Assessment of Excited Oscillation in Controller Parameter Setting

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Abstract: In industrial practice there is an ongoing search for methods that make controller parameter setting easier. Optimization procedures that do not require any mathematical theory are strongly preferred. The most popular existing methods are experimentally based. The classical Ziegler and Nichols method retains its popularity, but some knowledge of tuning is necessary in order to obtain good results in certain cases. The Relay Feedback method, which avoids critical controller setting by making temporary use of a relay in the control loop, has become popular nowadays. However, it is a pity to give up all the available theoretical methods, especially those based on linear control theory, simply due to reluctance to use mathematical modelling in industrial practice. Global optimization methods are an example of theoretically based controller setting that can be used with low requirements on the operator's knowledge. They can be implemented fully automatically without the need for human participation. Although that they were developed for linear models of control loops, they can provide satisfactory results when applied to real control loops. The optimal controller setting indicators that these methods offer, can be evaluated experimentally by additionally exciting the control loop by a sinusoidal signal whose amplitude does not greatly disrupt the controlled process but enables these experiments to be carried out on-line. Some of these evaluation methods are presented here with the motivation to overcome one of the possible disadvantages of the Relay Feedback method – a restricted control function during the identification phase, when critical oscillations are generated in the control loop. The idea underlying the investigation carried out here is to design an adaptive PID controller ready for operating in real conditions and making use of global optimal setting methods.

Keywords: PID controller tuning, critical oscillation, relay, phase margin, gain margin

1 Control performance assessments

There are many ways to set a PID controller in a control loop that is referred to be optimal. The first natural question is what we regard as optimal in control loop behaviour. In principle, the control quality can be assessed either from the course of the control loop output time responses, or, from some indicators of good control loop behaviour. The first, response-based approach requires the response to be excited, then measured (computed, simulated) and after that evaluated (i.e. by means of the control error area). The second group comprises global evaluation methods. These draw conclusions on optimal behaviour without depending on a specific response course. The control performance is assessed by means of various indicators (e.g. relative damping factor, phase margin, etc.), very often based on frequency response evaluation.

Choice of a global assessment method is influenced by the conditions under which auto-oscillations in the control loop can be evoked. These auto-oscillation conditions can be found solely from experiments, as in the case of Ziegler - Nichols tuning, or by a calculation from the frequency transfer functions. Classical linear control theory uses the Nyquist curve not only for finding the critical parameters of the controller important for the purposes of stability, but also for effective tuning based on a global approach. Oscillations in a control loop can be evoked not only by a critical controller parameter setting, but also by adding a nonlinear element – a relay in the feedback loop. The typical solution with a relay in a control feedback loop from which the controller has been temporary disconnected can be considered as a hybrid system consisting of a linear and a nonlinear part. Frequency analysis based on transfer of the first harmonic through this connection enables to determine the conditions when a state of self-sustained oscillation arises. Graphically expressed this state corresponds to the intersection of the Nyquist curve representing the linear part ($G_L(j\omega)$) with the negative inverse describing function representing the relay ($-1/N(A)$). Relay feedback estimation of the critical point characterizes the steady state oscillation with the ultimate frequency ω_π and the ultimate gain k_π

$$k_\pi = \frac{4h}{\pi A} \tag{1}$$

where $\pm h$ is the height of relay saturation and A is the amplitude of the oscillation. Knowing k_π and ω_π, the Ziegler and Nichols rules can be applied.

A great disadvantage of the relay method is that the control function is interrupted, while the critical parameter identifying process is performed. The controller must be disconnected and reconnected without any bump during the interval when a steady state is achieved. The amplitude of the oscillation added to the controlled variable can be influenced by the parameters of the relay, but it is difficult to forecast its size in advance. Excitation of the oscillation often requires changes from the manipulated variable that are easy to simulate but difficult to execute technically.

