Numerical Computation of Flows Field Caused by Vortices Chain

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Abstract: - In this paper we calculate the velocity field and distribution of stream function for ideal incompressible fluid, induced by a different system of vortex threads in a finite cylinder, in a finite frustum of the cone and in a channel. An original method was used to calculate the components of the velocity vectors. Such a procedure allows us to calculate the velocity fields inside the domain depending on the arrangement, on the intensity and on the radius of circular vortex lines. In this paper we have developed the first mathematical model for the process in the element of Hurricane Energy Transformer. This element is central figure in so called RKA (ReaktionsKraftAnlage) used on the cars' roof.

Key-Words: -incompressible fluid, finite cylinder, finite cone, channel, velocity field, vector potential, circular vortexes, spiral vortex threads, vortexes chain.

1 Introduction

In new technological applications it is important to use vortex distributions in area for obtaining large values of velocity. The effective use of vortex energy in production of strong velocity fields by different device is one of the modern areas of applications, developed during the last decade. Such processes are ecologically clean; there is no environment pollution. Although, on the other hand the aspect of energy is very important: the transformation process should be organized in such way that vortex energy is effectively transformed into heat or mechanical energy. In our previous papers [1]-[3] we have mathematically modeled the process how to transform the alternating electrical current into heat energy.

The goal of this paper is to develop the mathematical models for new type of ecologically clean and energetically effective devices [4]-[6]. Such type of devices firstly was developed by I. Rechenberg [9]. Now the continuator of the work is one of authors J. Schatz. The devices of such type can be considered as the energy source of the new generation. The practical aim of our investigation is to try to understand the process in the element of Hurricane Energy Transformer [4]. This element is central figure in so called RKA (ReaktionsKraftAnlage) used on the cars' roof for substantial reducing the airs' drag.

2 General Mathematical Formulation of the Problem

2.1 Description of Geometry

We will investigate the flow of incompressible fluid in finite circular frustum of cone $\Omega_{r,z}(\varepsilon)=\{(r,z,\varphi):\ 0<r<a-\varepsilon z,\ 0<z<Z,$ $0<\varphi<2\pi(M+1)\}$, with the parameter ε under the condition: $0\le\varepsilon Z<a$. The cone transforms to circular cylinder with the radius a for $\varepsilon=0$. Parameter M gives the number of circulation periods.

We will start with some geometrical descriptions of placement of the vortexes. We will consider the situation, when N discrete circular vortexes L_i, where $L_i=\{(r,z):r=a_i,z=z_i\},i=\overline{1,N}$ with intensity $\Gamma_i\left(\frac{m^2}{s}\right)$ and radii $a_i\left(m\right)$ are placed in the cylinder. The system of circular vortexes creates the radial v_r and axial v_z components of the velocity field in ideal incompressible liquid.

Similarly can be considered the system of N discrete spiral vortex threads ($i=\overline{1,N}$) $S_i=\{(r,z,\varphi):r=a-\varepsilon t,z=a\tau t,\varphi=t+i\delta\}$ with parameters $\delta = \dfrac{2\pi}{N}, \tau = \dfrac{Z}{2\pi a M}, t \in [0, 2\pi M]$. The argument φ fulfills the following enclosure: $\varphi \in \left[\dfrac{2\pi}{N}, 2\pi(M+1)\right]$. Parameter τ gives the rise (step) of the vortex threads. The system of vortex threads creates the radial v_r, axial v_z and azimuthal v_φ components of the velocity field in ideal incompressible liquid.

Unlike our previous papers [5], [6] here we additionally consider the chain of linear vortexes lines in the plane channel $\Omega_{x,y} = \{(x,y): x \in [0,L], y \in [0,2], z \in R\}$. The vortices chain creates the v_x, v_y components of the velocity field.

The main goal of this work is to analyze how different displacements of the vortices influence the maximal value on the velocity field.

2.2 Mathematical Statement of the Problem

The vortex motion of ideal incompressible fluid will be determined from the equations for the vector potential A [5]-[8]

$$\begin{cases} div\, v = 0, \\ rot\, v = \Omega. \end{cases} \tag{1}$$

in following form:

$$\Delta A = -\Omega. \tag{2}$$

Here

$$v = rot\, A \tag{3}$$

and v, Ω are the vectors of velocity and vortex fields and Δ is the Laplace operator.

