Modeling of the Temperature Dependence of the Surface Impedance in High-Tc Superconducting Microstrip Lines

M. I. Hussein[1], M. Benkraouda[2], H. Ghamlouche[2], and T. Badernah[1]
Elec. Engineering Department(1) and Physics Department(2)
UAE University
P.O.Box 17555, Al Ain
UAE

Abstract: - The surface impedance of high-Tc superconducting microstrip lines and its temperature dependence is studied using the full-wave Finite Difference Time Domain (FDTD) technique. Maxwell's equations are modified to incorporate the two-fluid London model that describes superconductivity. The supercurrent density and the average current along the microstrip line are calculated as function of temperature and the width of the microstrip line. In addition, the surface resistance and reactance as function of temperature have been calculated and compared to published results.

Key-Words: - Superconductors, Microstrip line, Surface impedance, FDTD

1 Introduction

High Tc superconducting (HTS) microstrip transmission lines are extensively used in microwave devices and applications [1]. This is due to the fact that the surface resistance of superconducting materials is at least one order of magnitude smaller than that of conventional conductors at an operating temperature of 77K.
The BCS, the London, and the Ginzburg–Landau (GL) theories [1, 2] are among the well known basic theories that describe conventional superconductivity. However, phenomenological models are needed to describe the properties of HTS. In particular, the two-fluid model states that the current in the HTS is composed of normal and superconducting components. The superconducting current is governed by the London equations [2], which is incorporated into Maxwell's equations.
These equations have been solved using several different numerical techniques. The most notable in this respect is the finite difference time domain (FDTD) method [3-5]. In reference [4], a full-wave FDTD is used to study nonlinearity in HTS planar structures, where Ginzburg-Landau equations are solved numerically. Even though this may be needed to investigate nonlinearity in HTS microstrips, it remains that it involves a larger number of equations to be solved simultaneously and heavy computations.
Maxwell's equations, in conjunction with the London equation, are numerically solved using a FDTD code for a 3-D microstrip line to calculate the current density distribution and the surface impedance as function of temperature and the width of the microstrip line. In this way our simulations are carried out using a PC rather than workstation or supercomputer machines.

2 Theory and Problem Formulation

According to the two-fluid model the total current density consists of two components; the superconducting current density $\vec{J}_s$

:

$$\vec{J} = \vec{J}_s + \vec{J}_n \tag{1}$$

where

$$\begin{aligned} \vec{J}_s &= q n_s \vec{v}_s \\ \vec{J}_n &= q n_n \vec{v}_n = \sigma_n \vec{E} \end{aligned} \tag{2}$$

$$\sigma_n(T) = \sigma_n(T_c)\left\{\left(\frac{T}{T_c}\right)^{\gamma-1} + \alpha\left(1-\left(\frac{T}{T_c}\right)^{\gamma}\right)\right\} \tag{3}$$

where, $\sigma_n(T)$ is the normal conductivity of the superconductor as function of temperature [6], n_n, v_n, n_s, and v_s are the charge density and velocity for the normal and superconducting currents, respectively. The simplest form of n_s and n_n which are related to the total electron density (n_0) and temperature (*T*), are given by the following relations [7]:

$$n_s(T) = n_0 \begin{cases} 0, \ for T \geq T_c \\ 1-(T/T_c)^{\gamma}, \ for T \leq T_c \end{cases} \quad (4)$$

$$n_n(T) = n_0 \begin{cases} 1, \ for T \geq T_c \\ (T/T_c)^{\gamma}, \ for T \leq T_c \end{cases} \quad (5)$$

where T_c is the critical temperature and γ (with values $1.3-2.1$) is an exponent. $n_s(T)$ is related to London penetration depth $\lambda_L(T)$ as follows [6]:

$$\frac{1}{\lambda_L^2(T)} = \frac{q^2 n_s(T)\mu_0}{m_s} \quad (6)$$

The temperature dependence of $\lambda_L(T)$ is given by:

$$\lambda_L(T) = \frac{\lambda(0)}{\left[1-(\frac{T}{T_c})^{\gamma}\right]^{1/2}} \quad (7)$$

where, $\lambda(0)$ is the penetration depth for $T = 0K$.

For HTS, Maxwell's equations are modified as follows:

$$\nabla \mathrm{x} \vec{H} = \varepsilon \frac{d\vec{E}}{dt} + \vec{J}_S + \vec{J}_n$$
$$\nabla \mathrm{x} \vec{E} = -\mu_0 \frac{d\vec{H}}{dt} \quad (8)$$

On the other hand, $\vec{J}_S$ is related to the electric field via the equation:

$$\frac{d\vec{J}_s}{dt} = \frac{1}{\mu_0 \lambda_L^2} \vec{E}. \quad (9)$$

Equations (7) and (8) are discretized and solved simultaneously using the FDTD method. To resolve the problem of a very thin superconductor thickness without using excessive computer memory and space, a variable mesh with second order accuracy is applied. The geometry of the microstrip line under consideration is shown in Fig.1. The (YBCO) HTS microstrip line has a 50 Ω impedance with a strip width of 7.5 μm, and a thickness of 1 μm.

