INITIAL FAILURE AND FRACTURE MECHANICS ANALYSES OF WOOD CONSTRUCTIONS

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Summary:-The paper presents the analysis methodology on strength of wood girders, both from aspect of strength and fracture mechanics analysis. The paper also considers two aspects of strength analysis on structures made of glued laminated wood. The first one is related to determining of load (stress) level, where initial failure of wood structure occurred. The method of finite elements has been used for that purpose in correlation with appropriate fracture criteria related to orthotropic structure. The second research aspect is aimed to strength analysis from aspect of fracture mechanics. Wood structure is observed as orthotropic structure in this research, along with occurrence of initial damages in form of cracks. The wood structure behavior along with occurrence of cracks has been analyzed by means of Stress Intensity Factors. Stress Intensity Factors have been determined by applying the Finite Elements Method. Special singular finite elements have been used for that purpose. The results of numerical simulation in applying finite elements have been compared with experimental results.

Key-Words: - Finite Element Method (FEM), Timber structures, Failure Analysis, Stress Intensity Factors (SIFs)

1 Introduction

The application of numerical methods certainly represents reliable and efficient procedure for analyzing complex wooden structures, especially for simulation on behavior of wood structures, both under applying static load until initial failure occurrence and in analysis of the structure behavior with initial damages. It is well known that geometrical discontinuities within the structure are sources of stress concentration and potential zones of initial failure occurrence and subsequently final failure of the structure.

It shall be mentioned that fracture mechanics-related analysis of structures is also necessary for structures without geometrical discontinuities. The critical zones, i.e. potential failure zones within structure are necessary to be determined for these structures. Upon determining of these critical zones, the damage occurrences in form of initial cracks are assumed in them. It is necessary to define the parameters of fracture mechanics for aforementioned cracks.

Critical parts are, by rule, determined based on maximum stress values. Since wood structures with orthotropic mechanical properties are in question here, the critical parts can be determined through maximum values of Initial Failure Index (IFI). This basically requires fracture mechanics-related analysis of structure. This paper also takes into consideration curved members made of glued laminated wood, the stresses of which as well as Initial Failure Coefficients have been determined using Finite Elements Method (FEM) in correlation with appropriate initial failure criteria, used for anisotropic materials. The initial cracks are defined on critical zones of curved members. Stress Intensity Factors (SIF) are determined for aforementioned cracks. There are several methods for determining Stress Intensity Factors. SIF's have been also determined using Finite Elements Method in this paper. Special singular finite elements have been used for purpose of determining Stress Intensity Factors. Singular finite elements have proved to be very reliable in determining parameters of fracture mechanics. As the wood is anisotropic material, there is a considerable difference for wood structures regarding the wood strength in direction of fibers related to direction perpendicular to fibers. It has to be specially pointed out that wood structures are very sensitive regarding tensile strength applied perpendicular to fibers direction, that is, they have very unfavorable properties related to tensile strength applied perpendicular to

direction of fibers. Therefore, special attention is paid to precise determination of tensile strength perpendicular to direction of fibers in strength analysis of wood girders.
In order to determine structure-related stresses as precisely as possible, the improved finite elements of shells have been used in this paper, which can describe rotations within the girder's plane. Therefore, 4-node finite elements of shells with 6 degrees of freedom have been used in this paper, including the appropriate degree of rotation in the finite element plane.

2. Finite Element Analysis

In this investigation is used FEM for stress analyzes and displacement of complex glulam structures. Definition of FEM model is based on assumed displacement. Constructional model includes orthotropic characteristics of the material. For problems considered in this paper, improved three and four-node shell finite elements[1] are included, based on Discrete Kirchoff Theory (DKT). These finite elements posses all six degrees of freedom in every node: three translations and three rotations. Those same elements can be used as for working stress determining, so for failure (stability loss) analyzes. Improved behavior of finite elements makes introduced correct degree of rotation (θ_z) in the plane of finite element. Finite element defined in this way possesses exceptionally high precision in describing behavior within its plane, which is primary goal in analyzes of curved glulam members
The 4-node shell finite elements [1,2] are used in this investgation.These elements present eight degress of freedom at each node: three translaions, three rotations about the nodal *x*, *y* and *z* axes and two higher order terms.

