Determination of Fracture Mechanics Parameters using FEM and J-Integral Approach

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Abstract: Finite element method is used to study cracked structural components in domain of linear elastic fracture mechanics (LEFM). For determination of stress intensity factors in this paper J-integral method is used. The stress intensity factor derived from J-integral method is less sensitive than that from the displacement method, to the finite element mesh size. Therefore, a coarser mesh, less degrees-of-freedom can be used to save the time of computation. To illustrate accuracy and efficiency of J-integral approach in fracture mechanics computations here are various numerical examples are included.

Key words: LEFM-linear elastic fracture mechanics, SIF-stress intensity factors, J-integral, FEM-finite element method

1. Introduction

In fracture mechanics, we try to correlate analytical predictions of crack propagation and failure with experimental results. The analytical predictions made by calculating fracture parameters such as stress intensity factors in the crack region, and later we can use it to estimate crack growth rate. Although several stress intensity factor handbook [1, 2] have published, the available solutions are not always adequate for particular engineering applications. This is especially true for cracks subjected to non-uniform stress fields near notch or thermal stresses [3]. Stress intensity factor calculation is an important issue for numerical analysis of fracture problems and there exist many approaches for instance, the extrapolation techniques, J-integral approach [4], the Virtual Crack Extension (VCE) technique etc. Amongst these methods, a displacement extrapolation method sometimes called crack opening displacement correlation technique used specially with the quarter-point elements (QPE) is most widely used due to its high accuracy and simplicity.

Typical fracture parameters of interests are:

- Stress intensity factors (KI, KII, KIII), which correspond to three basic modes of fracture
- J-integral [3], a path-independent line integral that measures the strength of the singular stresses and strains near a crack tip
- Energy release rate(G), which represents the amount of work associated with a crack opening or closure

Firstly, we have to perform a linear elastic or elastic-plastic static analysis and then use postprocessing commands to calculate fracture parameters.

Finite element method (FEM) and boundary element method (BEM) are the most widely used techniques for evaluating stress intensity factor (SIF). The most important region in modeling the fracture region is the region around the crack. While the domain is meshed, we are using crack tip elements with nodal singularity[5,6]. Those elements remove the nodal singularity at the crack tip. Those elements remove the nodal singularity at the crack tip. Displacement correlation was employed to determine stress intensity factors. Atluri at al.[7] employed hybrid finite elements with a square root surroundings the crack tip and regular elements in the rest of the domain to determine stress intensity factors.

One main objective of this paper lies in developing a complet J-integral approach to analyse cracked structural components.

2. Determination of Stress Intensity Parameters

2.1 Fracture Analysis using FEM

In 2-D problems of fracture mechanics, the quadratic elements PLANE 82 or Plane 2 are common in use. After meshing the crack tip, elements surrounding the crack tip are shown in Fig. 1. Their's sides, emanating from the crack tip, have nodes at quarter points.