2 Frequency Response Based Indicators of Optimal Controller Setting

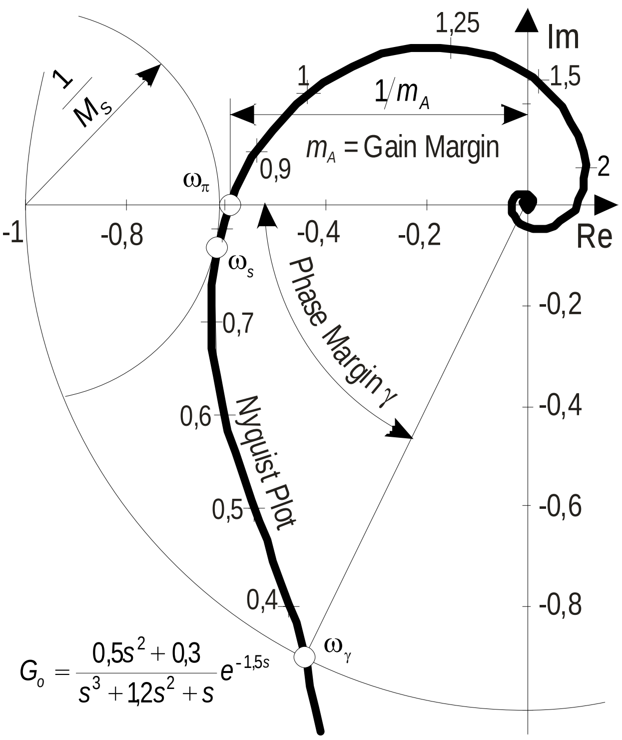

Fig. 1 Some indicators of optimal controller setting coming from the open loop frequency response (Nyquist plot)

Maximum Sensitivity

The open loop transfer function $G_o(s)$ (product of the controller function $G_R(s)$ and the controlled plant $G_S(s)$) enables the transfer function of the disturbance to be defined

$$G_{dy}(s) = \frac{1}{1+G_o(s)} \tag{2}$$

from which it follows that a (load) disturbance is transferred with maximum sensitivity M_s if in the sensitivity function $S(\mathrm{j}\omega) = G_{dy}(\mathrm{j}\omega)$ the absolute value of its denominator $|1+ G_o(\mathrm{j}\omega)|$ achieves its minimum. The reciprocal value $1/M_s$ is equal the length of the vector sum -1+j.0 and $|G_o(\mathrm{j}\omega)|$ and it corresponds to the radius of a circle with the centre in the critical point touching the Nyquist plot.

Gain Margin

The gain margin, denoted m_A, is the factor which, multiplying the amplitude of the Nyquist plot characterized by the phase angle $-\pi$, causes the plot to pass the critical point -1+0.j. This expresses how safe against stability loss the control loop is.

The recommended value of the gain margin ranges from 2 – 2,5.

Phase Margin

The phase margin, denoted γ, expresses the amount of phase shift that can be tolerated before the control loop becomes unstable. It is defined through the angle γ given in degrees (see Figure 1) appertained to the frequency ω_γ, sometimes known as the gain crossover frequency because this is the frequency at which the loop gain is one (the Nyquist plot passes the unit circle).

Recommended values of the optimal phase margin are quoted in the range from 30° to 60°, but according to our own experience a higher upper limit is usually more suitable.

The following relationships between gain and phase margin and the value of maximum sensitivity have been derived by Skogestad and Postlethwaite

$$m_A \geq \frac{M_S}{M_S - 1} \quad \gamma \geq 2\arcsin\left(\frac{1}{2M_S}\right) > \frac{1}{M_S} \tag{3}$$

3 Evaluation of Excited Frequency Responses

To evaluate the frequency response based indicators of an optimal controller setting, two principles can be used:

- phase-locked loop (PLL) identifier module
- direct frequency response assessment.

Phase-locked method

Phase-looked loops have been used for a long time in FM radio receivers. Their application for identifying the gain and the phase angle in control loops is quite new. It can be explained as follows.