Description of layered shell finite element

Based on the higher order shear deformation plate theory in the present analysis, a four-noded quadrilateral element with 8 degrees of freedom per node [1] is used. The formulation of a 4-noded shell finite element that can be good enough also if applied to the thin multilayered plates/shells is by no means an easy matter. The authors' experience has shown that a good approach to the formulation of a 4-noded shell finite element can be based on the application of the Discrete Kirchhoff's Theory (DKT) [3] for bending behavior. DKT ensures C^1 continuity at discrete points on inter-element boundaries. The improved 4-noded layered shell finite element is derived combining HOST and DKT, Fig.1.

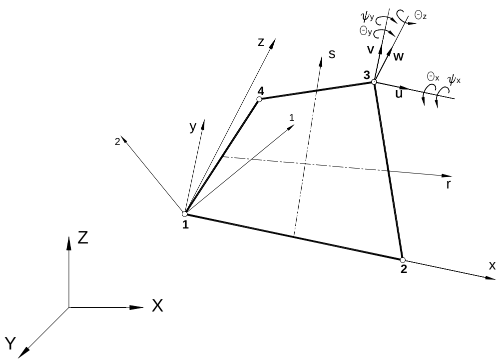


Fig. 1: Description of 4-noded shell finite element

More details about that element can be found in [1] and [7]. In the C^o finite element theory the continuum displacement vector within the element is defined by

$$a = \sum_{i=1}^{M} N_i(r,s)a_i \qquad (1)$$

where $N_i\,(r, s)$ is the interpolation function associated with the node *i* and expressed through the normalized coordinates (r, s); *M* is the number of nodes in the element and a_i is the generalized displacement vector in the mid-surface. In the case of the negligible mid-surface normal stress σ_z the stress-displacement relationships, stress resultants and the constitutive equations associated with the higher-order shear deformation theory are given in [1] and [7]. The total stiffness matrix of the element is obtained by the linear superposition of the following three independent parts:

i. Membrane stiffness matrix K_M
ii. Bending stiffness matrix K_B, and
iii. Rotational stiffness matrix $K_{\Theta z}$

Four-node quadrilateral layered shell element, for geometrical nonlinear analysis, is derived combining a higher order shear deformation theory and membrane elements with drilling/rotational degrees of freedom. This finite element has 8 degrees of freedom (DOF) per node

$$a^T = (u_0, v_0, w_0, \upsilon_x, \upsilon_y, \upsilon_z, \chi_x, \chi_y) \qquad (2)$$

where u_0, v_0, w_0, υ_x, υ_y represent conventional degrees of freedom, υ_z are in-plane vertex rotations. The terms χ_x and χ_y are the corresponding higher order terms in the Taylor's series expansion used in the theory and are also defined at the reference plane. This element is obtained by superposition of the refined membrane element with rotational degrees of freedom and discrete Kirchhoff model for bending. In order to avoid singularity in the assembled matrix using flat elements in a global coordinate system here is used membrane element, which includes in-plane nodal rotations, υ_z, as degree of freedom. In this work Allman's approach is used. Allman's approach [4] begins by selecting a quadratic form for the normal component of displacement, U_n, and a linear form for the tangential component of displacement, U_t, along each element edge i –j, Fig. 2.

$$U_n = \left(1-\frac{\xi}{l_{ij}}\right)U_{n1} + \left(\frac{\xi}{l_{ij}}\right)U_{n2} + \frac{\xi}{l_{ij}}\left(1-\frac{\xi}{l_{ij}}\right)(\theta_{z2}-\theta_{z1}) \quad (3)$$

$$U_t = \left(1-\frac{\xi}{l_{ij}}\right)U_{t1} + \left(\frac{\xi}{l_{ij}}\right)U_{t2} \quad (4)$$

where φ is the running distance from one end and (U_{n1}, U_{t1}, υ_{z1}), (U_{n2}, U_{t2}, υ_{z2}) are the translational and rotational components displacements at each end of the edge.

