Multilevel Optimization Approach Applied to Structural Design Including Material Consolidation

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Abstract: - Optimization approach is applied to multidisciplinary structural design problems like: minimum weight of aircraft nose landing gear structure under various strength and stiffness constraints (including material characteristics consolidation), directional aircraft stability and control during taxiing and take off. Optimality criteria approach (Dual algorithms) and finite element method (FEM) for stress analyzes subjected to strength constrains, in system level, are applied to achieve minimum weight of nose landing gear structure. In local levels the nose wheel castering length and damping of damper are considered as optimization parameters in stability maximization and controlability during taxiing and take off. The use of finite element methods in parallel with optimization techniques such as dual and multicriterion optimization techniques make it possible to attack large-scale and complex structural problems such as aircraft landing gear or composite structures.

Key-Words: - Optimal design**,** Minimum weight, Multilevel Optimization, Aircraft landing gear, FEM, Dual methods, Sizing optimization, Multicriterion optimization method.

1 Introduction

Structural optimization has been an active area of research since the early 1970s. The two basic optimization problems typically addressed in structural optimization have been sizing and shape optimization [1-3, 14]. In sizing optimization, variables define local geometric characteristics. In shape optimization, the optimum shape of structure is sought by varying the boundary shape defined by an appropriate spline function, the design variables defined in a function form [5, 14]. In the designing of large-scale structural systems such as aircraft structures, the major tasks is the sizing of the structural members to obtain the desired performances, strength, weight, and stiffness characteristics. The use of finite element methods (FEM) in parallels with optimization techniques such as nonlinear mathematical programming (NMP) or optimality criteria (OC) make it possible to attack large and complex aircraft structural problems [13].

The motivation of this study is to come up with a multilevel optimization method using optimality criteria and mathematical programming techniques. Multilevel optimization permits a large problem to be broken down into a number of smaller ones, at different levels according to the type of problem being solved. This approach breaks the primary problem statement into a system level design problem and set of uncoupled component level problems. Results are obtained by iteration between the system and component level problems. The decomposition of a complex optimization problem into a multilevel hierarchy of simpler problems often has computational advantages. It makes the whole problem more tractable, especially for the large engineering structures, because the number of design variables and constraints are so great that the optimization becomes both intractable and costly.

In the designing aircraft nose wheel it is necessary to consider many different (sometimes – conflicting) requirements [4, 10]. Mainly, the requirements are: the good aircraft behavior during ground motions; mass minimization; convenient design and technology; easy maintenance; etc.

The investigation of nose wheel behavior during aircraft taxiing and take-off is very important within designing and testing fazes. From the pilot point of view, the way the aircraft respond to command and/or disturbances during taxiing and take-off is very important. Two deferent requirements might be of interest: good stability on runway (low deviation from the path to outside noise) and easy control to obtain desired aircraft pointing.

The mass is prime interest while aircraft designing. Bother, the nose wheel position and nose wheel design effect the mass of nose wheel. In this paper, the nose wheel design to the mass minimization will be considered. Nose wheel geometry is important because it affect load distribution, nose wheel cinematic, volume in retracting position, etc.
Certain nose wheel parameters like the castering length, spring and damping have significant influence to the aircraft motion and parameters like: stability and control during ground motion; to the mass of the wheel; to the geometric values and other requirements. So, the optimization of those parameters during design phase is essential. For the parameter analysis and optimization, it is necessary to have convenient mathematical form.

The separate optimization problems like stability and controllability, the nose wheel mass minimization, the convenient geometry are functions of nose wheel parameters (especially the nose wheel castering length). Each of those optimizations gives the different optimization point. That is why combined multicriterion optimization is considered. Different waiting coefficients are addressed to each separate optimization problem, and the effect of change of waiting matrix to optimization point is analyzed. The method is illustrated by numerical example for light training aircraft.

2 THEORY OF MULTYLEVEL OPTIMIZATION

Today, it is a common practice to use optimization methodologies to deal with multidisciplinary industrial design. Let D and d represent the sets of system and component design variables, respectively. Then the problem can be stated as: Find vectors D and d such that

$$W(D) \Rightarrow \min \tag{1}$$

subject to

$$G_q(D,d) \geq 0 \quad , \quad q \in Q \tag{2}$$

and

$$g_{lj}(d_j, D) \geq 0, \; l \in L \quad ; \quad j \in M \tag{3}$$

The G_q (D,d) represents constraints that are strongly dependent on the D vector and they are implicit functions except for the side constraints. The $g_{lj}(d_j\ ,D)$ represent constraints that are primarily dependent on the j component variables d_j , and they are either explicit or implicit functions of d_j, depending on the type of constraints. The symbols Q and L denote the set of system and component level constraints respectively, M denotes the number of components and $d^T = [d_1^T, d_2^T,..., d_M^T]$. Then system and local analyses and optimizations are carried out separately and tied together by an iterative scheme going from one level of design modification to the other and vice-versa seeking an overall optimum design.