Let us consider a dynamical system, e.g. a control loop, whose transfer properties for certain frequency can be characterized by the frequency transfer function $G(j\omega)$. In a steady state, the transfer of a frequency signal is described by the magnitude gain $M(\omega) = |G(j\omega)|$ and by the phase angle $\varphi(\omega)$ = $\arg(G(j\omega))$ even when $G(j\omega)$ cannot be expressed analytically due to control loop nonlinearities. Both data items can be identified by the PPL identifier module whose block scheme is depicted in Fig. and its function is based on an assessment of the product of two harmonic signals. By means an oscillator, whose frequency is controlled by an external signal, two signals are generated:

$$u_1(t) = a\cos\omega t \quad u_2(t) = b\sin\omega t \tag{4}$$

The first signal $u_1(t)$ is used to excite the dynamic system (e.g. the control loop) and then as an output of the system (after amplitude and phase changes have come out), the output is brought to the multiplier whose second input is the directly brought signal $u_2(t)$. The steady state output from the multiplier is described by the formula

$$\begin{aligned} &G\{u_1(t)\}u_2(t) = M(\omega)a\cos(\omega t + \varphi(\omega))b\sin\omega t = \\ &abM(\omega)\left[\cos\omega t\sin\omega t\cos\varphi(\omega) - \sin^2\omega t\sin\varphi(\omega)\right] \end{aligned} \tag{5}$$

The expression in parentheses can be rearranged

$$f(\omega t) =$$

$$\frac{2\sin\omega t\cos\varphi(\omega) - (1-\cos^2\omega t)\sin\varphi(\omega) - \sin^2\omega t\sin\varphi(\omega)}{2}$$

$$f(\omega t) = \frac{2\sin\omega t\cos\varphi(\omega) + \cos 2\omega t\sin\varphi(\omega) - \sin\varphi(\omega)}{2}$$

$$f(\omega t) = \frac{\sin(2\omega t + \sin\varphi(\omega)) - \sin\varphi(\omega)}{2} \tag{6}$$

The derivative function of $f(\omega t)$ yields turning points at

$$2\omega t + \varphi(\omega) = \left\langle \begin{array}{l} \dfrac{\pi}{2} + 2k\pi \\ \\ \dfrac{3}{2}\pi + 2k\pi \end{array} \right. \qquad k = 0,1,2... \tag{7}$$

Using the second derivative of the function, it can be confirmed that the maxima occurring at

$$t_{\max} = \frac{\pi/2 + 2k\pi - \varphi(\omega)}{\omega}$$

the dynamical system output values are

$$y_{\max} = abM(\omega)f(\omega t_{\max}) = abM(\omega)\frac{1-\sin\varphi(\omega)}{2} \tag{8}$$

and the minima

$$y_{\min} = abM(\omega)f(\omega t_{\min}) = -abM(\omega)\frac{1+\sin\varphi(\omega)}{2} \tag{9}$$

