# Using Wavelet Transform on Suspended Particulate Matter Time Series

MOKHTAR SHAHARUDDIN Faculty of Civil Engineering Mara University of Technology Shah Alam MALAYSIA

AZAMI ZAHARIM Basic Engineering Unit National University of Malaysia 43600 Bangi MALAYSIA

MOHD. JAILANI MOHD. NOR Department of Mechanical and Materials Engineering National University of Malaysia 43600 Bangi MALAYSIA

### OTHMAN A. KARIM Department of Civil and Structural Engineering, Faculty of Engineering National University of Malaysia 43600 Bangi MALAYSIA

*Abstract:* - Malaysia has experienced several haze periods since early 1980s in which suspended particulate matter (SPM) was the major components. The temporal variations observed are the result of interactions of various meteorological variables and pollutants emissions in different time scale. In order to study the temporal variation, Morlet wavelet transform is applied to airborne particulate matter (PM10) daily data observed at Petaling Jaya monitoring station. The wavelet power spectrum shows most of the power is concentrated in 2-16 days band and 256 to 1024 days band. An increased in wavelet power as to the end of the time series is also evident in those scales.

Key- Words:- PM10, wavelet, wavelet power spectrum, global wavelet power, Morlet

## **1** Introduction

Haze episode in Malaysia has been recorded since early 1980s. During these episodes, national air quality guideline has been exceeded and airborne suspended particulate matter (SPM) was the main pollutant. Other pollutants were remained within standards [3]. Suspended particulate matter is of concern due to it adverse health effects, reducing visibility, socio economic effects and could deteriorate the image a country.

SPM temporal fluctuations are due to various processes of pollutant emissions and dilutions operated in different time scale such as intraday, diurnal, several days, seasonal, annual and long term [8]. Short term variations (intraday, diurnal, several days) may be due to local meteorological factors and changes in emission rate from sources such as traffic. Long term changes could be due to economic and policy factors which effect the emission of pollutant in a long run, thus may present nonstationary characteristics. In this paper, SPM with diameter less than 10µm (PM10) is chosen as it gained more concern due to its health effects [3].

Wavelet transform has been used in data analysis since its application in Geophysics in early 1980s [8]. Using wavelet transform, one can depicts the frequency content of a signal at every time of data [2]. Wavelet transform is used in this study as it can revealed the variances in a time series in their corresponding temporal scales and how they evolved in time.

Additional knowledge gained from wavelet analysis could enhance our understanding on the important of various factors that affecting air quality fluctuations and thus be used for air quality management.

### 2 **Problem Formulation**

PM10 time series consists of daily observation of PM10 concentration in microgram/m<sup>3</sup> ( $\mu$ g/m<sup>3</sup>) recorded by Malaysian Meteorological Department (MMS) from year August 2000 to December 2004. The station is located in a residential and commercial area of Petaling Jaya district which

located 15 km to the south-west of Kuala Lumpur. Petaling Jaya and Kuala Lumpur are situated in highly populated Klang Valley. The Klang Valley is surrounded by hills to the north, east and south, and this impedes the horizontal transport of surface generated pollutants. Hence, meteorological variables have a strong influence on the concentration and transport of PM10. These factors have resulted in the Klang Valley becoming the most polluted area in Malaysia [6]. The valley is also frequently subjected to stable weather inversions which effectively prevent the escape of pollutants by ventilating the atmosphere [1].

In this paper, Matlab codes and interactive wavelet developed and provided by Torrence and Compo, [7] is used. Morlet wavelet has been chosen as it has been used extensively in meteorology [eg. 5,7,9 and 10]. Fluctuations in PM10 observations are due to complex interaction of meteorological factors of different scales. Morlet is used for PM10 in this study to determine the dominant scale in the time series and how they change over time.

Morlet mother wavelet is Gaussian modulated of a plane complex wave. The different scales in a time series are estimated by translating and dilating the base wavelet.

