High rate LDPC codes using OFDM and Equalisation Techniques for WiMAX Systems

M K Khan[1], R.A. Carrasco[1] and I.J.Wassell[2]
1 School of Electrical, Electronics and Computer Engineering, University of Newcastle-upon-Tyne
Newcastle-upon-Tyne, NE1 7RU, UK

2 Digital Technology Group, Computer Laboratory, University of Cambridge
Cambridge CB2 1PZ, UK

Abstract: - In areas lacking pre-existing physical cable or telephone networks, WiMAX could allow broadband access that has previously been unavailable. However, the physical limitations of wireless medium still present a fundamental technical challenge to reliable communication and throughput. Thus, the future wireless broadband network requires greater capacity, more data throughput, low latency, improvements in coverage etc. This can be achieved by using advance modulation, coding and equalisation techniques to effectively reduce the adverse effects introduced by wireless medium. To help overcome the undesirable effects of the channel, equalisation techniques can be used in conjunction with high rate LDPC codes using QPSK modulation scheme. The equaliser single carrier approach is then replaced with an orthogonal frequency division multiplexing (OFDM) and the performance of these two approaches is evaluated in terms of bit-error rates. The simulation result shows that equalisation with LDPC decoding has a measurable performance gain over LDPC coding with OFDM, but at the cost of increased computational complexity.

Key- Words: - LDPC codes, OFDM, IEEE 802.16 or WiMax, Equalisation, ISI

1. Introduction

The rapid growth in demand for high speed internet access for the majority of UK population has created a challenge for last mile expansion. It is obvious that the low bandwidth dial-up access is holding back the services and applications that can be offered. The main challenge is to increase the bandwidth available to end user so that the services offered can be accessed easily, freely and as quickly as possible. The IEEE 802.16 (WiMax) Broadband Fixed Wireless Access (BFWA) [1] systems offer a solution to the problem of providing inexpensive broadband services from the local exchange to the customer. It can be deployed quickly, covering a large area with low cost compared to cable installations. The main challenge now for wireless broadband lies in providing a comparable quality of service (QoS) for similar cost as competing wired technologies. Many wireless broadband systems are being foreseen for different users with different needs.

However, achieving high data rates in wireless communication environments is limited by multipath fading between transmitter and receiver degrading the system performance. The transmitted data through the Broadband fixed wireless channel is subject to attenuation and distortion by various factors such as foliage, buildings, precipitation, and vehicles etc. Therefore, there is a need to use a model which is parameterised according to the various terrain and environments experienced by BFWA systems. The Stanford University Interim (SUI) channel model [2] has been developed specifically for fixed wireless systems and takes into account a number of parameters for example, Doppler effect, path loss, multipath delay spread and fading characteristics. Consequently it been chosen to model the BFWA channel in this paper.

The IEEE working group on Broadband Wireless Access Standards are developing IEEE 802.16 [3], which provides the standards for broadband wireless system implementation. As a part of specification the use of Orthogonal Frequency Division Multiplexing (OFDM) has been proposed in order to combat frequency selective fading in the BFWA channel. Also, the use of space diversity for capacity improvement using OFDM has been analysed in [4]. Therefore, the high data rate can be achieved in part via OFDM which splits broad channel into multiple narrowband channels, each using different frequencies thus carrying different parts of message simultaneously.

Moreover, there has been a lot of emphasis on joint equalization and decoding of coded systems. The application of the turbo principle [5] to iterative decoding and demodulation has provided significant gains [6]. Such systems are essentially serial concatenation schemes [7] with the ISI channel as the inner code. However, the aim of this paper is to design and analysed the performance of two single input single output SISO BFWA systems. The first system uses a single carrier approach with square root kalman (SRK) in conjunction with some newly generated BIBD-LDPC codes [8]. The second scheme replaces the single carrier approach with one employing OFDM i.e., a SISO OFDM BFWA system. The scheme can easily be extended to MIMO BFWA systems.

This paper is organised as follows. Section 2 presents the overall description of the system. The construction method of LDPC codes is explained in Section 3. In Section 4, we present the discussion of the simulation results of equalised system and un-equalised system. Finally, section 5 concludes the contribution of the paper which follows references and figure captions.