at the time

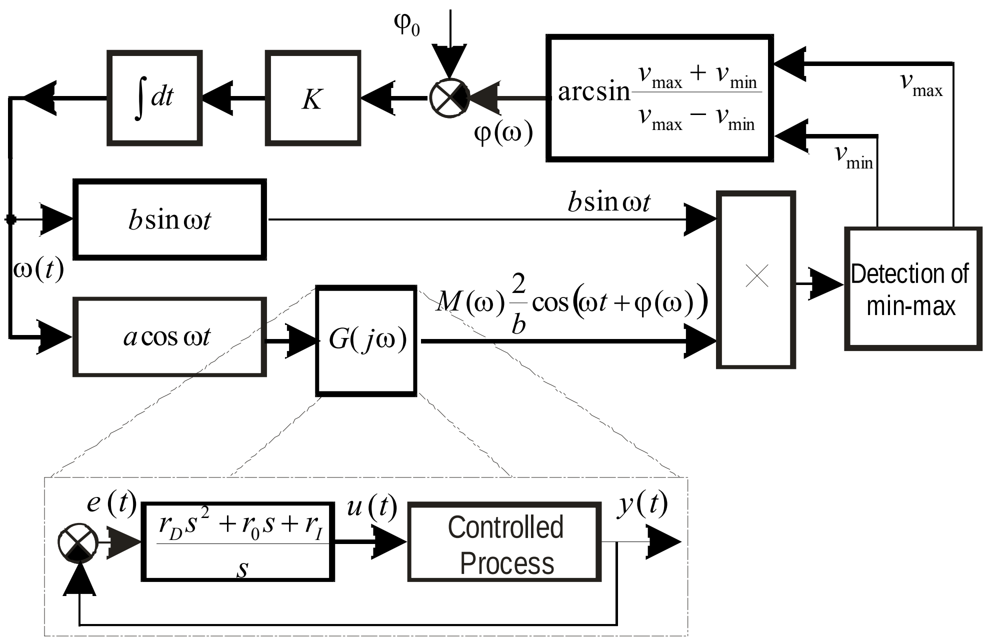

Fig. 2 Block scheme of phase angle identification via PPL method

$$t_{\max} = \frac{\pi/2 + 2k\pi - \varphi(\omega)}{\omega}$$

From these two values $y_{\min}$, $y_{\max}$ we can determine the magnitude $M(\omega)$ in a very simply way by subtracting the two output extreme values, especially if a convenient choice for the amplitude $a = 2/b$ is made

$$M(\omega) = \frac{y_{\max} - y_{\min}}{2} \tag{10}$$

The phase angle results from adding $y_{\min}$, $y_{\max}$ (again when $a = 2/b$)

$$\varphi(\omega) = \arcsin\frac{y_{\max} + y_{\min}}{y_{\max} - y_{\min}} \tag{11}$$

The block scheme in Fig. 2 displays a connection for the case when an automatic search for the frequency at which the desired value of the phase angle is achieved. The velocity of finding this frequency depends on constant *K,* which also ensures conversion of the dimensions. A search can be carried out for the frequency at which the phase angle is fixed on the value $-\pi$, and the achieved margin gain is provided by a practically identical scheme. The gain margin is calculated from the limit values $v_{\min}$, $v_{\max}$ using formula (10).

Direct frequency response assessment

Although the idea is the same – to add an exciting harmonic signal to the control loop from an oscillator with controllable frequency - the assessment procedure is different. By analogy with the previous case, a steady state of oscillation must be reached. Then we can compute the amplitude ratio and the time shift from recorded values of the control error $e(t)$ (these contain the exciting sinusoidal signal) and the controlled (process) variable $y(t)$. The basic idea is to perform the same operations as are done when a Nyquist plot is experimentally determined, but disconnecting the closed loop. Then, some of the indicators of globally considered optimal behaviour, specifically the phase margin, or the margin gain, can be obtained without difficulty from the measurement of the control variable $y(t)$ and the error $e(t)$.

The amplitude of the added sinusoidal signal can easily be checked, in order not to exceed the admissible control tolerance.

The proposed algorithm for the time and consequently for phase shift evaluation does not require any frequency transfer function model. The signals can be taken from real control loops, even from those controlling processes with nonlinear properties. Simultaneously, it provides information about the period T, the gain A for frequency ω. The desired phase margin γ can be defined as a phase angle

$$\varphi = -180° + \gamma \tag{12}$$

The complete arrangement of a system for direct frequency response assessment connected to a control loop whose controller (PI) parameters should be set according to the selected indicator of optimal behaviour is depicted in Fig. 3 . The block called the Harmonic Signal Generator in this scheme provides sine waves with frequency ω. The required value ω.

