Implementation of an Interior Point Multidimensional Filter Line Search Method for Constrained Optimization

M. FERNANDA P. COSTA
University of Minho
Dep. of Math. for Science and Technology
Campus de Azurem, 4800-058 Guimarães
PORTUGAL

EDITE M. G. P. FERNANDES
University of Minho
Dep. of Production and Systems
Campus de Gualtar, 4710-057 Braga
PORTUGAL

Abstract: Here we present a primal-dual interior point multidimensional filter line search method for nonlinear programming. The filter relies on a multidimensional feasibility measure enforcing every constraint violation to zero. To prevent convergence to feasible nonoptimal points a sufficient reduction criterion is also imposed on the barrier function. The evaluation of the method is until now made on small problems and a comparison is provided with a merit function approach.

Key–Words: Nonlinear optimization, Interior point method, filter line search method

1 Introduction

A primal-dual interior point method for nonlinear optimization that relies on a filter line search strategy to allow convergence from poor starting points is proposed. Some line search frameworks use penalty merit functions to enforce progress toward the solution. Recently Fletcher and Leyffer [4] proposed a filter method, as an alternative to merit functions, as a tool to guarantee global convergence in algorithms for nonlinear constrained optimization [3, 5]. This technique incorporates the concept of nondominance to build a filter that is able to accept trial points if they improve the objective function or the constraints violation instead of a combination of those two measures defined by a merit function. Thus the filter replaces the use of a merit function. The update of a penalty parameter that is associated with the penalization of the constraints in a merit function is avoided. The filter technique has been already adapted to unconstrained optimization [6], as well as to interior point methods in different ways. A filter with two entries in a trust region context is presented in [8], while authors in [10, 11, 12] propose a filter line search approach. In these papers, the filter components are the barrier function and the norm of the constraints violation. Another extension of the filter approach to interior point methods defines three components for the filter measuring feasibility, complementarity and optimality [1, 2]. These measures come naturally from the optimality conditions of the barrier problem.

In this paper we propose a primal-dual interior point method with a multidimensional filter technique in a line search implementation. The goal of the multidimensional filter is to encourage convergence to primal feasibility points by enforcing every constraint violation to zero.

The paper is organized as follows. Section 2 briefly describes the interior point framework, Section 3 presents the herein proposed multidimensional filter line search method and Section 4 summarizes the numerical experiments. The conclusions and future developments make Section 5.

2 The Interior Point Framework

Here we consider the formulation of a nonlinear constrained optimization problem with simple bounds as follows:

$$\begin{array}{ll} \min_{\boldsymbol{x}\in \mathbb{R}^n} & F(\boldsymbol{x}) \\ \text{s.t.} & \boldsymbol{b} \leq \boldsymbol{h}(\boldsymbol{x}) \leq \boldsymbol{b}+\boldsymbol{r} \\ & \boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u} \end{array} \quad (1)$$

where $h_j : \mathbb{R}^n \rightarrow \mathbb{R}$ for $j = 1,\ldots,m$ and $F : \mathbb{R}^n \rightarrow \mathbb{R}$ are nonlinear and twice continuously differentiable functions. $\boldsymbol{r}$ is the vector of ranges on the constraints $\boldsymbol{h}$, $\boldsymbol{u}$ and $\boldsymbol{l}$ are the vectors of upper and lower bounds on the variables and $\boldsymbol{b}$ is assumed to be a finite real vector. Elements of the vector $\boldsymbol{r}$, $\boldsymbol{l}$ and $\boldsymbol{u}$ are real numbers subject to the following limitations: $0 \leq r_j \leq \infty$, $-\infty \leq l_i, u_i \leq \infty$ for $j = 1,\ldots,m$, $i = 1,\ldots,n$. Constraints of the form $\boldsymbol{b} \leq \boldsymbol{h}(\boldsymbol{x}) \leq \boldsymbol{b}+\boldsymbol{r}$ are denoted by range constraints. When upper and lower bounds on the $\boldsymbol{x}$ variables do not exist, the vector $\boldsymbol{x}$ is considered free. Equality constraints are treated as range constraints with $\boldsymbol{r} = \boldsymbol{0}$.