2. System Model

The overall general system diagrams are shown in Figs.1 (a) and (b). In fig 1 (a), single carrier transmission with SRK equalisation has been used and in fig 1 (b), OFDM has been used with the aim of overcoming the ISI introduced by the BFWA channel. In the transmitter the data bits b_k, generated by the source, are encoded by the LDPC encoder into encoded bits c_n. The block interleaver re-orders the encoded bits and the modulator maps them into QPSK symbols s_n. In fig 1(a) this signal (represented by x_n) is modulated onto a single carrier for use in the equalised system, whereas in fig 1(b) the signal is fed into an OFDM modulator. In the initial stage of OFDM modulation, the symbol stream from the mapper is converted from serial to parallel and then an Inverse Fast Fourier Transform IFFT is applied. Finally a cyclic prefix (CP) is added to the signal x_n before transmitting over the SUI-3 multipath channel with three resolvable paths with tap spacings of 500ns and a total delay of 1000ns. The model can be considered as a 3-tap transversal filter with a finite impulse response. While calculating the taps coefficient, Doppler effect specified by SUI channel specification is also taken into account. The transmitted signal is also corrupted by Additive White Gaussian Noise (AWGN). The total power of all paths of the channel is normalized to unity, so that there is no gain provided by the dispersive channel. The channel filter coefficients are calculated and the transmitted sequence from antennae is multiplied by these coefficients.

There are number of SUI channel models specified in [2] depending upon the different terrain conditions. All of them have three resolvable paths with either a Rician or Rayleigh amplitude distributions. The amplitude distribution of the SUI-3 channel model is Ricean for the first path and Rayleigh for the other two paths. The complex values coefficients are expressed as follows:

$$\alpha_n \qquad , \quad n = 0,1,2$$

Hence the received signal r_n can be expressed by following equation

$$r_n = \alpha_0\, x_n + \alpha_1\, x_{n-1} + \alpha_2\, x_{n-2} + n_n \qquad (1)$$

where z_n is the complex additive white Gaussian noise (AWGN).

The receiver aims to recover the original information bits from the received samples corrupted by the channel. For the equalised system, the received samples, r_n, are passed directly to the SRK equaliser to attempt to reduce ISI. After equalisation/demodulation of the received samples, the log-likelihood (LLR) values, $\hat{d}_n$, of each bit are calculated and passed to the block de-interleaver. The decoder applies the message passing algorithm on the de-interleaved bits, $\hat{c}_n$, in an iterative manner to extract estimates of the original information bits, $\hat{b}_k$.

For the OFDM system the received samples r_n are passed directly to the OFDM demodulator. After OFDM demodulation the received symbols are passed to the soft demapper where the LLR values of the received symbol are calculated before being passed to the de-interleaver and LDPC decoder.

3. Combinatorial Construction of LDPC codes

The great deal of research started in the construction, decoding, performance analysis, application and structural study of LDPC codes. There are numerous construction method has been invented and designed. The main idea is to construct parity check matrix **H** of LDPC code, which defines interconnections between check and variable nodes.

Combinatorial mathematics [9] deals with the theory of enumeration, permutation, combination and the arrangement of objects in order to satisfy certain conditions. Informally, one may define a combinatorial design to be a way of selecting subsets from a finite set in such a way that some specified conditions are satisfied. The detailed study of combinatorial mathematics shows that the parity check matrix **H** can be constructed using BIBD having co-valency λ =1. The co-valency condition λ =1 makes the code free of cycle 4 and results in a girth of at least 6. The method presented here is based on the construction of structured LDPC codes known as the balance incomplete block (BIBD) design. Structured LDPC codes based on the BIBDs have an encoding advantage over random codes, particularly cyclic or quasi cyclic LDPC codes.

Over the years many BIBD designs have been constructed and they are represented by mathematical equations so that variable length designs with different parameters can be constructed with ease. In this paper, we have selected the Bose BIBD design method [10], [11] to generate high rate LDPC codes. Bose constructed many classes of BIBD design. The following example shows one of the types of Bose-BIBD design.