3 The Description of the Problem

It is well known that if we replace the velocity vector v with the magnet field induction vector B and the vortex vector Ω with the electrical current vector j then the system of equations (1) is identical with steady-state Maxwell's equations. Here we will apply our mathematical investigations to the vortexes influence on the distribution of velocity field.

3.1 Solution for the Frustum of the Cone

Applying the Biot-Savart law [7], [8] we receive the following form of representation for the vector potential created by the vortex thread $W_i = S_i$ or circular vortex $W_i = L_i$:

$$A(P)_i = \frac{\Gamma_i}{4\pi} \int_{W_i} \frac{dl}{R(NP)_i}. \tag{4}$$

Here dl is an element of the curve W_i, $P = P(x,y,z)$ ($N = N(\xi, \eta, \varsigma) \in W_i$) is the fixed point (the integration point) in the fluid and

$$R(NP)_i = R_i = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\varsigma)^2}. \tag{5}$$

For the **spiral** vortex $W_i = S_i$ we have expressions:

$$\xi = a_*(t)\cos(t + i\delta), \eta = a_*(t)\sin(t + i\delta),$$
$$\varsigma = bt. \tag{6}$$

Here $a_*(t) = a - \varepsilon t$, $b = a\tau$, $t \in [0, 2\pi M]$.

This gives following expressions for the components of vector potential:

$$A_{x,i} = \frac{\Gamma_i}{4\pi} \int_{S_i} \frac{d\xi}{R_i}, A_{y,i} = \frac{\Gamma_i}{4\pi} \int_{S_i} \frac{d\eta}{R_i}, A_{z,i} = \frac{\Gamma_i}{4\pi} \int_{S_i} \frac{d\varsigma}{R_i}.$$

Therefore finally [5]

$$A_{x,i} = -\frac{\Gamma_i}{4\pi} \int_0^{2\pi M} \frac{a_*(t)\sin(t + i\delta) + \varepsilon\cos(t + i\delta)}{R_i} dt,$$

$$A_{y,i} = \frac{\Gamma_i}{4\pi} \int_0^{2\pi M} \frac{a_*(t)\cos(t + i\delta) - \varepsilon\sin(t + i\delta)}{R_i} dt,$$

$$A_{z,i} = \frac{\Gamma_i b}{4\pi} \int_0^{2\pi M} \frac{dt}{R_i}.$$

In accordance with formulae (3) we have following expressions for the components of velocity field:

$$\begin{cases} v_{r,i} = -\dfrac{\partial A_{\varphi,i}}{\partial z} + \dfrac{\partial A_{z,i}}{r\partial\varphi}, \\ v_{z,i} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(rA_{\varphi,i}\right) - \dfrac{1}{r}\dfrac{\partial A_{r,i}}{\partial\varphi}, \\ v_{\varphi,i} = \dfrac{\partial A_{r,i}}{\partial z} - \dfrac{\partial A_{z,i}}{\partial r}. \end{cases} \tag{7}$$

It gives following expressions for last two components of vector potential:

$$A_{r,i} = \frac{\Gamma_i}{4\pi} \int_0^{2\pi M} \frac{a_*(t)\sin(\psi(t)) - \varepsilon\cos(\psi(t))}{R_i} dt,$$

$$A_{\varphi,i} = \frac{\Gamma_i}{4\pi} \int_0^{2\pi M} \frac{a_*(t)\cos(\psi(t)) + \varepsilon\sin(\psi(t))}{R_i} dt.$$

Here was used short notation $\psi(t) = \varphi - t - i\delta$.

