A Procedure for Calculating an Approximate Analytical Response in a Large Class of Mechanical Systems

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Abstract: - Dynamical response in closed form is always a major goal when one wants to analyze the behaviour of a dynamical system. Nowadays, as is well known, we are able to calculate analytical response of mechanical systems only in a limited number of classes. So it is highly relevant to develop powerful methodologies which can help us in the dynamic analysis or in the non-linear identification processes of those systems. In this paper we present a direct and elegant procedure for calculating an approximate response of a large class of evolutionary equations which are of primary importance by the viewpoint of physics and engineering people. For instance this method could be used for identification of damping parameter of rotors, friction parameters in sliding process, and so on. The procedure consists of four steps: a Taylor transform, the use of a Lie operator followed by a linearization and finally the application of the corresponding Taylor's inverse transform. A peculiarity of this method is that the Lie operator can be directly built by the Taylor transformation of the assigned evolutionary equation.
The most important characteristic of this method is that it allows to calculate the response including, in explicit form, the whole parameters system. Consequently, this feature allows one to use very simple algorithms, as the Least Square Method for example, in order to develop quick non-linear identification processes.

Key-Words: - Non-linear identification processes, Mechanical systems, PDEs, Lie Series, KdV.

1 Introduction

We describe, from an operative point of view, a procedure for solving in an easy and direct way some linear or nonlinear PDEs we can find in the Classical or Quantum Mechanics [1-11]. From a mathematical point of view, we are highly interested in the solution of Cauchy problems relevant in the field of dynamical systems.

In this paper, after describing in some details, how the method works, we choose to find the solution to a Korteweg-de-Vries equation: an equation which is very important in many scientific fields, both classical and modern, both physics and engineering. In particular, so to cite a few, in fluid dynamics and in the supersymmetric quantum mechanics.

The method (already successfully applied by us to simpler cases) needs four steps to be operative:

1) a Taylor transformation of the starting Cauchy problem.
2) the construction of a suitable Lie operator, both in the autonomous and non-autonomous case.
3) a linearization of the transformed system in order to obtain the propagator.
4) and a Taylor inverse transformation, at the end.

Let us start with a nonlinear evolutionary equation, simbolically written as follows:

$$\frac{\partial P}{\partial t} = A\left(x, t, P, P', P'', \ldots P^{\mu}\right) \tag{1.1}$$

where A is a differential operator generally depending on: *x,t,P* and the first μ derivatives of *P* w.r.t. *x*.

The basic hypothesis is the analyticity of A w.r.t. its arguments in a bounded region R of $E^{\mu+3}$. Our task is to find out the function solution of the (Cauchy's) problem:

i) *P solves eq.(1) when x and t are in R;*
ii) *P(x,0) ≡ $P_o(x)$ is an assigned analytical function when x is in R and t=0.*

In preceding papers [1, 5] we thoroughly studied this initial value problem. We showed the problem is well posed: the solution exists and is unique if x and t are in R, and depends continuously upon the initial data. Generally speaking, it is representable by a double series, i.e. a power series in x, whose coefficients are Lie series of t. [1-9].

2 Procedure

We introduce a one-to-one application, (T, T^{-1}), between F and S. F is the linear space of infinitely many differentiable functions f (good functions), w.r.t. x, with domain $\Omega \in R$. They are supposed developable in absolutely convergent power series at an initial point, e.g. $x=0$. S is the linear space of sequences whose terms are functions depending on t and convergent to zero on the complex field:

$$f(x,t) \in F \underset{T^{-1}}{\overset{T}{\leftrightarrow}} (f_n)_{n=0}^{\infty} \equiv (T_n(f(x,t)))_{n=0}^{\infty} \in S \tag{2.1}$$

More in details:
T, directly maps F in S, as the Taylor transformation which associates every point of F to the sequence of its Taylor series coefficients:

$$f_n = \frac{1}{n!}\left[\frac{d^n f}{dx^n}\right]_{x=0} \tag{2.2}$$

T^{-1}, inverse of T, maps S in F, as the operator which starts from any sequence $(f_n)_{n=0}^{\infty}$ of S and associates to it the sum, let us say $f(x,t)$, of the Taylor series whose coefficients are the correspondent terms of the starting sequence.
The operator T can be utilized as a first step towards the linearization of the assigned problem.