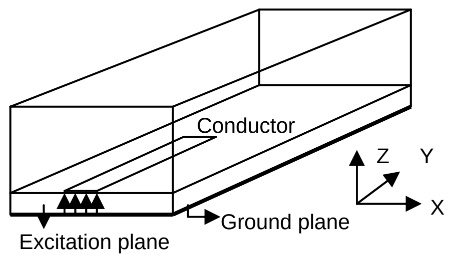

Fig.1 Microstrip geometry and computational domain.

Penetration depth and normal conductivity are equal to 0.2 μm and 1.0x10^6 S/m , respectively, at 77 K. The substrate thickness is 10 μm and ε_r = 13. The dimension of the computational domain is (64x100x30). The time step size is chosen based on the smallest mesh step size following the Courant stability condition. For simplicity, the ground plane is chosen to be a perfect conductor. The computational domain is terminated by Mur absorbing boundary conditions [8].

3 Results and Discussion

Temperature is a critical factor that affects the superconducting properties of the microwave devices. For this reason, the temperature dependence of the supercurrent density and the surface impedance of the microstrip line is investigated.

3.1 Supercurrent Density

The normalized supercurrent density (J_{SY}) distribution as function of width at different temperatures is shown in Fig.2. It is clear from the figure that for all temperatures, the supercurrent density along the microstrip is constant throughout the width except at the edges where a sharp increase occurs. Also in this figure, the supercurrent density decreases as the temperature increases. To further investigate the temperature dependence, the supercurrent density is averaged over the width for each temperature, as shown in Fig.3. In this figure, the average supercurrent density decreases slowly with temperature well below T_c followed by a sharp decrease in the vicinity of T_c. This is due to the fact that the charge density n_s decreases as temperature increases according to equation (3).

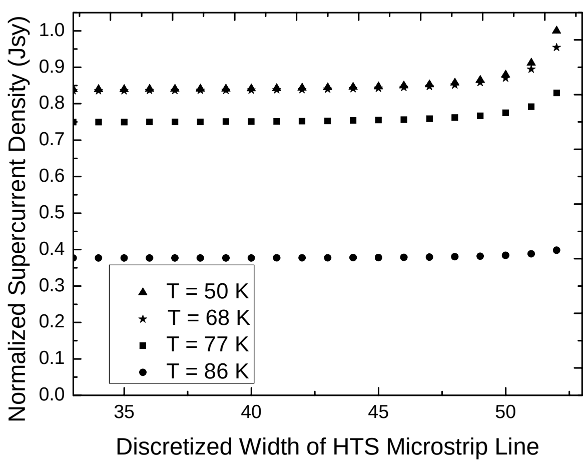

Fig.2 Normalized supercurrent density as function of HTS width at different temperatures.

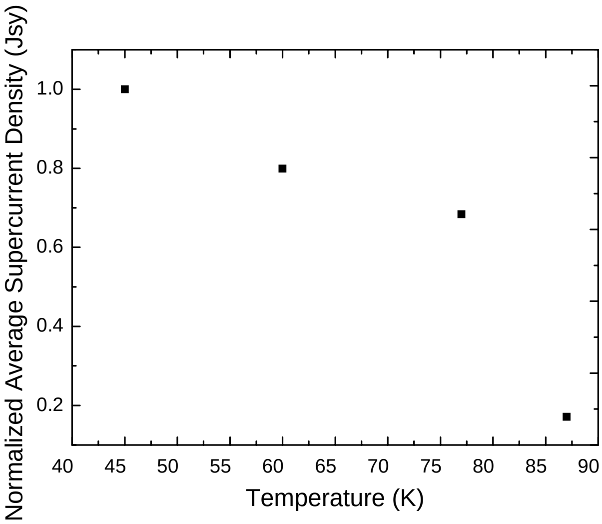

Fig.3 Normalized average supercurrent density with temperature.

3.2 Surface Impedance

The calculated surface impedance Z_s is defined as the ratio of the tangential electric field E_t and magnetic field H_t at the surface of the microstrip line [9]:

$$Z_s = \frac{E_t}{H_t}, \tag{10}$$

where $E_t = E_y$ and $H_t = H_x$ for the geometry given in Fig.1.