Let t be a positive integer that satisfies $12t + 1= p$, where p is a prime number. There exists a Galois Field GF ($12t$+1) with $12t$ +1 elements. Suppose GF ($12t$+1) has a primitive root, x, such that $x^{4t} - 1 = x^{c}$, where c is an odd integer less than $12t$+1. Then there exists a BIBD design with the following parameters:

No of rows, $v = 12t + 1$, Ojects
No of columns, $b = t\,(12t+1) = tv$, Blocks
Column weight, $\gamma = 4$,
Row weight, $\rho = \gamma t$
Co-valency, $\lambda = 1$

The design block is generated using the building blocks $B_i = \{0, x^{2i}, x^{2i+4t}, x^{2i+8t}\}$ for $0 \leq i < t$. From B_i we can form $12t$+1 blocks by adding each element in the Galois Field GF ($12t$+1). The incident matrix of this BIBD is a $(12t + 1) \times t\,(12t+1)$ matrix. It can be written in cyclic form consisting of t, $(12t + 1) \times (12t + 1)$ circulant sub matrices as follows:

$$\mathbf{Q} = [\mathbf{Q_1}, \mathbf{Q_2}, \ldots\ldots\ldots, \mathbf{Q_t}] \qquad (2)$$

The incident matrix of Bose-BIBD consists of a row of t circulants. For $1 \leq z \leq t$ the parity check matrix is

$$\mathbf{H}\,[z] = [\mathbf{Q_1}, \mathbf{Q_2}, \ldots\ldots \mathbf{Q_z}] \qquad (3)$$

The advantage of BIBD scheme is that we can generate very high rate LDPC codes, which can be used to increase the bandwidth efficiency of the overall system.

4. Analysis of Simulation Results

The simulation results presented here use LDPC codes constructed using the BIBD design method. The LDPC codes are free of cycle four and the LDPC decoder utilizes the message passing algorithm to decode the information bits. The maximum number of decoder iterations in all simulations is set to 7, since experiments showed that negligible additional coding gain was achieved when iterations greater than 7 were performed. These codes are tested using QPSK modulation (gray coded) over a channel simulated by the SUI-3 BFWA. The specified antenna correlation coefficient is set to the nominal value of 0.4.

Figure 2 compares the performance of LDPC (2715, 2535) code using equalisation and OFDM transmission scheme with QPSK modulation. The most immediate observation is that without equalisation, the ISI dominates performance and the LDPC coding alone provides negligible gain over uncoded QPSK, in contrast to its very promising performance over the AWGN channel. A small improvement in performance over the uncoded system can be achieved using either equalisation or OFDM, with SRK equalisation slightly outperforming OFDM, but both schemes suffer from an error floor. By including the LDPC code the error floor is ‘broken’ and significant coding gains are achieved over the uncoded equalised and OFDM systems. The results show that the LDPC code with SRK equalisation outperforms the scheme with the LDPC code and OFDM, particularly at lower signal-to-noise ratios, with a coding gain of around 1dB at a BER = 10^{-3}.

Figure 3 shows the simulation results of the LDPC (4351, 4123) code with equalisation and with OFDM over the BFWA channel. Once again, this longer code still does not perform well over the BFWA channel on its own and when used with SRK equalisation has approximately 1 dB coding gain at BER = 10^{-3} over the OFDM system.

5. Conclusion

This paper discussed the performance comparison of two different signal processing schemes to remove the adverse effect produced by the wireless media as laid down by the Broadband Fixed Wireless Access (WiMax or IEEE 802.16) specifications. The schemes, namely SRK equalisation and OFDM modulation, are combined with LDPC-BIBD forward error correction (FEC) to improve the performance of the system. The simulation result shows that the LDPC codes on its own do not improve the overall performance over fixed wireless channel. However, the better performance can be achieved using the SRK equalisation than OFDM modulation scheme.