This gives following representations for the components of the velocity field:

$$v_{r,i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}[(z-bt)(a_*(t)\cos(\psi(t)) \tag{8}$$

$$+\varepsilon\sin(\psi(t))) - ba_*(t)\sin(\psi(t))]\frac{dt}{R_i^3},$$

$$v_{z,i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}[a_*(t)(a_*(t)- \tag{9}$$

$$r\cos(\psi(t))) - \varepsilon r\sin(\psi(t))]\frac{dt}{R_i^3},$$

$$v_{\varphi,i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}[b(r-a_*(t)\cos(\psi(t)))- \tag{10}$$

$$(z-bt)(a_*(t)\sin(\psi(t))+\varepsilon\cos(\psi(t)))]\frac{dt}{R_i^3}.$$

The $v_{z,x}$ on the axis is as follows [5], [6]:

$$v_{z,i}(0,z) = \frac{\Gamma_i\varepsilon^2}{4\pi}\int_{a-2\pi M\varepsilon}^{a}\frac{q^2dq}{R(q)^3}.$$

Here

$$R(q) = \sqrt{a_1 + b_1 q + c_1 q^2}, a_1 = b^2 z_0^2,$$

$$b_1 = -2b^2 z_0, c_1 = \varepsilon^2 + b^2, z_0 = a - \frac{z\varepsilon}{b}.$$

This integral can be written in closed form:

$$v_{z,i}(0,z) = \frac{\Gamma_i}{4\pi c_1}[\frac{d_2 a_2 - 2a_1 b_1}{d_1 R(a_2)} - \frac{d_2 a - 2a_1 b_1}{d_1 R(a)} - \tag{11}$$

$$\frac{\varepsilon^2}{\sqrt{c_1}}\ln\frac{\sqrt{c_1}R(a_2) + c_1 a_2 + b_1/2}{\sqrt{c_1}R(a) + c_1 a + b_1/2},$$

$$a_2 = a - 2\pi\varepsilon M, d_1 = 4b^2 z_0^2, d_2 = d_1(\varepsilon^2 - b^2).$$

3.2 Solution for the Cylinder

Now we will concentrate our attention on the case of the circular cylinder ($\varepsilon = 0$) [5].

For the cylindrical coordinates we have:

$$x = r\cos\varphi,\ y = r\sin\varphi,\ z = z_i. \tag{12}$$

It is easy to proof that for the cylinder with the radius a all components of velocity are even functions according to middle point $z = \frac{Z}{2}$ of the cylinder, i.e.:

$$v_i(r, \frac{Z}{2} - z, \varphi) = v_i(r, \frac{Z}{2} + z, \varphi). \tag{13}$$

The representations for the components of vector potential in case of cylinder take a simplified form:

$$A_{x,i} = -\frac{\Gamma_i a}{4\pi}\int_0^{2\pi M}\frac{\sin(t+i\delta)}{R_i}dt,$$

$$A_{y,i} = \frac{\Gamma_i a}{4\pi}\int_0^{2\pi M}\frac{\cos(t+i\delta)}{R_i}dt,$$

$$A_{z,i} = \frac{\Gamma_i b}{4\pi}\int_0^{2\pi M}\frac{dt}{R_i}.$$

Respectively, both components of the vector potential in cylindrical coordinates simplify:

$$A_{r,i} = \frac{\Gamma_i a}{4\pi}\int_0^{2\pi M}\frac{\sin(\psi(t))}{R_i}dt,$$

$$A_{\varphi,i} = \frac{\Gamma_i a}{4\pi}\int_0^{2\pi M}\frac{\cos(\psi(t))}{R_i}dt.$$

The components of the velocity field now look as follows:

$$v_{r,i}(r,z,\varphi) = \frac{\Gamma_i a}{4\pi}\times \tag{14}$$

$$\int_0^{2\pi M}[(z-bt)\cos(\psi(t)) - b\sin(\psi(t))]\frac{dt}{R_i^3},$$

$$v_{z,i}(r,z,\varphi) = \frac{\Gamma_i a}{4\pi}\times \tag{15}$$

$$\int_0^{2\pi M}[a - r\cos(\psi(t))]\frac{dt}{R_i^3},$$

$$v_{\varphi,i}(r,z,\varphi) = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}[b(r-a\cos(\psi(t)) \tag{16}$$

$$-a(z-bt)\sin(\psi(t))]\frac{dt}{R_i^3}.$$

On the axis of the cylinder, the second component (15) of velocity reduced to simple expression in closed form [5], [6]:

$$v_{z,i}(0,z) = \frac{\Gamma_i M}{2Z}\left[\frac{z}{\sqrt{a^2+z^2}} + \frac{Z-z}{\sqrt{a^2+(Z-z)^2}}\right].$$