2.1 First step: Taylor transform

The application of T to eq. (1.1) gives (at the (n+1) step):

$$\frac{dp_n}{dt} = \Theta(p_{-1}, p_0, ..., p_{\mu+n}) \tag{2.1.1}$$

being

$$\Theta(p_{-1}, p_0, ..., p_{\mu+n}) \equiv T_n(AP) \tag{2.1.2}$$

In this procedure, time t can be considered, just as one as the other $\mu+n+1$ functions involved. So we put:

$$p_{-1} = t \quad \text{obtaining} \quad \frac{d}{dt}p_{-1} = 1 \tag{2.1.3}$$

We note that by just adding this last simple differential equation we make autonomous the equivalent initial value problem. By this means it is always possible to change the evolution operator A into an autonomous one.
In compact form system (2.1.1) and (2.1.3) can be written:

$$\frac{dp}{dt} = \Theta(p) \tag{2.1.4}$$

With $p = (p_{-1}, T_0(P), T_1(P), ...)$

$$\Theta(p) = (1, T_0(AP), T_1(AP), ...)$$

2.2 Second step: Lie operator

Now we construct the Lie exponential operator:

$$e^{tD_\pi} = \sum_{\nu=0}^{+\infty} \frac{t^\nu}{\nu!} D_\pi^\nu \tag{2.2.1}$$

by means of, what we could name, the Groebner-Lie operator:

$$D_\pi = \frac{\partial}{\partial \pi_{-1}} + \sum_{n=0}^{+\infty} \Theta_n(\pi_{-1}, \pi_0, \pi_1, ... \pi_{\mu+n}) \frac{\partial}{\partial \pi_n} \tag{2.2.2}$$

Notice that it has inside it the same function Θ_n but with the arguments π_j instead of the p_j. In other words here the Θ_n depend on parameters which take the place of the unknown functions. The coefficient 1 of $\frac{\partial}{\partial \pi_{-1}}$ in D_π is a consequence of the coefficient we found in the r.h.s. of eq (2.1.3).

2.3 Third step: Linearization

Now we are able to linearize Cauchy problem (1.1) and ii). In fact we can write a linear problem [1-9]:

$$\frac{dp}{dt} = D_\pi p \tag{2.3.1}$$

with $p_{-1}(0) = 0, \quad p_n(0) = a_n, \quad a_n = T_n(P_0(x))$
which is equivalent to (2.1.4), since:

(2.3.2) $$D_\pi p = [[D_\pi \pi]]_{\pi=p} = \Theta(p).$$

By symbol [[ ]] we want to stress that we have to substitute p to π only at the end and just on the image given by D_π

2.4 Forth step: Taylor anti-transform

Having proved in [1-9] that:

(2.4.1) $$p_n(t) = [[e^{tD}\pi_n]]_{\substack{\pi_{-1}=0\\ \pi=a}}$$

we can, finally, perform the inverse transform of p and obtain the searched solution:

(2.4.2) $$P(x,t) = \sum_{n=0}^{\infty} p_n(t)x^n \equiv T^{-1}(p)$$

3 Example: Korteweg - de Vries equation

In order to give a flash on the functioning of the method we are going to apply it to a well known non linear equation relevant in the fluid dynamic field as well as in modern physics (supersymmetric quantum mechanics) [12]: a Korteweg-de Vries equation.