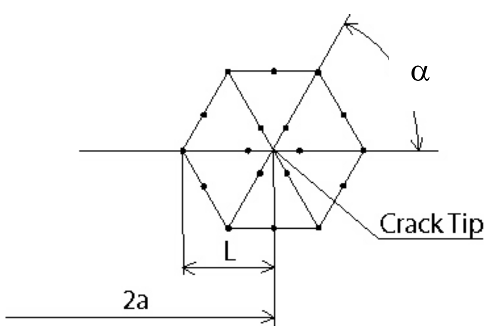

Fig. 1 Quarter-point triangular elements surrounding the crack tip

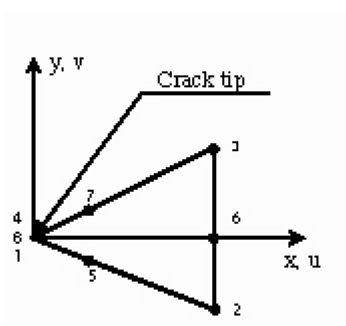

Fig. 2 Singular elements PLANE 82, Cartesian space

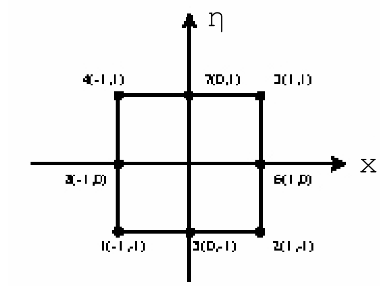

Fig. 3: Singular elements PLANE 82, Natural space

2.2 Calculating of SIFs

There are different techniques for evaluating SIFs [3,5]. In this work, we only concern evaluating SIF using J-integral.

For a 2-D body with surface traction components tx and ty prescribed over a portion of the bounding surface, the J-integral is defined for path independent contours э encircling the crack tip as shown in Fig. 4.

$$J = \int_{\Gamma} W dy - \int_{\Gamma} \left(t_x \frac{\partial u}{\partial x} + t_y \frac{\partial v}{\partial y} \right) d\Gamma \quad (1)$$

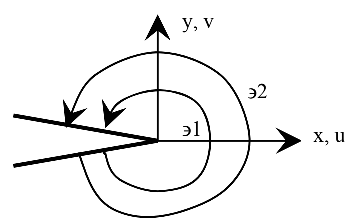

Fig. 4 Path independent contours

W is the strain energy density (strain energy per unit volume). In linear elastic fracture mechanics (LEFM) J- integral is equal to the energy release rate:

$$J=G \quad (2)$$

Commonly, several contours эi are constructed, and the average value is used,

$$J = \frac{1}{n}\sum_{i=1}^{n} J_i \qquad (3)$$

Calculating stress intensity factor (KI), follows:

$$K_I = \sqrt{JE} \qquad \text{Plain strain,} \quad (4)$$

$$K_I = \sqrt{\frac{JE}{1-\nu^2}} \qquad \text{Plain stress} \quad (5)$$

For the fairly complicated geometry and loading conditions in the specimen of this work, the finite element techniques is used to calculate the stress intensity factor. The finite element method offers two advantages in the numerical evaluation of the J-integral: (1) No specialized idealization is required near crack tip and (2) A coarse mesh can be used to achieve an accurate solution.

3. Numerical examples

To illustrate efficient of J-integral approach, two examples presented. In this work, 8 node PLANE 82 was used as an element type with plain stress option. In order to calculate fracture mechanics parameters, it was used software package ANSYS 9.0. Because of symmetry, a one-quarter model is used for calculating fracture parameters. Thus, three main aspects have been concentrated on:

- Modeling the crack region
- Calculating J-integral
- Calculation of stress intensity factor (KI) using J-integral

Example 1: Evaluation Stress Intensity Factor (KI), in plate with central crack

In this example, a finite plate in tension σ0= 9 daN/mm^2 (90 MPa) with a central crack 2a= 4mm is presented as shown in Fig. 5. The plate is made of aluminum alloy with Young's modulus E= 7400 daN/mm^2 (74000 MPa) and Poisson's ratio $\nu = 0.3$.

The dimensions of plate are: width 2b= 90mm and height h= 45 mm, as shown on Fig. 5. Calculations of fracture parameters are performed and results are given in Table 1.