2.1 The System Level Optimization

The two optimization problems typically addressed in structural optimization have been sizing and shape optimization. In sizing optimization, the variables define local geometric characteristic such as thickness, width, etc. In shape optimization, the optimal shape of a structure is sought by varying the boundary shape defined by an appropriate spline function, with the design variables defined in a function form. To achieve minimum weight of nose wheel structure here both types optimization (sizing and shape optimization) are included. Using standard optimization procedure based on combining OC and finite elements the weight optimization can be expressed as [1-3] : Find vector D such that

$$W(D) = \sum_{i=1}^{N} \frac{w_i}{D_i} \Rightarrow \min \tag{4}$$

and

$$G_q(D, d^*) \geq 0; \quad q \in Q \tag{5}$$

where d^* implies that the parameters strongly dependent on the detail design variables d (i.e, directional aircraft stability and control during taxiing and take off) do not change during a system level design modification stage. The w_j are positive fixed constants corresponding to the weight of the set of finite elements in the j-th linking group when $D_j=1$. The set of independent design variables after linking is denoted by N. The selection of design variables, especially in shape optimization, is very important in the optimization process. One has to decide a priory where to allow for design changes and to evaluate how these changes should take place by defining the location of the design variables and the moving directions. In many investigations, the design variables were chosen as the positions of the nodes on the boundary, or the coefficients of polynomials defining the boundary and control points of the Bezier and B-spleens. In the present study, the coordinates of the key points are specified as design variables. The use of the coordinates at key points as design variables leads to fewer design variables and more freedom in controlling the shape of the structure. Shape design sensitivity analysis is an important part of optimization. The exact semi analytical sensitivity analysis method[5] is used (the exact derivative of $\partial k / \partial a_i$ can be evaluated – where k is elemental stiffness matrix and a_j is the nodal coordinate of the element).

2.2 The Local level – Multicriterion Optimization

In many engineering applications, including mechanical and structural design problems, however, there often exist several, usually conflicting, criteria to be considered by the designer. It has been a common practice in the literature to represent the objective function as a weighted sum of those desirable properties. Multicriterion optimization seems to offer a very promising possibility to consider effectively all the different, mutually conflicting requirements inherent in the design problem. The recent emergence of the multicriterion approach in structural mechanics can be seen from the author's knowledge, were published in second half of the 1970s[7,8] applied the control theory approach to a bicriterion problem with weight and stored energy as criteria, and obtained analytic solutions for some structural elements, calling the results `natural structural shapes`. Baier[9] studied multicriterion optimization of structures from a general point of view, choosing weight and stored energies in separate loading conditions as design criteria. Several techniques for solving multicriterion nonlinear vector optimization problem have been presented in the literature. Usually they turn the original problem into a sequence of scalar optimization problems, which can be solved numerically by applying adapted methods of nonlinear programming. For this purpose weighting method is used in solving multicriterion optimization problem. Perhaps one of the most commonly used approaches to problems with several criteria is to form one scalar objective function as a weighted sum of the criteria. One drawback of this technique is the difficulty involved in choosing the weights for the criteria. In convex multicriterion problems, however, it is possible to apply the method in a parametric form to the determination of a pareto-optimal set. If the notation $a^T = \lfloor a_1 \; a_2 \; ... a_n \rfloor$ is used for the vector of weighting coefficients, the problem takes the form

$$\min_{x \in \Omega} a^T \; J(x) \, .$$

Without loss of generality, a can be normalized so that the sum of its components, which are non-negative and not all zero, is equal to one. Now Pareto-optimal solutions can be generated by parametrically varying the weights a_i in the objective function. In this paper, the compromise between more optimization tasks is proposed as multicriterion optimization in the form

$$J^m(x) = c_1 \left|J_1(x)\right|_N + c_2 \left|J_2(x)\right|_N + ... + c_n \left|J_n(x)\right|_N , \quad x \in R^n \qquad (6)$$

subject to

$$f_i(x) \le 0$$

where is: $J^m(x)$ - multicriterion optimization form,

$\left|J_k(x)\right|_N$ - absolute norm of each single optimal criteria (in this formula, norm means that the maximum value of the referred criteria is bring to one),

$c_1, \ldots c_n$ - weighting coefficient, ($\sum_1^n c_i = 1$). By these coefficients c_i, the designer (according to his judgment) gives more or less significance to the certain single optimization criteria $J_i(x)$,

$f_i(x) \le 0$ - posed constraints.