The Morlet wavelet is given by

$$\psi(t) = \pi^{-1/4} e^{-i\omega 0t} e^{-t^2/2} \tag{1}$$

The wavelet function,  $\psi(t)$  depends on non dimensional time parameter, *t*. To be admissible as wavelet, the function must has zero mean and be localize in both time and frequency. Parameter  $\omega_0$  is the non dimensional frequency equal to 6 to satisfy admissibility condition [7].

The scaled wavelet is given by :

$$\psi\left[\frac{(n'-n)\delta t}{s}\right] = \left(\frac{\delta t}{s}\right)^{1/2} \psi \left(\frac{(n'-n)\delta t}{s}\right)$$
(2)

where changing *s* and *n* will dilate and translate the mother wavelet. Given a time series X, with values of  $x_n$ , each value is separated in time by time

interval  $\delta t$ . The wavelet transform is the result of convolution of the wavelet function with the original time series:

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \psi * \left[ \frac{(n'-n)\delta t}{s} \right]$$
(3)

where \* denotes complex conjugate.

The choice of scale based on fractional powers of two;

 $S_j = S_0 2^{j\delta j}$  j=0,1,2...,J, $J = \delta j^{-1} log(N\delta t/S_0),$ 

where  $S_0=2dt$  is the smallest resolvable scale and J is the largest scale. The letter N denotes the length of data,  $\delta t$  for sampling intervals and  $\delta j$  stands for spacing between the discrete scales. For Morlet wavelet,  $\delta j$  less than about 0.5 for adequate scale resolution [7].

The wavelet power spectrum is defined as the absolute valued squared of the wavelet transform and gives the measures of variance in the time series at each scale and time. The wavelet power spectrum is defined as  $|W_n(s)|^2$ . The (absolute value)<sup>2</sup> gives information of relative power at certain time and scale.

To examine the fluctuations in power over a range of particular scales, scale-average wavelet power is used which is the weighted sum of the wavelet power spectrum over scales  $S_1$  to  $S_2$  [7]:

$$\overline{W}_{n}^{2} = \frac{\delta j \delta t}{C_{\delta}} \sum_{j=j_{1}}^{j_{2}} \frac{|W_{n}(S_{j})|^{2}}{S_{j}}$$
(4)

The scale-average wavelet power can be considered as a time series of the average variance in a certain scale. The period-averaged wavelet power may be significant in a particular localized time interval only, even it is not significant in the global wavelet power. Details in wavelet analysis can be found in [4].

#### **3 Problem Solution**

An overall increased in PM10 concentration as well as it's fluctuation over time is evident from

the series (Figure 1a). Wavelet transform is applied to study the variation of power contained within the different scales. Due to the daily data used, the smallest resolvable fluctuation is two days.

The wavelet power spectrum exhibits more power concentrated in short term, annual and long term scale (Figure 1b). Power in short term scale exhibits changes as the number of significant peaks increased over time. This is later confirmed in scale-average wavelet power (Figure 2). Short term fluctuations are usually related to local weather regime which often changes within several days. Annual and inter-annual scale between 512 to 1024 days present an increasing in power and become significant from the second quarter of the series. Both scales has a similar peak of wavelet power. Larger scale usually associated with more regional meteorological factors and long term changes in pollutant emission [8].

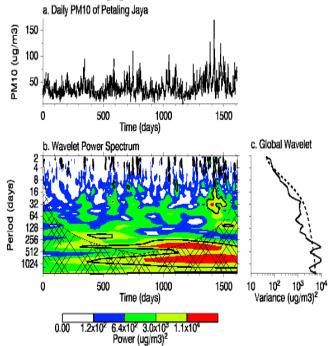


Figure 1: (a)PM10 daily observations at Petaling Jaya city from Aug 2000 to Dec.2004. (b) The wavelet power spectrum using Morlet. The contours levels show 75%, 50%, 25% and 5% of the wavelet power is above each level respectively. Black contour indicates 5% significant level. (c) global wavelet power spectrum. The dash line is 5% significance level for the global wavelet spectrum.

The black contour in wavelet power spectrum indicates 5% significant level using red noise as the background spectrum. The lag-1 autocorrelation,  $\alpha$  from the series is assumed to be 0.72.