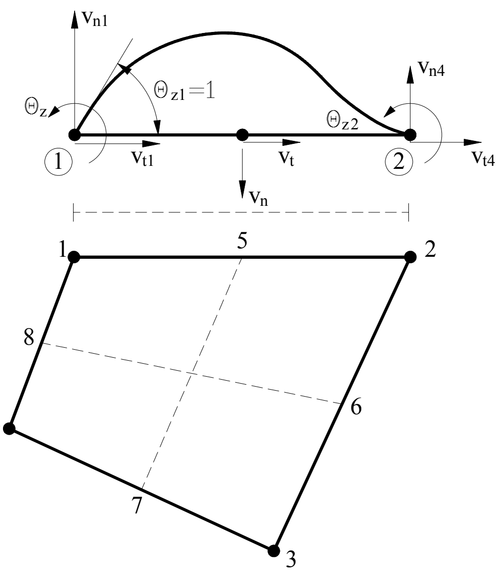


Fig. 2: Membrane part of shell element

As noted Harder and Mac Neal [5] eqns. (3) and (4) can be used to eliminate the transitional displacements at the midpoint of the edge in favor of the degrees of freedom at the adjacent corner points so that, in this way, any eight-noded membrane quadrilateral can be converted into an element with corner translations and rotations as DOF's.

4. FAILURE CRITERION

Failure theories are functions of the stresses and strengths of the material are assumed to represent failure under all loading conditions without regard to failure mechanism r failur mode. For isotropic materials there are three well-known strength theories maximum principal stress, maximum shear stress (Tresca), and distortional energy (Mises-Hencky). In each case a function of the stresses is equated with a single parameter, the tensile yeald strength, or the fatigue strength of the material. Recognition of the fact that anisotropic materials have more than one strength parameter has led to numerous proposals for failure theories. More than 40 such theories have been proposed for wood, reinforced plastics, etc. Many of the theories are quadratic functions of the stresses and strengths. Failure theories for plane stress involve σ_1, σ_2 and σ_6. Thorough ivestigation of a failure theory involves comparison of experimental data for complex stress loading with the chosen failure surface. This requires an experimental facility capable of changing the relative of σ_1, σ_2 and σ_6 over a wide range.

Along with some practical glulam structures failure estimations, which are mostly based on allowed stresses in fiber direction and perpendicular to fiber direction, there are other failure criterions, based on interactive expresions of work stress and correspondent strength of materials with orthotropic characteristics. For this purpose Tsai-Wu initial failure criterion [6] is used. In tensor notation, initial failure is predicted to occur when failure index (F.I):

$$F.I = F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 \geq 1 \quad (5)$$

where i,j=1,2,…6, and F_{ij} are 2nd and 4th rank strength tensors, respectively. For the case of plane stress equation (5) can be expressed in the form. The coefficients are described in terms of the strength of timbers.

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c}, \quad F_{11} = \frac{1}{X_t X_c}, \quad F_{22} = \frac{1}{Y_t Y_c},$$

$$F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}, \quad F_{66} = \frac{1}{T^2}, \quad F_{12} = -\frac{1}{2}(X_t X_c Y_t Y_c)^{1/2}$$

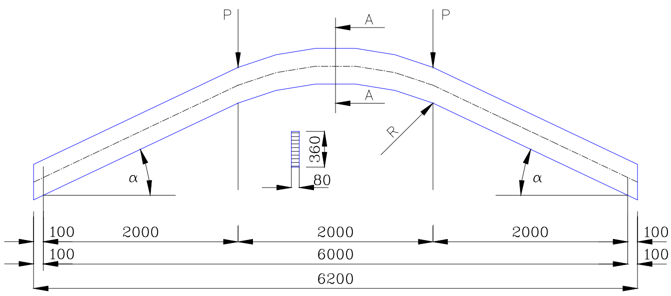
where: X_t and X_c are the tensile and compressive strength parallel to the direction of the fibre, Y_t and Y_c are the tensile and compressive strength transverse to the direction of the fiber and T is shear strength.

5. Determining Parameters of Fracture Mechanics

As it has been mentioned in the introduction, the key analysis factors with regard to fracture mechanics are Stress Intensity Factors. As for wood structures, orthotropic properties of the material shall be included in its determination. Regarding the compact girders made of glued laminated wood, that is, girders without holes and other geometrical discontinuities, which are described in this paper, the single mode fracture-based analysis (mode I) can be successfully used related to fracture mechanics. Not considering all aspects in determining parameters of fracture mechanic, related to orthotropic materials, this paper will show only basics of theoretical considerations, which can be applied to wood structures. The special singular finite elements are used in determining SIF. As illustrative example of determining SIF for considered girders made of glued laminated wood, the board was taken with initial crack along direction of fibers, whereas load was applied perpendicular to direction of fibers. This is at the same time a critical form in analysis of considered compact girders made of glued laminated wood related to fracture mechanics. Of course, the initial cracks can be initiated within the glued laminated wood girders itself and SIF to be determined along with it.