This section briefly describes an infeasible primal-dual interior point method for solving (1). We refer to [9] for details. Adding nonnegative slack variables $\boldsymbol{w}$, $\boldsymbol{p}$, $\boldsymbol{g}$ and $\boldsymbol{t}$, problem (1) becomes

$$\begin{array}{ll} \min & F(\boldsymbol{x}) \\ \text{s.t.} & \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{w} = \boldsymbol{b} \\ & \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{p} = \boldsymbol{b} + \boldsymbol{r} \\ & \boldsymbol{x} - \boldsymbol{g} = \boldsymbol{l} \\ & \boldsymbol{x} + \boldsymbol{t} = \boldsymbol{u} \\ & \boldsymbol{w}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{t} \geq \boldsymbol{0}. \end{array} \tag{2}$$

Incorporating the nonnegativity constraints in logarithmic barrier terms in the objective function, problem (2) is transformed into

$$\min b_\mu \equiv F(\boldsymbol{x}) - \mu \sum_{j=1}^{m} \ln(w_j) - \mu \sum_{j=1}^{m} \ln(p_j) - \mu \sum_{i=1}^{n} \ln(g_i) - \mu \sum_{i=1}^{n} \ln(t_i) \tag{3}$$

subject to the same set of equality constraints, where b_μ is the barrier function and μ is a positive barrier parameter. This is the barrier problem associated with (1). Under acceptable assumptions, the sequence of solutions of the barrier problem converges to the solution of the problem (1) when $\mu \searrow 0$. Thus primal-dual interior point methods aim to solve a sequence of barrier problems for a positive decreasing sequence of μ values. Optimality conditions for the barrier problem produce the standard primal-dual system

$$\begin{aligned} \nabla F(\boldsymbol{x}) - \nabla h(\boldsymbol{x})^{\mathrm{T}} \boldsymbol{y} - \boldsymbol{z} + \boldsymbol{s} &= \boldsymbol{0} \\ \boldsymbol{y} + \boldsymbol{q} - \boldsymbol{v} &= \boldsymbol{0} \\ \mathbf{WV}\boldsymbol{e}_1 &= \mu \boldsymbol{e}_1 \\ \mathbf{PQ}\boldsymbol{e}_1 &= \mu \boldsymbol{e}_1 \\ \mathbf{GZ}\boldsymbol{e}_2 &= \mu \boldsymbol{e}_2 \\ \mathbf{TS}\boldsymbol{e}_2 &= \mu \boldsymbol{e}_2 \\ \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{w} - \boldsymbol{b} &= \boldsymbol{0} \\ \boldsymbol{w} + \boldsymbol{p} - \boldsymbol{r} &= \boldsymbol{0} \\ \boldsymbol{x} - \boldsymbol{g} - \boldsymbol{l} &= \boldsymbol{0} \\ \boldsymbol{x} + \boldsymbol{t} - \boldsymbol{u} &= \boldsymbol{0}, \end{aligned} \tag{4}$$

where $\boldsymbol{v}$, $\boldsymbol{q}$, $\boldsymbol{z}$ and $\boldsymbol{s}$ are the dual variables, $\nabla F(\boldsymbol{x})$ is the gradient vector of $F(\boldsymbol{x})$, $\nabla h(\boldsymbol{x})$ is the Jacobian matrix of the constraints $\boldsymbol{h}$, $\mathbf{W} = \mathrm{diag}(w_j)$, $\mathbf{P} = \mathrm{diag}(p_j)$, $\mathbf{G} = \mathrm{diag}(g_i)$, $\mathbf{T} = \mathrm{diag}(t_i)$, $\mathbf{V} = \mathrm{diag}(v_j)$, $\mathbf{Q} = \mathrm{diag}(q_j)$, $\mathbf{Z} = \mathrm{diag}(z_i)$ and $\mathbf{S} = \mathrm{diag}(s_i)$ are diagonal matrices, $\boldsymbol{y} = \boldsymbol{v} - \boldsymbol{q}$, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are m and n vectors of all ones. The first two equations define the conditions of dual feasibility, the next four equations are the μ-complementarity conditions and the last four define the primal feasibility. This is a nonlinear system of $5n + 5m$ equations in $5n + 5m$ unknowns.