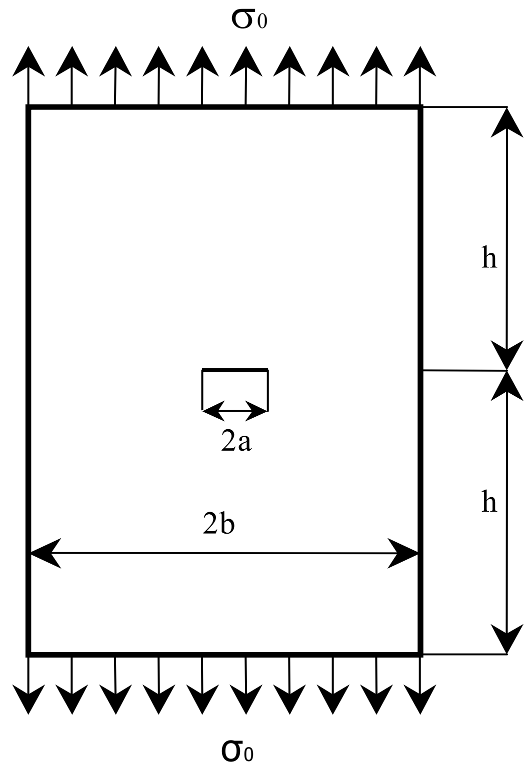

Fig. 5 Geometric properties of specimen

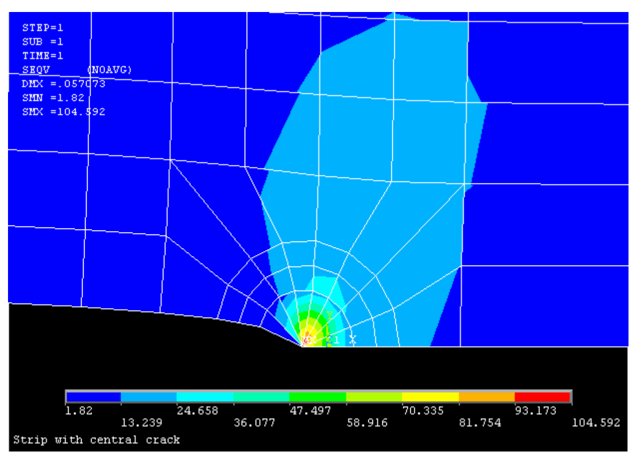

Fig. 6: Element stresses

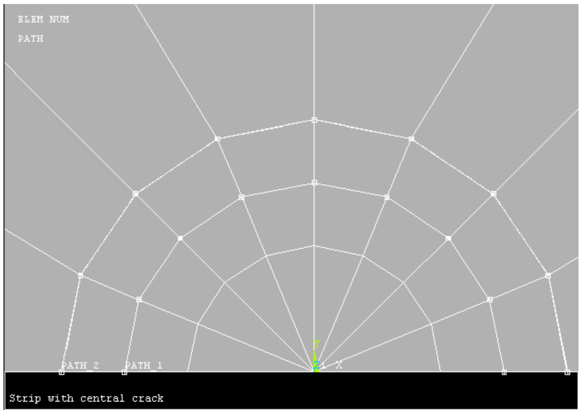

Fig.7: Mesh around crack tip and two independent paths

Table 1: Results of SIF for center-cracked plate using J-integral method

Plate	Path 1	Path 2
JA	0.0064067	0.0061066
JB	-0.0280248	-0.0277251
J=2*(JA-JB)	0.0688632	0.0676635
KI(daN/mm^2)	23.664	23.457

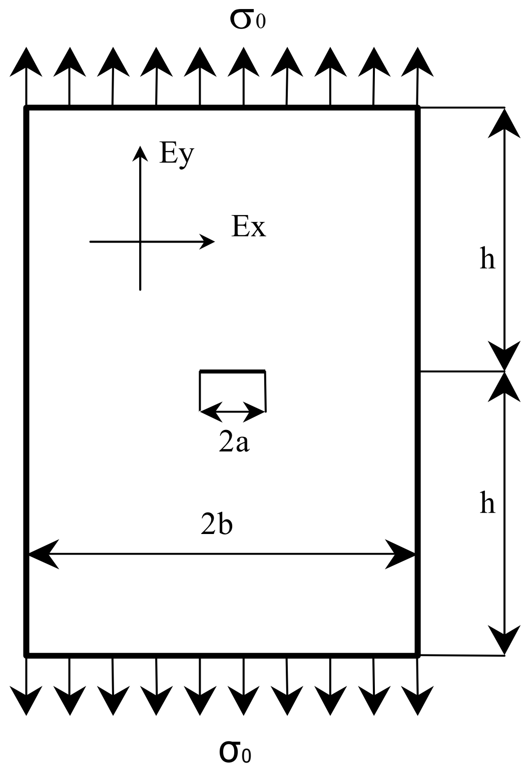

Fig. 8 Evaluation stress intensity factor (KI), in plate with central crack- orthotropic material