This function takes its maximal value in middle point of cylinder axis $z = Z/2$ [6]:

$$v_{z,i}(0, Z/2) = \frac{\Gamma_i M}{2a\sqrt{1+(Z/(2a))^2}}. \tag{17}$$

We obtain the minimal values of the z component of the velocity in two end points of cylinder axis:

$$v_{z,i}(0,0) = v_{z,i}(0,Z) = \frac{\Gamma_i M}{2a\sqrt{1+(Z/a)^2}}. \tag{18}$$

The integral averaged value of the axial velocity component is equal to:

$$v_{av,i} = \frac{\Gamma_i M}{2a} \frac{2}{1+\sqrt{1+(Z/a)^2}} . \tag{19}$$

The whole solution can be written now as the sum of separate vortexes:

$$\begin{aligned} v_r(r,z,\varphi) &= \sum_{i=1}^{N} v_{r,i}(r,z,\varphi), \\ v_z(r,z,\varphi) &= \sum_{i=1}^{N} v_{z,i}(r,z,\varphi), \\ v_\varphi(r,z,\varphi) &= \sum_{i=1}^{N} v_{\varphi,i}(r,z,\varphi), \\ A_\varphi(r,z,\varphi) &= \sum_{i=1}^{N} A_{\varphi,i}(r,z,\varphi). \end{aligned} \tag{20}$$

In general case we calculated all needed integrals with the trapezoid formulas.

In case of circular vortex we have following expressions instead of (6):

$$\xi = a_i \cos\alpha, \eta = a_i \sin\alpha, \varsigma = z_i.$$

Therefore

$$A_{z,i} = 0. \tag{21}$$

The circular vortex originate axially-symmetric conditions; at $\varphi = 0$ we have

$$A_{x,i} = 0. \tag{22}$$

Then it follows that

$$A_{y,i} = A_{\varphi,i} = A_i(z,r) = \frac{\Gamma_i a_i}{4\pi} I_i ,$$

where

$$I_i = \int_0^{2\pi} \frac{\cos\alpha d\alpha}{\sqrt{(z-z_i)^2 + a_i^2 + r^2 - 2a_i r\cos\alpha}} . \tag{23}$$

We have:

$$I_i = \int_0^{\pi/2} \frac{(1-2\sin^2 t)dt}{\sqrt{(z-z_i)^2+(r+a_i)^2}\sqrt{1-k_i^2\sin^2 t}} =$$

$$\frac{2}{\sqrt{ra_i}}\left[\left(\frac{2}{k_i}-k_i\right)K(k_i) - \frac{2}{k_i}E(k_i)\right].$$

Here

$$t = (\alpha-\pi)/2 ,$$

$$k_i = 2\sqrt{ar}/c_i , \tag{24}$$

$$c_i = \sqrt{(z-z_i)^2+(r+a_i)^2} . \tag{25}$$

Further $K(k)$ and $E(k)$ are the total elliptical integral of first, respectively second kind:

$$K(k) = \int_0^{\pi/2} \frac{dt}{\sqrt{1-k^2\sin^2 t}},$$

$$E(k) = \int_0^{\pi/2} \sqrt{1-k^2\sin^2 t}dt.$$

Therefore the azimuthal component of vector potential A_i induced by the circular vortex L_i is:

$$A_i(r,z) = \frac{\Gamma_i}{2\pi}\sqrt{\frac{a_i}{r}}F(k_i) . \tag{26}$$

Here

$$F(k_i) = \left[\left(\frac{2}{k_i}-k_i\right)K(k_i) - \frac{2}{k_i}E(k_i)\right]. \tag{27}$$

The two non-zero components of the velocity field for the circular vortex according the formulas (7) reduce to expressions:

$$v_{r,i} = -\frac{\partial A_i}{\partial z},\ v_{z,i} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_i\right). \tag{28}$$