6. Numerical and experimental results

In practical designing of wooden structures, radial stresses represent critical components for dimensioning. Purpose of these experiments was to perform measurements of stresses analyzis and shifting, which would serve for verification of methods and software for curved members strenth analysis, and beside that, to clarify influence of member curvature to the radial stresses. For this purpose, for testing were defined five groups of specimens. All specimens were of same span L=600 cm and height H=36 cm, and constant cross section 36x8 cm, with variety in curvature, Fig. 3 and Table 1.



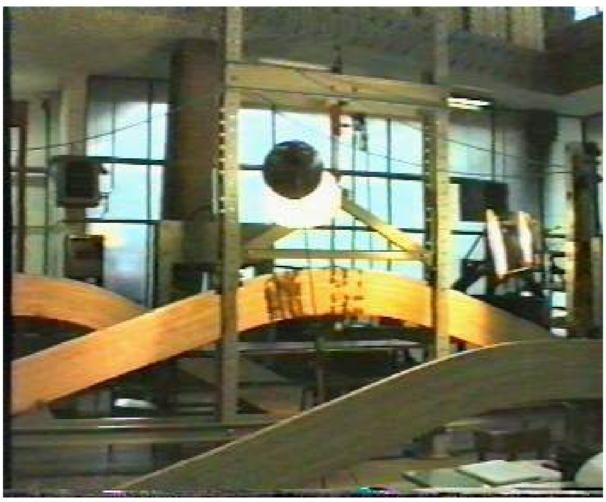


Fig. 3 Curved timber specimens

Specimens were made of constant thickness lams (24 lams of 1.5 cm thickness). Lams were made of first class conifer. During testing, measured dampness of wood was 11.8-12.8%. Figure 4 shows good agreement finite element results with experiments of radial stresses at the curved laminated structures.

To determine initial failure loads of curved wood members here Tsai-Wu initial failure criterion is used. Computation failure load level of curved timber member, defined using Tsai Wu criterion, is in good agreement with experiment too (See Table 1 and Figure 5).

Table 1: *The effect of curvature of specimens (R) on failure load (F_f)*

Type specimens	No. of spec.	j_f	F_f [kN]	$(j_f)_{av}$
D-1 (R=2460)	D-1.1	1.5	33.60	1.6
	D-1.2	1.1	24.64	
	D-1.3	1.7	38.08	
	D-1.4	1.7	38.08	
	D-1.5	1.6	35.84	
D-2 (R=3240)	D-2.1	2.6	58.24	2.4
	D-2.2	2.2	49.28	
	D-2.3	2.2	49.28	
	D-2.4	2.6	58.24	
	D-2.5	2.5	56.00	
D-3 (R=4810)	D-3.1	3.5	78.40	3.2
	D-3.2	3.2	71.68	
	D-3.3	2.6	58.24	
	D-3.4	3.4	76.16	
	D-3.5	3.2	71.68	
D-4 (R=9570)	D-4.1	3.2	71.68	3.4
	D-4.2	3.6	80.64	
	D-4.3	2.8	62.72	
	D-4.4	3.6	80.64	
	D-4.5	3.6	80.64	
Straight member D-5 (R=∞)	D-5.1	3.8	85.12	3.7
	D-5.2	2.5	56.00	
	D-5.3	3.8	85.12	
	D-5.4	3.6	80.64	
	D-5.5	3.8	85.12	

The corresponding strengths that are necessary for initial failure analyses are experimental determined. In this investigation Tsai-Wu initial failure criterion is used to study initial failure load.

$$X_t = 8.7\,kN/cm^2$$

$$X_c = 5.63\,kN/cm^2$$

$$Y_t = 0.10\ kN/cm^2$$

$$Y_c = 0.72\ kN/cm^2$$

$$T = 1.00\ kN/cm^2$$

Combining high-quality shell finite elements and adequate initial failure criterions that is possible to study failure of complex timber structures. This shell finite element can be used in stability analysis of timber structures too.