Applying the Newton's method to solve (4), we obtain the system of equations

$$\begin{aligned} & -\mathbf{H}(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{\Delta} x + \nabla h(x)^{\mathrm{T}} \boldsymbol{\Delta} y + \boldsymbol{\Delta} z - \boldsymbol{\Delta} s = \\ & \quad = \nabla F(\boldsymbol{x}) - \nabla h(\boldsymbol{x})^{\mathrm{T}} \boldsymbol{y} - \boldsymbol{z} + \boldsymbol{s} \\ & \boldsymbol{\Delta} y - \boldsymbol{\Delta} q + \boldsymbol{\Delta} v = \boldsymbol{y} + \boldsymbol{q} - \boldsymbol{v} \\ & \boldsymbol{\Delta} w + \mathbf{V}^{-1}\mathbf{W}\boldsymbol{\Delta} v = \mu \mathbf{V}^{-1} \boldsymbol{e}_1 - \boldsymbol{w} \\ & \mathbf{P}^{-1}\mathbf{Q}\boldsymbol{\Delta} p + \boldsymbol{\Delta} q = \mu \mathbf{P}^{-1} \boldsymbol{e}_1 - \boldsymbol{q} \\ & \mathbf{G}^{-1}\mathbf{Z}\boldsymbol{\Delta} g + \boldsymbol{\Delta} z = \mu \mathbf{G}^{-1} \boldsymbol{e}_2 - \boldsymbol{z} \\ & \mathbf{T}^{-1}\mathbf{S}\boldsymbol{\Delta} t + \boldsymbol{\Delta} s = \mu \mathbf{T}^{-1} \boldsymbol{e}_2 - \boldsymbol{s} \\ & \nabla h(\boldsymbol{x})\boldsymbol{\Delta} x - \boldsymbol{\Delta} w = \boldsymbol{w} + \boldsymbol{b} - \boldsymbol{h}(\boldsymbol{x}) \\ & \boldsymbol{\Delta} p + \boldsymbol{\Delta} w = \boldsymbol{r} - \boldsymbol{w} - \boldsymbol{p} \\ & \boldsymbol{\Delta} x - \boldsymbol{\Delta} g = \boldsymbol{l} - \boldsymbol{x} + \boldsymbol{g} \\ & \boldsymbol{\Delta} x + \boldsymbol{\Delta} t = \boldsymbol{u} - \boldsymbol{x} - \boldsymbol{t} \end{aligned} \tag{5}$$

for the Newton directions $\boldsymbol{\Delta} x, \boldsymbol{\Delta} s, \boldsymbol{\Delta} z, \boldsymbol{\Delta} g, \boldsymbol{\Delta} t, \boldsymbol{\Delta} y, \boldsymbol{\Delta} w, \boldsymbol{\Delta} p, \boldsymbol{\Delta} q$ and $\boldsymbol{\Delta} v$, where $\mathbf{H}(\boldsymbol{x}, \boldsymbol{y}) = \nabla^2 F(\boldsymbol{x}) - \sum_{j=1}^{m} y_j \nabla^2 h_j(\boldsymbol{x})$.

For easy of presentation we introduce the following notations:

$$\begin{aligned} \boldsymbol{\rho} &\equiv \boldsymbol{w} + \boldsymbol{b} - \boldsymbol{h}(\boldsymbol{x}) \\ \boldsymbol{\alpha} &\equiv \boldsymbol{r} - \boldsymbol{w} - \boldsymbol{p} \\ \boldsymbol{\upsilon} &\equiv \boldsymbol{l} - \boldsymbol{x} + \boldsymbol{g} \\ \boldsymbol{\tau} &\equiv \boldsymbol{u} - \boldsymbol{x} - \boldsymbol{t}. \end{aligned} \tag{6}$$

In a line search context, and after the search directions have been computed, the algorithm proceeds iteratively, choosing a step size $\overline{\alpha}_k \in (0, \overline{\alpha}_k^{\max}]$ at each iteration and determining the new iterates by $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \overline{\alpha}_k \boldsymbol{\Delta} x_k$, $\boldsymbol{s}_{k+1} = \boldsymbol{s}_k + \overline{\alpha}_k \boldsymbol{\Delta} s_k$, $\ldots$, $\boldsymbol{v}_{k+1} = \boldsymbol{v}_k + \overline{\alpha}_k \boldsymbol{\Delta} v_k$. $\overline{\alpha}_k^{\max}$ is the longest step size that can be taken, with an upper bound of 1, along these directions to assure the nonnegativity of the slack and dual variables.

