algorithms [4]. The codeword matrix of these codes is constructed by orthogonal design theory [5] so that each two rows of the codeword matrix are orthogonal. The orthogonal property makes the code achieve maximum transmit diversity. It also simplifies the maximum likelihood receiver by separately decoding sequences transmitted from antennas and employing only linear operations on the received signals.

Alamouti code for two transmit antennas is the only space-time block code that is full rate for complex modulations. For more than two transmit antennas and complex modulations, there is no full rate space-time block code. However, Alamouti code does not provide any coding gain. That is, its performance is the same as that of the uncoded system with the same diversity gain.

This paper proposes a new space-repetition code for two transmit antennas and one receive antenna based on one bit feedback that provides a relatively significant coding gain while it's encoding and decoding complexity is less than Alamouti code. Although the orthogonal design theory does not satisfy, the new code achieves maximum transmit diversity. All the above advantages are achieved while preserving spectral efficiency with respect to Alamouti code. The only price paid for these advantages is one bit feedback from the receiver to the transmitter. The feedback bit provides partial channel state information (CSI) at the transmitter.

This paper is organized as follows. Section 2 gives an overview of space-time coding and well known Alamouti code. Then it presents basic ideas of the new code. Section 3 confirms theory results by simulations. Energy computations are also presented in this section and it is shown that the new code can save energy compared to Alamouti code. It is shown that even with unreliable reverse channel, the performance of the new code still remains better than that of the Alamouti code. Effect of imperfect channel estimation on performance is also considered. Section 4 concludes with a brief summary of results.

2 New Code Basics

Space-time codes are defined by a codeword matrix. Each of the rows determines sequences transmitted from a specific antenna at various time periods and each of the columns determines sequences transmitted from various antennas at a specific time period. It can be denoted as:

$$\mathbf{X} = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_L^1 \\ x_1^2 & x_2^2 & \dots & x_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{n_T} & x_2^{n_T} & \dots & x_L^{n_T} \end{bmatrix} \tag{1}$$

Where x_t^i is the modulated symbol transmitted from antenna i at time period t, L is the frame length and n_T is the number of transmit antennas.

For a space-time code, conditional pairwise error probability is given by [3],[6]:

$$P(\mathbf{X},\hat{\mathbf{X}} \mid \mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2N_0}\left\|\mathbf{H}(\mathbf{X}-\hat{\mathbf{X}})\right\|^2}\right) \tag{2}$$

where E_s is the symbol energy, N_0 is the noise power spectral density, $\mathbf{H}$ is the $n_R \times n_T$ channel matrix, n_R is the number of receive antennas, $\mathbf{X}$ and $\hat{\mathbf{X}}$ are two $n_T \times L$ codeword matrices and $\left\|\mathbf{H}(\mathbf{X}-\hat{\mathbf{X}})\right\|^2$ is defined as:

$$\left\|\mathbf{H}(\mathbf{X}-\hat{\mathbf{X}})\right\|^2 = \sum_{t=1}^{L}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T} h_{j,i}(x_t^i - \hat{x}_t^i)\right|^2 \tag{3}$$

where $h_{j,i}$ is the channel coefficient between transmit antenna i and receive antenna j.

2.1 A review of Alamouti space-time code

Alamouti space-time code for two transmit antennas is the most famous space-time block code. Its codeword matrix is defined as:

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \tag{4}$$

where x_1 and x_2 are two symbols modulated with some 2^m-level real or complex modulation technique.

Using (2), conditional pairwise error probability of Alamouti code for one receive antenna can be computed as:

$$P(\mathbf{X},\hat{\mathbf{X}} \mid \mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2N_0}(|h_1|^2+|h_2|^2)(|d_1|^2+|d_2|^2)}\right) \tag{5}$$

where $d_1 = x_1 - \hat{x}_1$ and $d_2 = x_2 - \hat{x}_2$, h_1 and h_2 are channel coefficients between transmit antennas 1 and 2 and the one receive antenna, respectively.

2.2 New space-repetition code

Consider a space-repetition code for two transmit antennas with the codeword matrix defined by:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ ax_1 & ax_2 \end{bmatrix} \tag{6}$$

where a is a real number. a^2 is chosen equal to 1 so that the energy transmitted from each transmit antenna at a time period be the same as the energy of x_1 or x_2. Hence, the transmitter transmits either:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ x_1 & x_2 \end{bmatrix} \tag{7.a}$$

or:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ -x_1 & -x_2 \end{bmatrix} \tag{7.b}$$

Using (2), pairwise error probability of this code, for one receive antenna is obtained as:

$$P(\mathbf{X},\hat{\mathbf{X}} \mid \mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2N_0}(|h_1|^2+|h_2|^2+2a\,\mathrm{Re}\{h_1h_2^*\})(|d_1|^2+|d_2|^2)}\right) \tag{8}$$