Finally we have:

$$v_{r,i}(r,z) = \frac{\Gamma_i}{2\pi r}\frac{z-z_i}{c_i}\times$$
$$\left[E(k_i)\frac{a_i^2+r^2+(z-z_i)^2}{(a_i-r)^2+(z-z_i)^2} - K(k_i)\right], \tag{29}$$

$$v_{z,i}(r,z) = \frac{\Gamma_i}{2\pi c_i}\times$$
$$\left[K(k_i) - \frac{a_i^2-r^2-(z-z_i)^2}{(a_i-r)^2+(z-z_i)^2}E(k_i)\right]. \tag{30}$$

We have on the axis of the cylinder:

$$v_{z,i}(0,z) = \frac{\Gamma_i a_i^2}{2\left[a_i^2+(z-z_i)^2\right]^{3/2}} .$$

This component of the velocity has the maximal value by $z = z_i, a_i = a$ on the axis and it is as follows:

$$v_{z,i}(0,z_i) = \frac{\Gamma_i}{2a} .$$

In the middle point of the z axis we obtain the value (for $a = a_i$):

$$v_{z,i}(0,Z/2) = \frac{\Gamma_i}{D\left[1+(Z-2z_i)^2/D^2\right]^{3/2}} .$$

Here $D = 2a$ is the diameter of the cylinder. For the integral averaged value of the axial velocity component we have following formula:

$$v_{av,i} = \frac{\Gamma_i a}{DZ} \times$$
$$\left[\frac{(Z - z_i)/a}{\sqrt{1 + ((Z - z_i)/a)^2}} + \frac{z_i/a}{\sqrt{1 + (z_i/a)^2}} \right]. \quad (31)$$

From here we have in the middle point $z_i = Z/2$:

$$v_{av,i} = \frac{\Gamma_i}{D} \frac{1}{\sqrt{1 + (Z/D)^2}}.$$

The total velocity field of all the circular vortexes and the vector potential A_φ we have as the sum:

$$v_r(r,z) = \sum_{i=1}^{N} v_{r,i}(r,z),$$
$$v_z(r,z) = \sum_{i=1}^{N} v_{z,i}(r,z),$$
$$A_\varphi(r,z) = \sum_{i=1}^{N} A_{\varphi,i}(r,z).$$

The hydrodynamic stream function $\psi = \psi(r,z)$ is given by relations:

$$v_r = -\frac{1}{r}\frac{\partial \psi}{\partial z}, v_z = \frac{1}{r}\frac{\partial \psi}{\partial r}.$$

Then we have from (28):

$$\psi(r,z) = r A_\varphi(r,z). \quad (32)$$

Important attribute of the process is the amount Q of substance which flows through the cross section $[z = z_0, 0 \le r \le a_0]$ of the cylinder, which is given by the integral:

$$Q(a_0, z_0) = \int_0^{2\pi} d\varphi \int_0^{a_0} v_z(r, z_0) r dr.$$

It is very easy to calculate the quantity:

$$Q(a_0, z_0) = 2\pi a_0 A_\varphi(a_0, z_0) = 2\pi \psi(a_0, z_0). \quad (33)$$

Then the amount Q_T of substance which flows trough the whole cylindrical domain is equal to:

$$Q_T(a_0) = \int_0^{Z} Q(a_0, z) dz = 2\pi \int_0^{Z} \psi(a_0, z) dz. \quad (34)$$

Proposed method allows calculating the velocity field for arbitrary number and location of circular vortexes or vortex threads in a finite cylinder. This approach is different from the usual methods [10].

3.3 Solution for the Channel

For this geometry we assume the symmetry condition in the middle of the channel:

$$\left. \frac{\partial v_x}{\partial x} \right|_{y=1} = 0$$

and formulate the slip-conditions for the velocity vectors on the line $y = 0$:

$$v_x\big|_{y=0} = v_y\big|_{y=0} = 0.$$

The flow in the channel is given by prescribed fixed amount of flow trough cross section of the channel:

$$Q = \int_0^1 v_x\big|_{x=0} dy. \quad (35)$$

We note that for $L = \infty$ we have Poiseuille flow with

$$v_x = u(y) = 3Q(y - 0.5y^2), v_y = 0. \quad (36)$$

On the wall $y = 0$ of the channel we placed linear vortexes chain with the axis orthogonal to the (x, y) plane. One linear vortex line in the point with coordinates (x_k, y_k) creates following velocity field:

$$v_{x,k} = \frac{\Gamma_k}{2\pi} \frac{y - y_k}{R^2}, v_{y,k} = \frac{\Gamma_k}{2\pi} \frac{x - x_k}{R^2},$$
$$R^2 = (x - x_k)^2 + (y - y_k)^2. \quad (37)$$