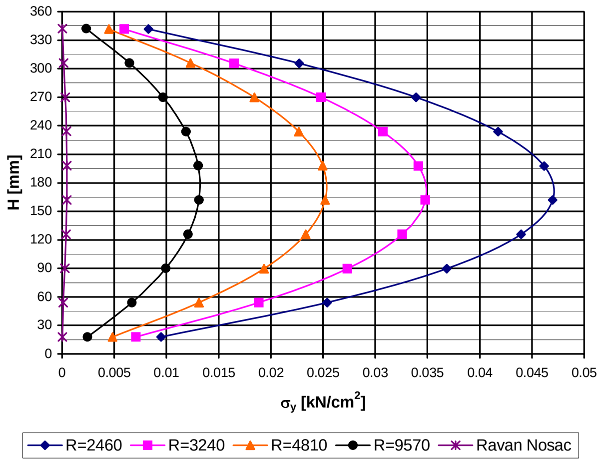
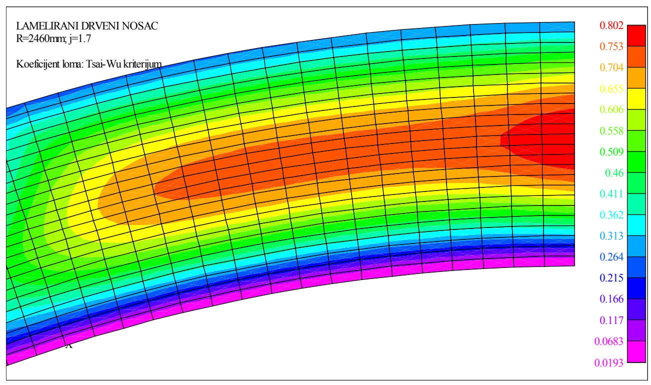


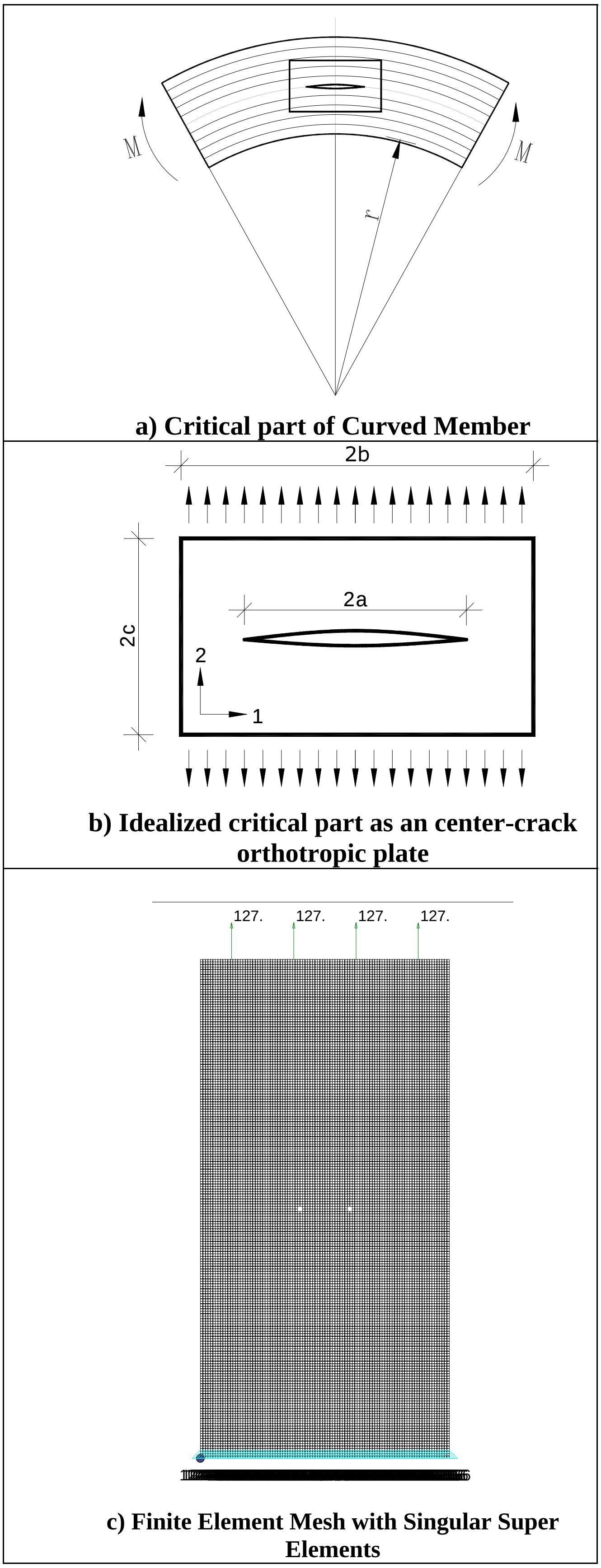
Fig. 4 The effect of radius of curvature (R) on maximal radial stresses using FEM, F=11.2 kN