It is clear that if the term $2a\,\mathrm{Re}\{h_1h_2^*\}$ always remains positive, then the argument of $Q(.)$ for the new code will always be more than the argument of $Q(.)$ in (5) for Alamouti code. Therefore, its error probability will always be less than that of the Alamouti code due to descending property of $Q(.)$. Hence, if $\mathrm{Re}\{h_1h_2^*\}$ is positive, a is chosen equal to 1 and if negative, a is chosen equal to -1. Therefore the receiver should compute the term $\mathrm{Re}\{h_1h_2^*\}$ and feedback one bit determining it's sign to the transmitter. If $\mathrm{Re}\{h_1h_2^*\}$ is positive the transmitter transmits (7.a) and if negative, it transmits (7.b). It is clear that fading should be slow enough so that the channel coefficients remain constant at least over one block of data.

From the special form of the codeword matrix, it is clear that we can transmit the codeword matrix:

$$\mathbf{X} = \begin{bmatrix} x \\ ax \end{bmatrix} \tag{9}$$

for every symbol x instead of (6). So the transmitter transmits either:

$$\mathbf{X} = \begin{bmatrix} x \\ x \end{bmatrix} \quad (10.a)$$

or

$$\mathbf{X} = \begin{bmatrix} x \\ -x \end{bmatrix} \quad (10.b)$$

In this case the channel coefficients should be constant over at least two blocks.

It should be noted that since one symbol of *m* bits is transmitted during one time period, spectral efficiency of the new code remains equal to *m*, which is the same as that of the Alamouti code.

3 Simulation Results

This section compares the new code to Alamouti space-time code by simulations. In the simulations, complex baseband linear system model described in discrete time is considered. Individual channels between transmit and receive antennas are modeled by independent Rayleigh, flat and slow fading processes. It is assumed that the channel coefficients change from frame to frame and the receiver can estimate perfect channel state information.

Fig.1 plots FER in terms of SNR for the new code, each of the component codes (10.a) and (10.b), as well as Alamouti code using complex QPSK modulation. Fig.2 shows the same plots considering real BPSK modulation. The frame length is assumed to be 320 symbols. Because channel coefficients vary from frame to frame, the feedback bit must be computed at every frame.

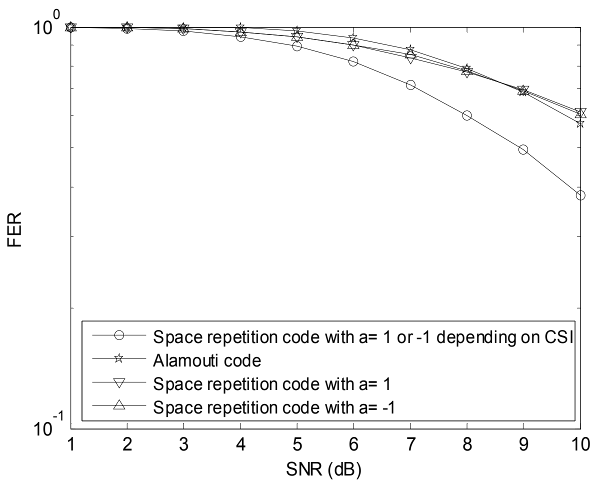

Fig.1 Performance comparison of the new code and Alamouti code for QPSK modulation and frame length of 320 symbols.

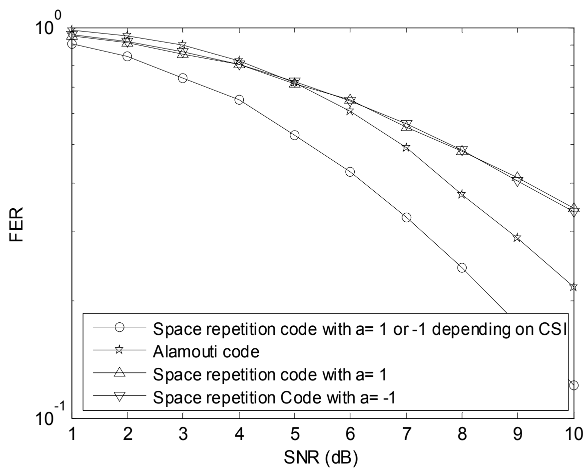

Fig.2 Performance comparison of the new code and Alamouti code for BPSK modulation and frame length of 320 symbols.

From the above simulations, it can be seen that the new code provides a coding gain of about 1.5dB at a FER of 0.4. While each of the component codes (10.a) or (10.b) achieves transmit diversity gain of 1, the new code achieves full transmit diversity gain of 2.