Because of infinity of velocity in the centre of point-wise vortex we consider the vortex line as circle with radii a. In such situation the formulae (37) are valid for $R \ge a$, but for $R < a$ we must use following expressions:

$$v_{x,k} = \frac{\Gamma_k}{2\pi} \frac{y - y_k}{a^2}, v_{y,k} = \frac{\Gamma_k}{2\pi} \frac{x - x_k}{a^2}. \quad (38)$$

The total velocity field of all the linear vortex lines we have as the sum:

$$v_x(x,y) = \sum_{k=1}^{N} v_{x,k}(x,y),$$
$$v_y(r,z) = \sum_{k=1}^{N} v_{y,k}(x,y).$$

3 Some Results of the Computations

We investigate the influence of 6 circular vortex lines in finite cylinder which are arranged in the axial direction at the fixed points $z_i = 0.2i, r_i = a_i, i = \overline{1,N}$. In fig. 1 circular vortices are placed in points $a_i = a = 1$; in fig. 2 – in points $a_i = c_i a$, $c_i = [0.75, 0.8, 0.85, 0.9, 0.95, 1.0]$.

We scaled all lengths to inlet radius of the tube $r_0 = a$, the axial and the radial components of velocity were scaled to $v_0 = \frac{\Gamma_0}{2\pi r_0}$. The results of numerical experiments are given for dimensionless values

$$Q_t = \frac{Q_T}{\psi_0 r_0}, \psi_0 = A_0 r_0, A_0 = \frac{\Gamma_0}{2\pi}.$$

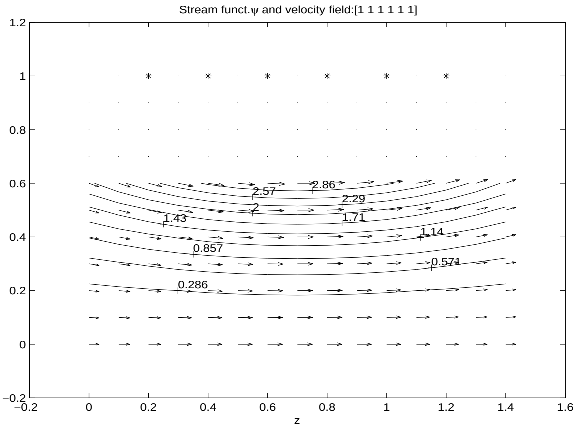

Figure 1. Distribution of the stream function and velocity; $v_{z,\max} = 16.21, Q_t = 25.12.$

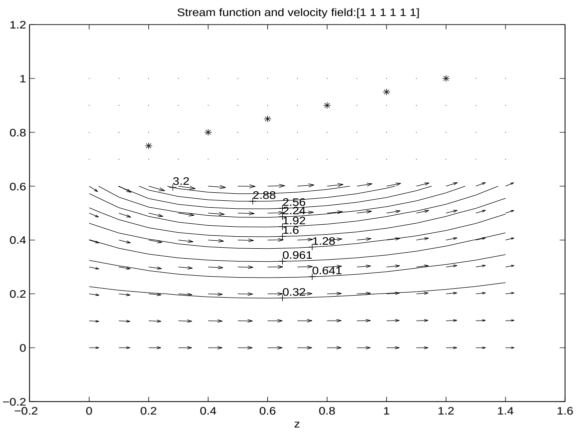

Figure 2. Distribution of the stream function and velocity; $v_{z,\max} = 17.98, Q_t = 27.96.$

4 Conclusion

Velocity fields of ideal incompressible fluid influenced by vortexes in a finite cylinder, finite cone and channel are investigated. The maximal value of the velocity induced by the spiral vortexes is in the middle of the cylinder. The behavior of circular vortexes in the ideal incompressible flow depends on the number, location and on the orientation of the vortexes. This approach can be generalized for the vortex threads on the surface of finite frustum of the cone.

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