(3.1) $$P_t + 6PP_x + P_{xxx} = 0$$

First step: Taylor transform
By applying the operator

(3.2) $$T_n^0 \equiv \frac{1}{n!}\frac{d^n}{dx^n}\bigg|_{x=0}$$

we obtain the Taylor transform of equation (3.1):

(3.3) $$\frac{d}{dt}p_n + p_{n+3} + 6\sum_{k=0}^{n}(k+1)p_{k+1}(n-k)p_{n-k} = 0$$

in which:

(3.4) $$p_n \equiv \frac{1}{n!}\frac{d^n}{dx^n}P(x,t)\bigg|_{x=0} \equiv T_n^0 P(x,t)$$

Eq. (3.3) written in normal form, becomes:

(3.5) $$\frac{d}{dt}p_n = \Theta_n(p_0, p_1, ..., p_{n+1}, p_{n+3})$$

Second step: Lie operator
Lie exponential operator is, now: e^{tD_π}

being:

(3.6) $$D_\pi \equiv \sum_{n=0}^{\infty}\Theta_n(\pi_0, \pi_1, ..., \pi_{n+1}, \pi_{n+3})\frac{\partial}{\partial \pi_n}$$

where Θ_n functions are now depending on parameters π instead of p.

Third step: Linearization
We can rewrite the eq.(3.5) in a linearized form,[1-6], i.e.:

(3.7) $$\frac{d}{dt}p_n = D_\pi p_n$$

since:

(3.8) $$D_\pi p_n = [[D_\pi \pi_n]]_{\pi=p} = \Theta(p_0, p_1, ..., p_{n+1}, p_{n+3})$$

with $\pi = (\pi_n)_{n=0}^{\infty} \qquad p = (p_n)_{n=0}^{\infty}$

The Lie operator is a propagator for eqs. (3.7) because it is able to give the set of all components of the solution,

(3.9) $$\left\{ \left(e^{tD_\pi}\pi_n\right)_{n=0}^{\infty} \right\}$$

In fact as a propagator it acts on the initial function, here a parametric sequence, and gives the components of the solution at time t.
Therefore, for our initial value problem, we have the solution:

(3.10) $$\begin{gathered}(p_n)_{n=0}^{\infty} = [[e^{tD_\pi}\pi_n]]_{\pi=a}\\ a = (a_n)_{n=0}^{\infty} \equiv \left(T_n^0(P_0(x))\right)_{n=0}^{\infty}\end{gathered}$$

Forth step: Taylor's inverse transform
Finally, the Taylor anti-transform gives the searched solution .

(3.11) $$T^{-1}(p) = P(x,t) = \sum_{n=0}^{\infty} p_n x^n$$

for the Cauchy problem with

(3.12) $$P_0 = P(x,0) = \sum_{n=0}^{\infty} a_n x^n$$

Summarizing, we have:

$$P(x,t) = \sum_{n=0}^{\infty} \left[\!\left[e^{tD_\pi} \pi_n \right]\!\right]_{\pi=a} x^n$$

$$D_\pi = \sum_{n=0}^{\infty} \Theta_n\left(\pi_0, \pi_1, \ldots, \pi_{n+1}, \pi_{n+3}\right) \frac{\partial}{\partial \pi_n}$$

being a_n the Taylor coefficients of the initial function $P_0(x)$ and Θ the Taylor transform of the r.h.s. of the starting eq, after having written it in normal form and with parameters π_j as its arguments:

$$\Theta_n = -\pi_{n+3} - 6\sum_{k=0}^{n} (k+1)\pi_{k+1}(n-k)\pi_{n-k}$$

4 Conclusion

In this paper we propose a methodology for calculating the dynamic response of dynamical systems based on the Lie series. As a result of the general forms of the solutions, the applicability of this approach is not restricted to certain types of non-linearity and/or certain number of degrees of freedom. This seems to be a distinctive feature of the approach, since many well-known analytical methods of non-linear dynamics are sensitive to both the degree of nonlinearity and the number of degrees of freedom. This method, in addition, allows one to set up quick algorithms in order to identify one or more parameters of non-linear systems such as damping parameter of rotors, friction parameters in sliding processes, stiffness in beams, plates, etc…

In order to give a flash on the functioning of the method we applied it to a well known non linear equation relevant in the fluid dynamic field as well as in modern physics (supersymmetric quantum mechanics) [11] Korteweg-de Vries equation.

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