Failure Indexes (at Failure load level of specimens)
F.I $_{(D-1)}$ = 0.802
F.I $_{(D-2)}$ = 0.951
F.I $_{(D-3)}$ = 0.967
F.I $_{(D-4)}$ = 0.785
F.I $_{(D-5)}$ = 0.825

Fig. 5 Failure index distributions at specimen D-1 using Tsai-Wu criterion for failure load (j=1.7)

To illustrate Fracture Mechanics Analysis of timber structures here is considered critical part of curved member in Fig. 5. As critical part of member, with aspect fracture mechanics, is part of member where are maximal normal stresses in direction perpendicular to fibers. In Figure 6 is shown the complete simplified algorithm for fracture mechanics analysis including: (a) Critical zone; (b) Simple cracked model of orthotropic specimen and (c) Finite element model of cracked specimen with singular super-element. Orthotropic plate is loaded with σ=10N/mm^2. Geometry of specimen is: 2a =25.4 mm; b =5a=63.5mm; c=63.5 mm, **d**=10 mm.



a) Critical part of Curved Member

b) Idealized critical part as an center-crack orthotropic plate

c) Finite Element Mesh with Singular Super Elements

Fig. 6 Simplified Model of Cracked Timber Structure

Cracked specimen is modeled as orthotropic material with special singular elements. In Figure 7 are shown stress distributions in finite element model.

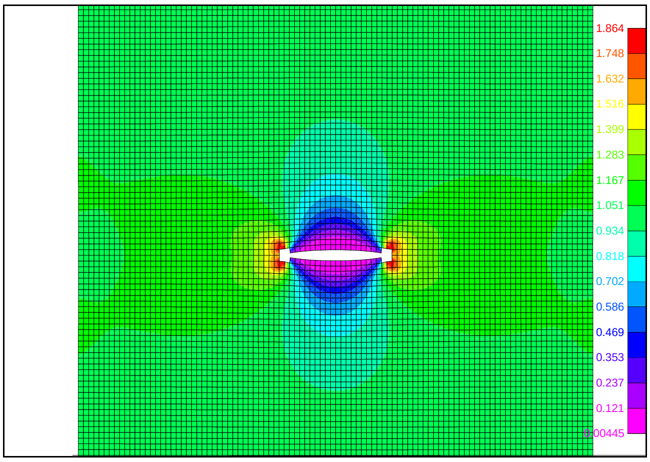


Fig. 7 Stress Distribution in Cracked Specimen

Material properties of orthotropic material are:

E_1=1250 daN/mm^2
E_2=48 daN/mm^2
G_{12}=53.85 daN/mm^2
ν=0.3
The Stress Intensity Factor is obtained using singular finite elements: $\mathbf{K_I}$**=7.66**.

7. CONCLUSIONS

This paper has presented the methodology of strength analysis of curved members made of glued laminated wood, both from aspect of strength and fracture mechanics analysis. It basically uses the Finite Elements Method in correlation with appropriate fracture criteria in determining load level when initial failure occurs as well as position of critical part on analyzed wood structure. The damages in form of initial cracks are assumed on critical parts of wood structure. Particular parameters of fracture mechanics are determined for aforementioned cracks. This paper also included determining of Stress Intensity Factors on the crack tip, using special singular finite elements. The results of numerical simulation structure strength capacity based on the use of improved 4-node shell finite elements and have been compared with experimental results, where good agreement has been obtained.

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