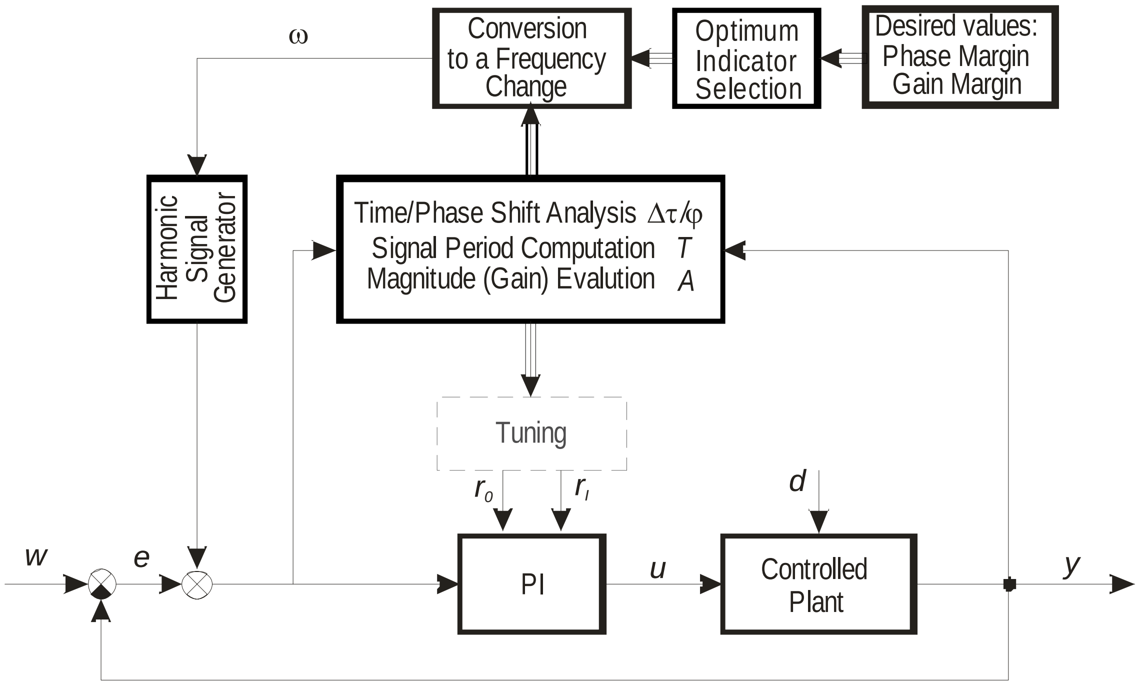

Fig. 3 PI control loop with a block scheme of the excitation by harmonic signal utilized in evaluation and controller optimal parameter tuning

of the frequency is passed to this generator from a block performing conversion to frequency changes as a result of frequency response evaluation. The processing part of this evaluating block contains a signal analyser which is used for obtaining the properties of the signals brought to the block and for evaluating and computing their mutual relations (gain, phase shift angle, amplitude and period of oscillation).

In the block called Conversion to a Frequency Change, the information obtained from the signal analysis is used for computing a new value of the frequency passed then to the Signal Generator. The Tuning block represents all kinds of changes in the controller parameter setting. In this representation, no account is taken of whether the changes are made manually (as in the results presented in this paper) or automatically (as a part of the intended autotuning function).

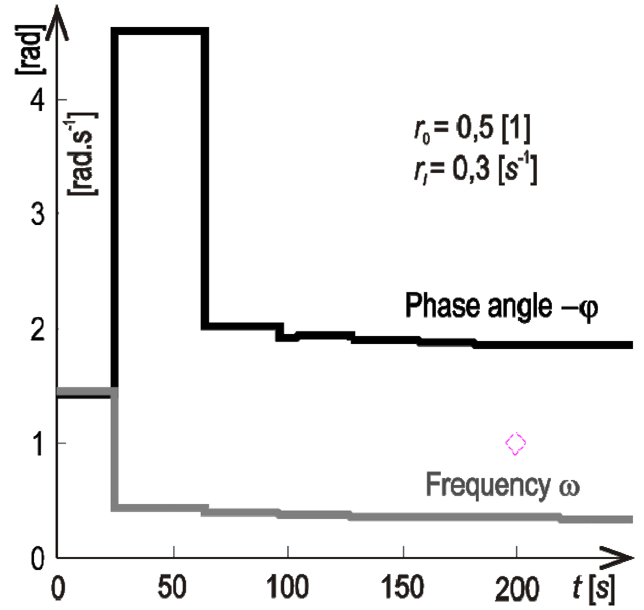

Fig. 4 The phase angle changes development process in a search of the frequency of the added exciting signal leading to the desired value of phase angle

Fig. 4 depicts two courses. One represents changes of the phase angle caused by changes in the exciting signal frequency ω depicted below the first course. This figure demonstrates the speed and convergence in finding a requested value of the phase margin.