The surface impedance of the HTS microstrip line is very sensitive to temperature variations near T_C as can be seen from Fig.4. However, at temperatures much below T_C, the surface impedance is much less sensitive. The increase in temperature induces an increase in the normal carriers (n_n) as given by equation (5). This in turn induces oscillations of theses carriers under the influence of the electromagnetic field. This oscillation or motion of the unpaired carriers will cause power dissipation which can be characterized by surface resistance.
Fig.4 shows also a good agreement of our results with those reported in [7]. The slight discrepancy between our results and those reported in [7] can be attributed to the fact that we used the basic definition of the surface impedance as given by equation (10) without any approximations.

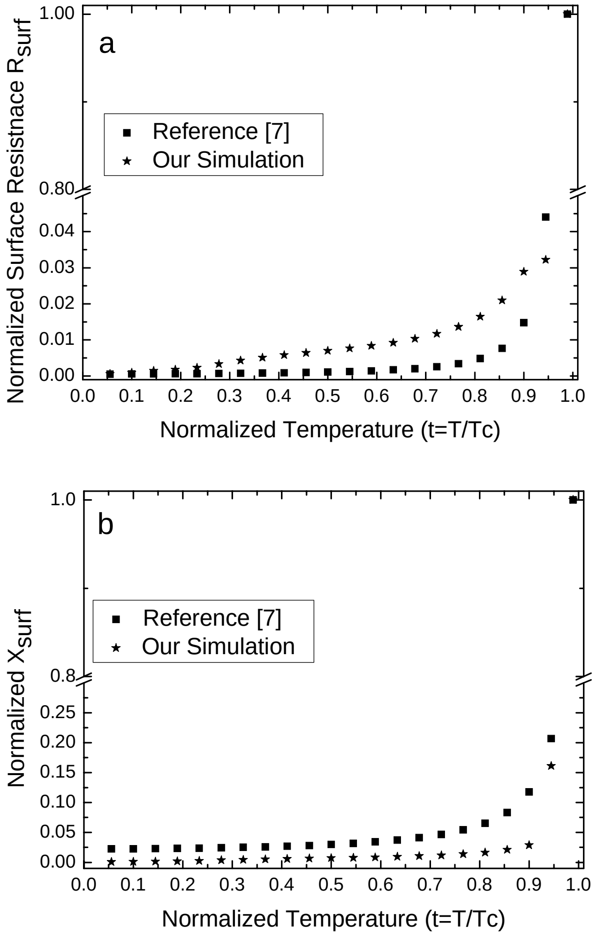

Fig.4 Temperature dependence of (a) real part (R_{surf}) and (b) imaginary part (X_{surf}) of the surface impedance for f = 10GHz.

4 Conclusion

In this paper, a theoretical analysis of HTS at high frequencies was discussed, based on the two-fluid model and the London theory in conjunction with Maxwell's equations. The implementation of this model in a FDTD code for a 3-D HTS microstrip line has been presented. Then the temperature dependence of the supercurrent density and the surface impedance of the microstrip line was investigated numerically using this technique. Our results show good agreement with those reported in the literature.

Acknowledgement
The authors wish to acknowledge the UAE University financial support under the IGRASP scholarship program for graduate studies.

References:

[1] T. P. Orlando and K. A. Delin, *Foundations of applied superconductivity,* Reading, MA: Addison-Wesley, 1991.

[2] T. Van Duzer and C. W. Turner, *Principles of superconductive devices and circuits*, Englewood Cliffs, NJ: Prentice-Hall, 1999.

[3] L. H. Lee S. M. Ali, and W. Gregory Lyons, Full-Wave characterization of High-T_c superconducting transmission lines, *IEEE Trans. Microwave Theory & Tech.,* Vol.2, No.2, June 1992, pp. 49-57.

[4] Mohamed A. Megahed, and Samir M. El-Ghazaly, Nonlinear analysis of microwave superconductor devices using full-wave electromagnetic model, *IEEE Trans. Microwave theory Tech*., Vol.43, No.11, November 1995, pp.2590-2599.

[5] Shujun Xiao and Ruediger Valdieck, 3D FDTD simulation of superconductors coplanar waveguides, *IEEE MTT-Digest*, 1995.

[6] I. B. Vendik, O. G. Vendik, A. N. Deleniev, V. V. Kondratiev, M. N. Goubina, and D. V. Kholodniak, Development of CAD tool for a design of microwave planar HTS filters, *IEEE Trans. Microwave Theory & Tech.,* Vol.48, No.7, July 2000, pp. 1247-1255.

[7] Orest G. Vendik, Irnia B. vendik, and Dimitri I. Kaparkov, Emperical model of the microwave properties of HTS, IEEE Trans. Microwave theory Tech., Vol.46. No.5, May 1998, pp. 469-478.

[8] G. Mur, Absorbing boundary conditions for the finite difference approximation of the time domain electromagnetic field equations, *IEEE Trans. Electromag. Compat.* Vol.23, 1981, pp. 377-382.

[9] M.J. Lancaster, *Passive Microwave Device of HTS*, Cambridge University Press, 1997.