If the complexity of encoding and decoding algorithm is measured as the time elapsed in simulations by MATLAB, it is seen that encoding and decoding complexity of the new code decreases by an amount of about 20% compared to Alamouti code because of the simple form of the codeword matrix.

It should be noted that the above simulations give a comparison of the new space-repetition code given that partial CSI is returned to the transmitter and Alamouti code which its performance is the same as that of the uncoded system when full CSI is available at the transmitter (transmit beamforming).

3.1 Energy discussion

It is clear that by increasing the symbol energy in Alamouti code, error probability can be decreased until it reaches the error probability of the new code. On the other hand, for correct operation of the new code, the transmitter must receive the feedback bit with a very low error probability. This requires that the feedback bit is sent with high energy. Now one may ask whether the energy consumed for accurately estimating the feedback bit be more that the extra energy for achieving the same error probability in Alamouti code and the new code. This question is answered by following computations.

From Figs.1 and 2, it is seen that for achieving the same error rate in Alamouti code and new code, Alamouti code needs about 1.5dB more SNR compared to the new code. So symbol energy should be increased from 1 unit (assumed for modulated symbol energy) to 1.4 units according to relationship:

$$\mathrm{SNR} = \frac{2E_s n_T}{N_0} \tag{11}$$

(where the factor 2 is inserted for expressing symbol energy in baseband form and n_T is 2 in this case). Since two symbols are transmitted at every time period, extra energy in each time period is 2×0.4=0.8 units.

If the forward channel is used as the reverse channel, the reverse channel becomes a 1×2 channel. Fig.3 plots BER in terms of SNR for a 1×2 uncoded system. For this channel, if BER of 10^{-4} is acceptable for estimating the feedback bit, it requires a SNR equal to about 17dB according to this figure.

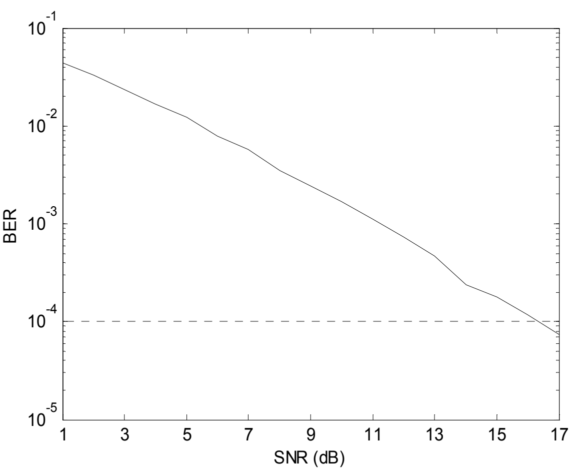

Fig.3 BER in terms of SNR for uncoded system with one transmit antenna and two receive antennas.

The above energy discussion does not apply to high SNRs, since at high SNRs, the channel noise is low and the receiver does not need to transmit the feedback bit with high energy. Therefore, if for example SNR is assumed to be 5dB for the new code (which is equivalent to a noise power spectral density of about 1.26 units according to (11) where n_T is 2 and E_s is 1 in this case), the feedback bit transmit energy should be about 32 units according to (11) where E_s is replaced by bit energy, SNR is 17dB, N_0 is 1.26 units and n_T is 1 in this case. Thus up to about 32/0.8=40 symbol times, increasing symbol energy in Alamouti code is preferred. After 40 symbol times, the new code will save energy of about 0.8 units in every symbol time.

In digital communication system design, symbol period is typically chosen so that the channel coherence time is between 10 to 100 symbol periods. For example if the channel coherence time is 100 symbol times, then using the new code will save energy of about 17dB. Clearly, with slower fading rate, more energy is saved.

By employing more equipment such as increasing the number of receive antennas in the reverse channel, more energy will be saved. For example if a 1×4 channel is used as the reverse channel instead of a 1×2 channel, the new code will save energy after 4 symbol times (instead of 40 symbol times).

If a SISO channel is used as the reverse channel, the new code will save energy after 1978 symbol times.

The feedback bit can be easily transmitted with the ACK signal from the receiver to the transmitter. Since it may not be possible to transmit the feedback bit with high energy due to, for example, interfering with adjacent stations, it can be transmitted repeatedly instead of increasing the energy. In this case the above energy discussion and slow fading constraint do not apply.

3.2 Effect of non ideal reverse channel

Simulations of Figs.1 and 2 are based on the assumption that the reverse channel is ideal. That is, the feedback bit is estimated accurately at the transmitter. However, in practice the reverse channel is not ideal. Therefore one may ask whether estimation error of the feedback bit at the transmitter may deteriorate the performance of the new code so that it becomes even worse than that of the Alamouti code.