To perform multicriterion optimization, the algorithm is as follows:

1. Each optimality criteria $J_i(x)$ has to be performed separately and optimization point b^i found which satisfy criteria $J_i(b^i) = [J_i(x)]_{\max} = J_i^{\max}$
2. The norm of each optimality criteria is defined as $$\left|J_i(x)\right|_N = \left|\frac{J_i(x)}{J_i^{\max}}\right|$$
3. The weighting coefficients $c_1, \ldots c_n$ are chosen,
4. The constrained $f_i(x) \le 0$ are imposed,
5. The multicriterion optimization $J^m(x)$ is performed, usually applying certain numerical methods.

By this process, the designer has optimal point b^i for each separate criteria and optimal point b as results of combined criteria or multicriterion problem. The optimal solution of the parameters $b = x_{opt}$ may be between separate optimization points, but close to some separate results, as a function of the chosen weighting coefficient.

2.3 Nose wheel parameters optimization

Both, aircraft and nose wheel equations of motion, as connected system, might be represented as liberalized second order system [4,10]. There are a lot significant parameters to the nose landing gear design, having into the consideration the system performances like stability, controllability, mass, technological aspects and the others. From the experience, the significant parameters are: castering length, spring stiffness, damping, tire, and others. In this paper, to show the optimization procedure, two parameters -the castering length and damping of damper- are selected, having the obvious effect to the stability, controllability, mass, and technology.

The different criteria are applied to castering length and damping optimization, as:

- stability index as $J_1(l,H_p) = \xi(l,H_p)\omega_n(l,H_p)$ (or σ_1 - lower periodic root)
- aircraft controllability index defined by transfer function gain as: $J_2(l) = K^{\psi}_{M_{Nu}}$
- geometry (technology) complexity index determined as $J_3(l,H_p) = T_0 - T_1 \cdot l^{0.5} - l \cdot H_p$
- mass defined as $$J_4(l,H_p) = 0.95/(1+200\cdot l^2) - 0.9/20 \cdot H_p \cdot l$$

Stability condition introduce positive nose wheel length as constraint, l>0.

Each of these optimization criteria gives different optimization point for nose wheel castering length and damping. Stability criteria determine lower castering length, aircraft controllability criteria has no significant effect. Geometry complexity index tends to decrease castering length and so on.

3 NUMERICAL EXAMPLES

To illustrate the application and versatile multilevel approach some aspects of the optimal design of nose wheel structure is considered, Fig 1. Let consider light training aircraft with the next parameters: I_z = 3812 kgm^2, $C_{n\beta}$ =0.152, d_N = 1.408, C_{nr} = -0.213, H_L+H_P = 500, $C_{n\delta r}$ = - 0.0018, b = 9m , S = 13m^2 , F_N = 2590 N $\rightarrow$ static nose wheel load, K_N = 4 1/rad $\rightarrow$ for dry surface, K_N =2.8 1/rad $\rightarrow$ for wet surface.

In this paper, to show the optimization procedure, the two parameters -the castering length and damping of damper- are selected, having the obvious effect to the stability, controllability, mass, technology.

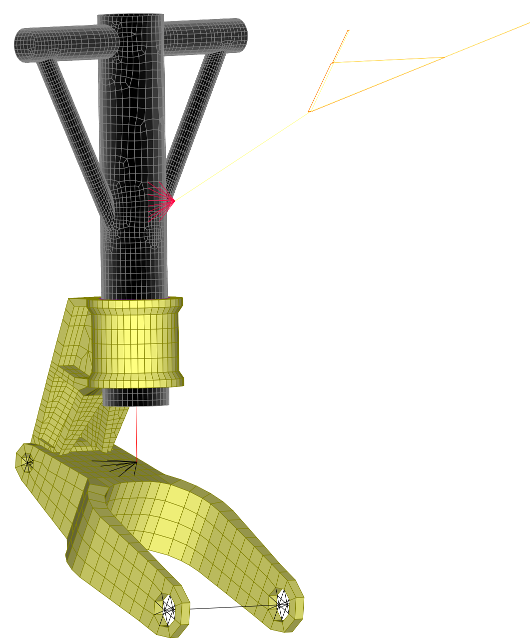

Fig 1 FE model of landing gear

Single optimization for different criteria gives four different optimization points for nose wheel castering length and the damping as: $l_1^{loc} = 22.0\,\text{mm}$ and $H_{p1}^{lpc} = 195.0\,\text{Nms}$, $l_2^{loc} = 80.0\,\text{mm}$, $l_3^{loc} = 0.0\,\text{mm}$ and $H_{p3}^{lpc} = 0.0\,\text{Nms}$, $l_4^{loc} = 0.0\,\text{mm}$ and $H_{p4}^{lpc} = 0.0\,\text{Nms}$,