Example 2

In this numerical example, an orthotropic plate with a central crack is presented as shown in Fig. 8. The orthotropic plate contains a central crack 2a= 25.4 mm and is loaded by uniform tensile load at the far ends σ_0= 10 daN/mm^2 (100 MPa). The height is taken as 2h= 127 mm and width 2b= 127 mm. The materials properties of a plate are Ex= 20685 daN/mm^2 (206850 MPa), Ey= 8274 daN/m^2 (82740 MPa), ν =0.3 and Gxy= 5047 daN/mm^2 (50470 MPa). Calculation of fracture parameters are performed and results are given in Table 2.

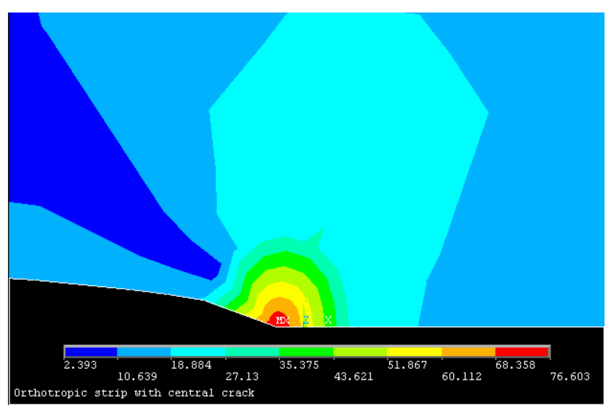

Fig 9. Nodal stresses

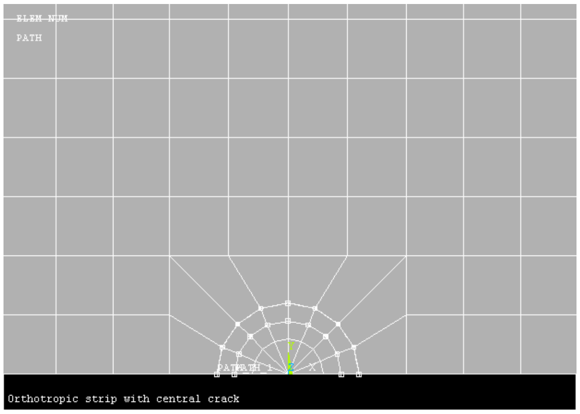

Fig.10 Mesh around crack tip and two independent paths

Table 2: Results of SIF for center- cracked plate using J-integral method

Plate	Path 1	Path 2
JA	0.04568145	0.04561297
JB	-0.16392799	-0.16391391
J=2*(JA-JB)	0.419218897	0.41905383
KI(daN/mm^2)	63.1275	63.1151

4. Conclusions

In order to analyse the fatigue behaviour of cracked structural components it is necessary to know the stress intensiy factors for the different crack fronts and and crack sizes that are originated in this geometry throughout the number of cycles. Thanks to the developing of finite element techniques, this task is now accessible by a LEFM analysis. Most of the authors use the extrapolation of displacements in the vicinity of crack tip and model the latter with singular Barsum elements. Hovever, this technique may be complex in the case of the fatigue growth of general cracks. In this paper the stress intensity factors for 2-D cracked structural components have been derived using J-integral approach. The J-integral method shows some advantages compared to the displacement extrapolation one. The knowledge of the exact displacement field in the vicinity of the crack tip is not required and the use of singular finite elements is not essential anymore.

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