References:

[1] W.Webb, “Broadband fixed wireless access as a key component of the future integrated communication environment,” *IEEE Communication Magazine*, vol. 39, pp. 115-121, Sept.2001.
[2] V. Erceg, K.V.S. Hari, M.S. Smith, D.S. Baum et al, “Channel Models for Fixed Wireless Applications”, Contribution IEEE 802.16.3c-01/29r1, Feb. 2001.
[3] IEEE Standard, “Part 16: Air interface for fixed broadband wireless access systems – Amendment 2: Media access control modifications and additional physical layer specifications for 2-11 GHz,” IEEE 802.16a, January 2003.
[4] G. L. Stuber, J. R. Barry, S. W. McLaughlin, Y. G. Li, M. A. Ingram, and T. G. Pratt, “Broadband MIMO-OFDM wireless communications,” *Proceedings of the IEEE*, vol. 92, pp. 271–294, Feb.2004.
[5] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit errorcorrecting coding and decoding: Turbo-codes,” in *Conference Record, IEEE International Conference on Communications, ICC-93*, May 1993, vol. 2, pp. 1064.1070.
[6] S. Benedetto and G. Montorsi, “Unveiling turbo codes: some results on parallel concatenated coding schemes,” *IEEE Transactions on Information Theory*, vol. 42, no. 2, pp. 409-428, March 1996.
[7] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, “Serial concatenation of interleaved codes: performance analysis, design, and iterative decoding,” *IEEE Transactions on Information Theory*, vol. 44, no. 3, pp. 909-926, May 1998.
[8] R.G. Gallager, Low Density Parity-Check Codes. MIT Press, Cambridge, MA, 1963.
[9] I. F. Blake and R. Mullin, *The Mathematical Theory of Coding*. NewYork: Academic, 1975.
[10] R. C. Bose, “On the construction of balanced incomplete block designs,” *Ann. Eugenics 9*, pp. 353–399, 1939.
[11] B.Ammar, B. Honary, “Construction of Low-Density Parity-Check Codes Based on Balance Incomplete Designs,” in *IEEE Transaction on Information Theory,* vol.50, pp. 1257-1268, June 2004.

Figure Captions:

Fig.1: The complete diagram of the single-input single-output LDPC coded BFWA system, (a) with equalisation and (b) with OFDM.

Fig. 2 Comparison of Simulation results of LDPC (2715, 2535) code using QPSK over BFWA channel.

Fig. 3 Comparison of Simulation results of LDPC (4351, 4123) code using QPSK over BFWA channel.

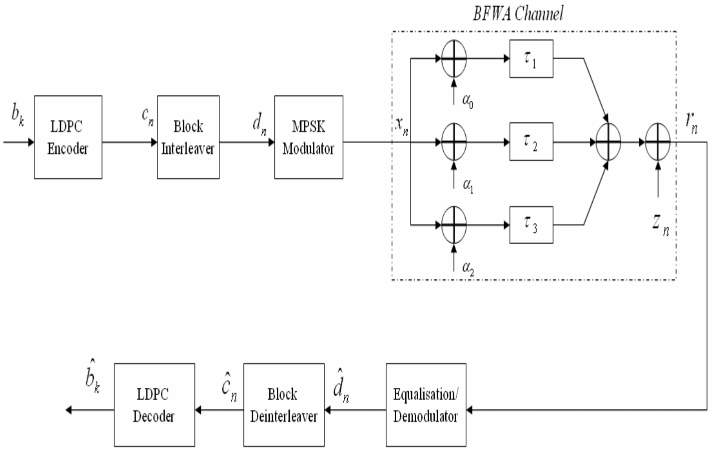

Fig. 1 (a)

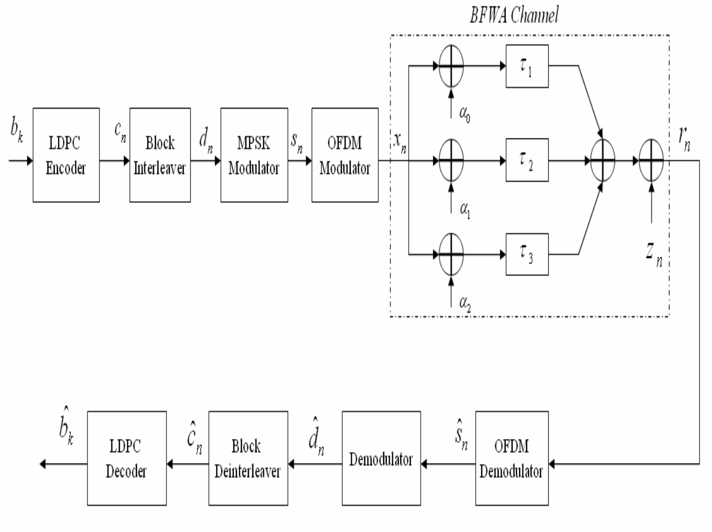

Fig. 1 (b)