Implementation details to provide initial values for all the variables in this interior point paradigm as well as to solve system (5) and to compute μ and $\overline{\alpha}^{\max}$ are described in [9].

The procedure that decides which trial step size is accepted in this interior point method is a multidimensional filter line search method.

3 Multidimensional Filter Line Search Method

In this section we briefly present the main ideas of a multidimensional entry filter method based on a line search approach. To abbreviate the notation we set

$$\begin{aligned}
&\boldsymbol{u} = (\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{t}, \boldsymbol{y}, \boldsymbol{v}, \boldsymbol{q}, \boldsymbol{s}, \boldsymbol{z}),\\
&\boldsymbol{u}^1 = (\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{t}),\\
&\boldsymbol{\Delta} = (\boldsymbol{\Delta} x, \boldsymbol{\Delta} w, \boldsymbol{\Delta} p, \boldsymbol{\Delta} g, \boldsymbol{\Delta} t, \boldsymbol{\Delta} y, \boldsymbol{\Delta} v, \boldsymbol{\Delta} q, \boldsymbol{\Delta} s, \boldsymbol{\Delta} z),\\
&\boldsymbol{\Delta}^1 = (\boldsymbol{\Delta} x, \boldsymbol{\Delta} w, \boldsymbol{\Delta} p, \boldsymbol{\Delta} g, \boldsymbol{\Delta} t),\\
&\overline{\boldsymbol{h}} = (\boldsymbol{\rho}, \boldsymbol{\alpha}, \boldsymbol{v}, \boldsymbol{\tau}).
\end{aligned}$$

To adapt the methodology of a multidimensional filter as given in [6] to this interior point method, and to encourage convergence to feasible points, we propose the use of the $2m + 2n$ elements of the vector $\overline{\boldsymbol{h}}$ as the filter entries. These components measure primal infeasibility as stated in the optimality conditions (4).

The notion of filter is based on that of dominance. In our case, a point $\boldsymbol{u}_+$ is said to dominate a point $\boldsymbol{u}_-$ whenever

$$|\overline{h}_i(\boldsymbol{u}^1_+)| \leq |\overline{h}_i(\boldsymbol{u}^1_-)| \quad \text{for all} \quad i = 1, \ldots, 2m + 2n.$$

Filter methods aim to accept a new trial point if it is not dominated by any other point in the filter.

After a search direction $\boldsymbol{\Delta}_k$ has been computed, the line search method considers a backtracking procedure, where a decreasing sequence of step sizes $\overline{\alpha}_{k,l} \in (0, \overline{\alpha}_k^{\max}]$ $(l = 0, 1, ...)$, with $lim_l\ \overline{\alpha}_{k,l} = 0$, is tried until an acceptance criterion is satisfied. Here, we use l to denote the iteration counter for the inner loop.

3.1 Sufficient reduction

Line search methods that use a merit function ensure sufficient progress toward the solution by imposing that the merit function value at each new iterate satisfies an Armijo condition with respect to the current iterate.

Adopting this idea, we consider the trial iterate $\boldsymbol{u}_k(\overline{\alpha}_{k,l}) = \boldsymbol{u}_k + \overline{\alpha}_{k,l}\boldsymbol{\Delta}_k$ during the backtracking line search technique to be acceptable if it leads to sufficient progress in one of the $2m + 2n$ following measures compared to the current iterate, i.e., if

$$|\overline{h}_i(\boldsymbol{u}^1_k(\overline{\alpha}_{k,l}))| \leq (1 - \gamma_f)\, |\overline{h}_i(\boldsymbol{u}^1_k)| \qquad (7)$$

holds for some $i \in I$ and fixed constant $\gamma_f \in (0, 1)$, where $I = \{1, \ldots, 2m + 2n\}$.