In order to introduce the effect of estimation error of the feedback bit in the above simulations, it is assumed that the transmitter estimates the feedback bit with an error probability of about 0.04. For example if the forward channel is used as the reverse channel, SNR in the reverse channel can be considered to be about 1dB according to Fig.3. Based on this assumption, the performance of the new code compared to Alamouti code is shown in Fig.4.

As seen in Fig.4, despite the feedback bit estimation error of about 0.04, the performance of the new code still remains better than that of the Alamouti code. For example at a FER of 0.4, the performance of the new code deteriorates about 0.3dB compared to the ideal reverse channel and it is still about 1.2dB better than Alamouti code. At a FER of 0.2, the performance of the new code deteriorates about 0.6dB compared to the ideal reverse channel and it is still about 0.9dB better than Alamouti code.

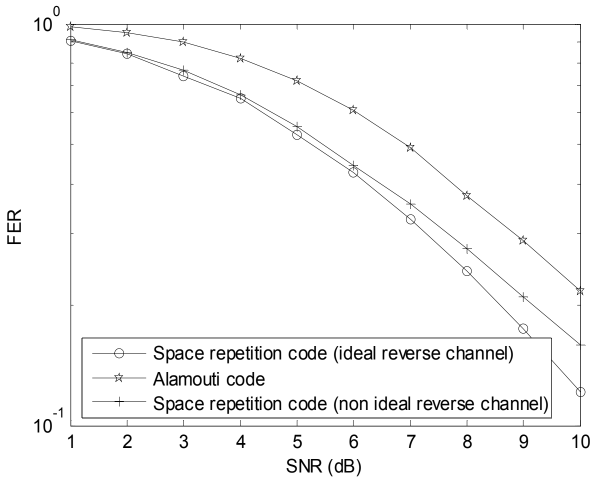

Fig.4 Performance comparison of the new code assuming non-ideal reverse channel and Alamouti code for BPSK modulation and frame length of 320 symbols.

3.3 Effect of imperfect channel estimation

So far, we have assumed that the receiver can estimate perfect channel state information. Now we consider imperfect channel state information. The channel fading coefficients are estimated by inserting pilot sequences at the beginning of each transmitted frame. It is assumed that the channel is constant over the duration of a frame and independent between the frames. With n_T transmit antennas we need to have n_T orthogonal pilot sequences $\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_{\mathbf{n_T}}$. The receiver estimates the channel fading coefficients $h_{j,i}$ by using the observed sequences $\mathbf{r}^j$. The minimum mean square error estimate of $h_{j,i}$ is given by [7] :

$$\widetilde{h}_{j,i} = \frac{\mathbf{r}^j . \mathbf{P}_i}{\left\| \mathbf{P}_i \right\|^2} \ , \quad i = 1,2,\ldots,n_T \qquad (12)$$

Fig.5 plots FER in terms of SNR of the new code and Alamouti code for real BPSK modulation and imperfect channel state information. It is assumed that the channel coefficients are constant over a frame of 320 symbols. The pilot sequences inserted in each frame have a length of 20 symbols.

The simulation results show that due to imperfect channel estimation, both codes have degradation in performance compared to the case of perfect channel estimation. The new code performance is degraded slightly more than Alamouti code. However its performance still remains better than that of the Alamouti code. For example at FER of 0.3, the performance of the new code is still about 1.4dB better than Alamouti code.

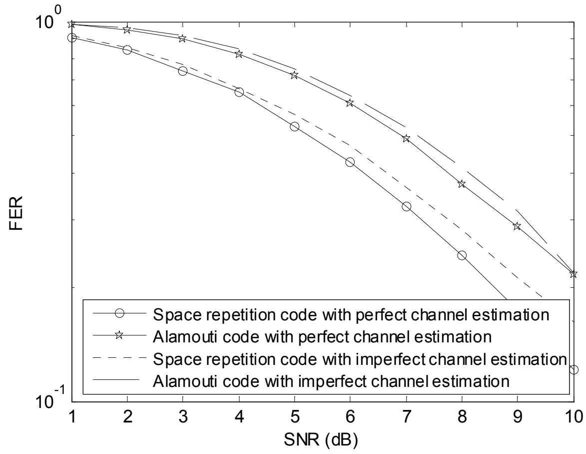

Fig.5 Performance comparison of the new code and Alamouti code for BPSK modulation and frame length of 320 symbols assuming imperfect channel estimation.

4 Conclusion

The new space-repetition code based on one-bit feedback presented in this paper was shown to be better than Alamouti space-time code in both performance and complexity even if the feedback is unreliable. The feedback bit can be easily returned with the ACK signal to the transmitter. All these advantages are achieved while preserving spectral efficiency compared to Alamouti code.

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