Local level optimization gives optimal point as $l^{loc} = 30.0\,\text{mm}$ and $H_p^{lpc} = 255.0\,\text{Nms}$, and optimization level

$$J^m(l = 0.030, H_p = 255) = 0.812\,.$$

Local level optimization gives optimal point for nose wheel castering length as 30.0mm, Fig. 2. The optimization point is determined for ground speed as 10 m/s.

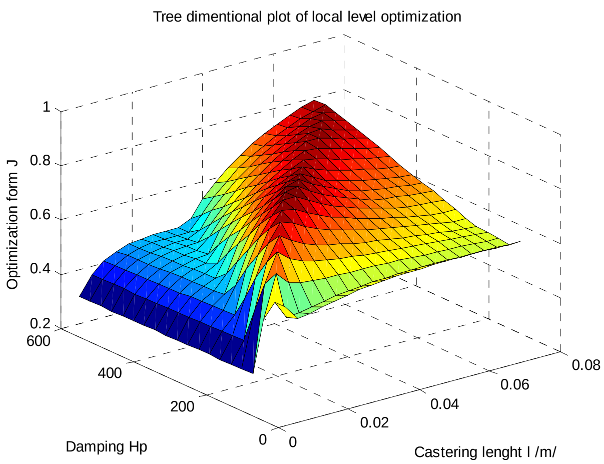

Fig. 2. Tree-dimensional plot of local level optimization form $J = J(l,Hp)=c_1J_1(l,Hp)+ c_2J_2(l,Hp)+ c_3J_3(l,Hp)+ c_4J_4(l,Hp)$. The optimization parameters are: l – Castering length, Hp – Damper damping

On the global level, material properties are described and introduced to the optimization process under various strength and stiffness constraints, to obtain mass and shape minimization. To achieve minimal mass of landing gear structure, Fig 1, the sizing and shape optimization techniques are used. Size optimization is applied on statical structural part that is modeled using shell finite elements. In order to reduce the computational burden to the size optimization problem approximation concept is used [11,12]. The number of the design variables was reduced by linking. The idea is reasonable as in practice some of the variables are the same as in this considered problem. The number of constraints was also reduced by considering only the critical or near critical constraints at each iteration. Detail description approximate concepts are given in reference [13]. Shape oprimization [14] is applied on structural part of landing gear that is shown in Fig. 3. Figure 3 shows the shape of the nose wheel structural element, before and after shape optimization, and fig. 4 shows stress distributions in this structural element after optimization.