Fig. 5 shows the gains (absolute values of the open loop transfer function) for different pairs of PI controller settings. Gain value one signals that the requested phase margin has been achieved. The requested phase margin can be achieved only for those pairs of controller parameter settings represented by the fat curve (Gain 1) in Fig. 5 . Although all of them guarantee that the required phase margin is achieved, the control responses may be quite different for each of the marked frequencies ω.

The proposed algorithm computes these indicators simultaneously. They can be exploited in tuning based on multicriterial evaluation.

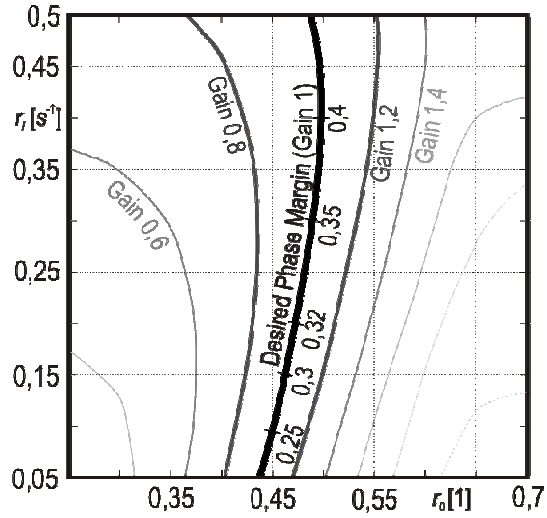

Fig. 5 Mapping of PI parameter pairs with the same open loop gain

4 Conclusion

This paper has introduced a method using a new mechanism for parameter evaluation of periodic signals. It can be used in PID controller tuning, where the achieved optimum is considered from a global viewpoint and not from the course of the response. Several indicators of optimal controller setting known from classical linear control theory can be exploited by this algorithm, even if the controlled plant is nonlinear. This enables connection to a real control loop or to a nonlinear control loop model without the need to have any mathematical models of the controlled process.

In practical implementation, it is assumed that the connection to the control loop will be done on request, whenever the operator feels that a new check of the controller setting is necessary.

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References

[1] Aström, K.J. and Hägglund, T., *PID Controllers. Theory, Design and Tuning*. ISA, Research Triangle Park, 1995,NC, USA.

[2] Šulc, B. and Vítečková M., *Teorie a praxe návrhu regulačních obvodů*. (Monograph in

Czech). Praha 2004: Vydavetelství ČVUT v Praze. ISBN 80-01-03007-5.

[3] Tan, K.K., Wang, Q.G. and Hang, C.C. with T. Hägglund, *Advances in PID Control.* Springer Verlag, London, 1999.

[4] Šulc, B., *Autotuned PI Level Control in a Two-Tank-Cascade Model with Sliding Control Error*. WSEAS Transactions on Circuits and Systems, 4, No 9, pp. 1077-1085. WSEAS Press 2005.

[5] Hlava, J., Šulc, B., Tamáš, J., *A Laboratory Scale Plant with Hybrid Dynamics and Remote Access via Internet for Control Engineering Education.* Preprints of the 16th World Congress of the International Federation of Automatic Control.

[6] S. Kowalewski et al., A Case Study in Tool-Aided Analysis of Discretely Controlled Continuous Systems: The Two Tanks Problem. Lecture Notes in Computer Science, Vol.1567, 1999, pp. 163-185.

[7] Clarke, D.W. and J.W. Park, *Phase-locked Loops for Plant Tuning and Monitoring.* IEE Proc. Control Theory and Applications, 150(1), pp.155-169, 2003.

[8] Šulc, B. and P. Neuman, *Engineering Models of a Coal Fired Steam Boiler in Teaching, Training and Research Applications.* ACE 2000, 2000, Gold Coast, Australia.