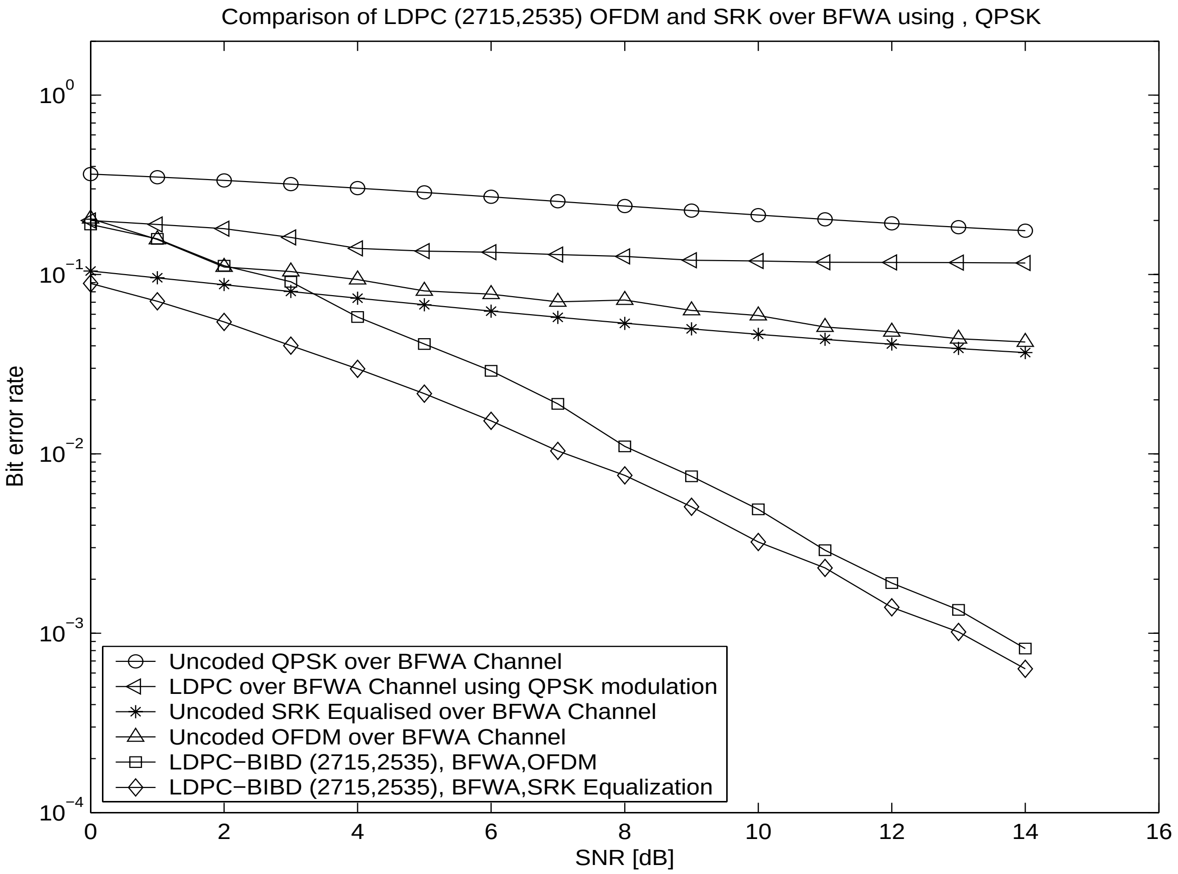
Comparison of LDPC (2715,2535) OFDM and SRK over BFWA using , QPSK
Bit error rate
SNR [dB]
10^0
10^−1
10^−2
10^−3
10^−4
0
2
4
6
8
10
12
14
16
Uncoded QPSK over BFWA Channel
LDPC over BFWA Channel using QPSK modulation
Uncoded SRK Equalised over BFWA Channel
Uncoded OFDM over BFWA Channel
LDPC−BIBD (2715,2535), BFWA,OFDM
LDPC−BIBD (2715,2535), BFWA,SRK Equalization