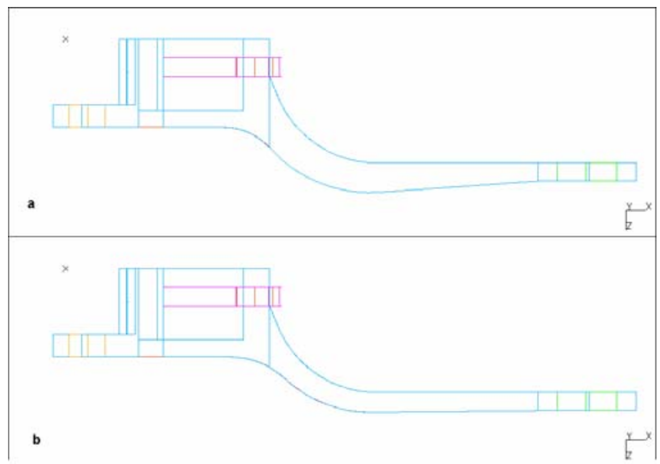

Fig. 3 System level – structural optimization

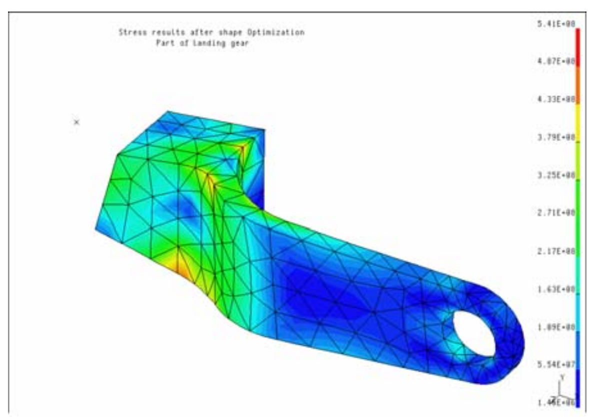

Fig. 4 Stress distributions in structural part after shape optimization

4 CONCLUSION

The obtained results demonstrate the practicality of multilevel optimization in the design of the multidisciplinary complex aircraft structures such as aircraft nose wheel. In this study two-level optimization algorithm is applied; system- and component level. Combining FEA, approximation concepts and OC or dual algorithms has led to a very efficient method for minimum weight sizing of large-scale structural systems. Finally, minimum weight designs obtained for the aircraft nose wheel structure illustrate the application of the multilevel approach to a relatively large structural system.

Recent optimization technique contributed a lot to the system parameters determination during the design process. On the other hand, the engineering judgment remains the design tool. The contribution in this paper is in the combined effect: application of optimization methods including the engineering preference and experience. The engineering judgment and influence is expressed trout the weighting coefficients c in the multicriterion optimization functional.

References:

[1] Berke, L and Khot, N. S., Use of Optimality Criteria Methods for Large Scale Systems, AGARD Lecture Series, No. 70 on Structural Optimization, AGARD-LS-70, pp 1-29, 1974.

[2] Maksimovic, S., Optimal Design of Composite Structures by Finite Elements, Proc. Int. Conf. Computer Aided in Composite Material Technology, Eds Brebbia, C. A., Wilde, W.P. and Blain, W.R., Springer Verlag, Southampton 1988

[3] Maksimovic, S., Some Computational and Experimental Aspects of Optimal Design Process of Composite Structures, Int. J. Composite Structures, Vol 17, pp. 237-258, 1990.

[4] Zeljkovic V., Maksimovic S. (1997) Investigation of nose wheel effect on aircraft stability, Transaction 2/1997 pp 27-32, MF Belgrade.

[5] Olhoff, N., Rasmussen, J. and Lund, E., Method of exact Numerical Differentiation for Error Estimation in Finite Element Based Semi-Analytical Shape Sensitivity Analysis, Special Report No. 10, Institute of Mechanical Engineering, Alborg University, Alborg, DK, 1992

[6] Maksimovic, S. and Zeljkovic, V., Multilevel Optimization of Large Scale Structures Based on Combining Optimality Criterion and Mathematical Programming Algorithms, Ed. B.H.V. Topping, Advances in Structural Engineering Optimization, Civil-Comp Press, Edinburgh, 1996.

[7] Stadler, W., `Preference optimality and applications of Pareto optimality`, Multi-Criteria Decision Making, G. Leitmann and Marzollo (Eds), *CISM Courses and Lectures* No. 211, Springer-Verlag, Wien and New York, 23, 1975.

[8] Stadler, W., Preference optimality in multi-criteria control and programming problems, Nonlinear Anal. Theory Methods, Appl., 4, No. 1, 51-65, 23, 1980.

[9] Baier, H., Mathematische programmierung zur Optimierung von Tragwerken insbesondere bei mehrfachen Zielen, Dissertation D17, Darmstadt, 23, 1978.

[10] Maksimovic, S., Zeljkovic, V., Multilevel Optimization Approach Applied to Aircraft Structures, *Computational Mechanics (CM'04)-The First International Conference*, November 15-17, 2004, Belgrade, Paper 55, 1-10. (Eds) Mijuca D and Maksimovic, S.

[11] Schmit, L. A. and Fleury, C., Discrete-Continuous Variables Structural Synthesis using Dual Methods, AIAA J, Vol. 18, 155-1524, 1980.

[12] Vanderplaats, G.N. and Thomas, H. L, an Improved Approximation for Stresses Constraints in Plate Structures, Structural Optimization, Vol. 6, No. 1, 1-7, 1993.

[13] Maksimovic, S., Optimal Design of Composite Structures by Finite Elements, Proc. Computer Aided Design in Composite Material Technology, held at Southampton, Eds. C. A. Brebbia, W.P de Wilde and W. R. Blain, Springer-Verlag Berlin, 1988.

[14] Hinton, E and Sienz, J., Aspects of Adaptive Finite Element Analysis and Structural Optimization, Advances in Structural Optimization, Eds B.H.V. Topping and M. Papadrakakis, Civil-Comp Press, 1994.

ACKNOWLEDGEMENTS: The research refering to material caracteristics and application are investigatied under the project: INVESTIGATION OF THE PROCESSES OF THE CONSOLIDATION OF MATERIALS, the Scientific Research Fund of the Serbien Academy of Sciences and Arts.