The cross hatch region is the cone of influence due to zero padding. Padding the time series with zeroes decrease the amplitude near edges towards larger scale. Thus the peaks in this region show a lower power spectrum are unclear weather due to lower variance or from the effect of zero padding.

The significant peaks in wavelet power spectrum (Figure 1b) is confirmed by an integration of wavelet power over time (Figure 1c). The global wavelet power spectrum shows three significant peaks: 1) 2-8 day 2) 256-512 day and 3)512-1024 day. Power for seasonal scale less than annual is not significant in global wavelet power spectrum although there is a peak in wavelet power presented at the end of year 2004 (Figure 1b).

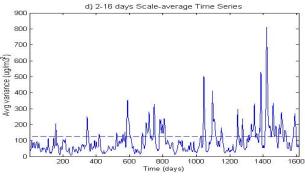


Figure 2. Scale-average wavelet power over 2-16 day band. The dash line indicates 95% confidence level assuming red noise  $\alpha = 0.72$ .

The changes in power for 2-16 day scale is further depicted in scale-average power spectrum (Figure 2). The Figure shows the average variance in the band scale of 2-16 day is increasing over time.

The null hypothesis is defined as assuming that the time series has a mean power spectrum. The 95% confidence interval implies that 5% of the wavelet power should be above the background spectrum.

In order to study the variance of PM10 at Petaling Jaya further, longer data is needed since

the wavelet power spectrum shows an increasing power in many scales.

#### Conclusions

Morlet wavelet is applied to PM10 daily observations in order to determine the dominant scale that influence it's temporal variability. During August 2000 to 2004, it is evident that wavelet power in many scales are in increasing trend. Wavelet power spectrum and global wavelet power spectrum show that PM10 variations are due to factors that have scale of 2-16 days and 256 to 512 days at 5% significant level.

#### Acknowledgement:

The writers wish to thank Dr. Christopher Torrence and Dr. Gilbert P. Compo for providing computer program for wavelet transform.

We also wish to thank Malaysian Meteorological Department for providing PM10 data used in this analysis.

References:

- [1]Abas, M.R.B., Simoneit, B.R.T., Composition of extractable organic matter of air particles from Malaysia: initial study. *Atmospheric Environment* 30, 1996, pp. 2779–2793.
- [2]Daubecies, I., The Wavelet Transform Timefrequency Localization and Signal Analysis, *IEEE Trans. Inform. Theory*, Vol 36, 1990, pp. 961-1004.
- [3]Department of Environment Malaysia, A Study to determine the Origin, Formation and Composition of Aerosol haze in Malaysia. Preliminary Report Sept. 1997.
- [4]Farge, M., Wavelet Transforms and their applications to turbulence. *Ann. Rev. Fluid Mech.*, Vol. 24, 1992, pp. 395-457.
- [5]Foufoula-Georgiou, E., and P. Kumar, Eds., *Wavelets in Geophysic,*. Academic Press, 1995.
- [6]Solieman,A., Othman,M., Samah,A.Z., Sulaiman,N.M., and Radojevic,M, The Occurance of Haze in Malaysia. A Case Study in an Urban Industrial Area. *Pure App. Geophys.*, Vol 160, pp. 221-238.
- [7]Torrence, C. and Compo, G.P.. A, Practical Guide to Wavelet in Analysis, *Bull. Amer. Meteor. Soc.*, Vol. 79, No 11, 1998, pp.61-78.
- [8]Wise,E.K. and Comrie,A.C.,Extending the Kolmogorov-Zurbenko Filter: Application to Ozone,

Particulate Matter and Meteorological Trends, *J. Air & Waste Management Associations*, Vol 55, 2005, pp. 1208-1216.

- [9]Morlet, J., G.Arens, E.Fourgeau, and D. Giard, Wave Propogation and Sampling Theory ,1, Complex Signal and Scattering in Multilayered Media, *Geophysics*, Vol. 47, No 2, 1982, pp. 203-221.
- [10]Santos, A.G., Galvao, Carlos d.O., Suzuki, K. and Trigo, R.M., Matsuyama City Rainfall Data Analysis Using Wavelet Transform, Annual Journal of Hydraulic Engineering, Vol 45, 2001.