Fig. 2

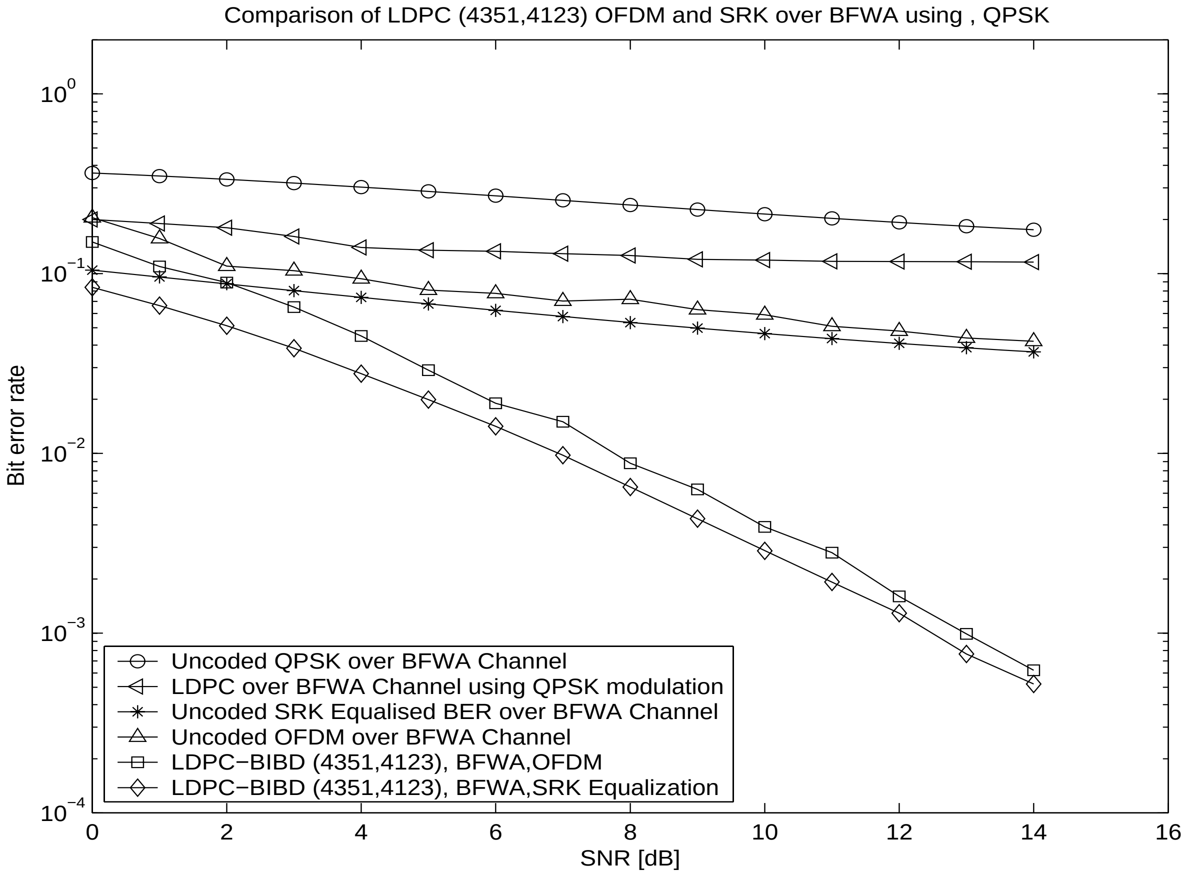
Comparison of LDPC (4351,4123) OFDM and SRK over BFWA using , QPSK
Bit error rate
SNR [dB]
10^0
10^−1
10^−2
10^−3
10^−4
0
2
4
6
8
10
12
14
16
Uncoded QPSK over BFWA Channel
LDPC over BFWA Channel using QPSK modulation
Uncoded SRK Equalised BER over BFWA Channel
Uncoded OFDM over BFWA Channel
LDPC−BIBD (4351,4123), BFWA,OFDM
LDPC−BIBD (4351,4123), BFWA,SRK Equalization

Fig. 3