However, to prevent convergence to a feasible point that is nonoptimal, and whenever, for the current iterate, we have

$$|\overline{h}_i(\boldsymbol{u}^1_k)| \leq \overline{h}_i^{\min} \text{ and } \boldsymbol{\nabla} b_\mu(\boldsymbol{u}^1_k)^{\mathrm{T}} \boldsymbol{\Delta}^1_k < 0 \qquad (8)$$

for some $\overline{h}_i^{\min} \in (0, \infty]$, for all $i \in I$, the following sufficient reduction criterion on the barrier function is imposed on the trial point $\boldsymbol{u}^1_k(\overline{\alpha}_{k,l})$:

$$b_\mu(\boldsymbol{u}^1_k(\overline{\alpha}_{k,l})) \leq b_\mu(\boldsymbol{u}^1_k) + \eta_b \overline{\alpha}_{k,l} \boldsymbol{\nabla} b_\mu(\boldsymbol{u}^1_k)^{\mathrm{T}} \boldsymbol{\Delta}^1_k \quad (9)$$

instead of (7). Here $\eta_b \in (0, 0.5)$ is a constant. According to a previous publication on filter methods [3], we call a trial step size $\overline{\alpha}_{k,l}$ for which (8) holds, a "b_μ-step". Similarly, if a "b_μ-step" is accepted as the final step size $\overline{\alpha}_k$ in iteration k, we refer to k as a "b_μ-type iteration".

3.2 The filter definition

At each iteration k, the algorithm also maintains a filter, here denoted by

$$\overline{F}_k \subseteq \left\{ \overline{\boldsymbol{h}} \in \mathrm{I\!R}^{2m+2n} : |\overline{h}_i| \geq 0, \text{ for all } i \in I \right\}.$$

Following the ideas in [10, 11, 12], the filter here is not defined by a list but as a set $\overline{F}_k$ that contains those combinations of values of $\overline{h}_1, \ldots, \overline{h}_{2m+2n}$ that are prohibited for a successful trial point in iteration k. Thus a trial point $\boldsymbol{u}_k(\overline{\alpha}_{k,l})$ is rejected, if

$$\left(\overline{h}_1(\boldsymbol{u}^1_k(\overline{\alpha}_{k,l})), \ldots, \overline{h}_{2m+2n}(\boldsymbol{u}^1_k(\overline{\alpha}_{k,l}))\right) \in \overline{F}_k.$$

At the beginning of the optimization, the filter is initialized to

$$\overline{F}_k \subseteq \left\{ \overline{\boldsymbol{h}} \in \mathrm{I\!R}^{2m+2n} : |\overline{h}_i| \geq \overline{h}_i^{\max}, \text{ for all } i \in I \right\} \qquad (10)$$

for sufficiently large positive constants $\overline{h}_1^{\max}, \ldots, \overline{h}_{2m+2n}^{\max}$, so that the algorithm will never accept trial points that have values of $\overline{h}_1, \ldots, \overline{h}_{2m+2n}$ larger than $\overline{h}_1^{\max}, \ldots, \overline{h}_{2m+2n}^{\max}$, respectively.

The filter is augmented, using the update formula

$$\begin{aligned}\overline{F}_{k+1} = \ & \overline{F}_k \cup \Big\{ \overline{\boldsymbol{h}} \in \mathrm{I\!R}^{2m+2n} : \\ & |\overline{h}_i| > (1 - \gamma_f)\, |\overline{h}_i(\boldsymbol{u}^1_k)|, \text{ for all } i \in I \Big\}\end{aligned} \qquad (11)$$

after every iteration in which the accepted trial step size satisfies (7). On the other hand, if (8) and (9) hold for the accepted step size, the filter remains unchanged.

Finally, in some iterations it is not possible to find a trial step size $\overline{\alpha}_{k,l}$ that satisfies the above criteria. If the backtracking multidimensional line search technique finds a trial step size $\overline{\alpha}_{k,l} < \overline{\alpha}^{\min}$, the filter is reset to the initial set.

Our interior point multidimensional filter line search algorithm for solving constrained optimization problems is as follows:

Algorithm 1

Given: Starting point $\boldsymbol{u}_0$, constants $\overline{h}_i^{\min} > 0, \overline{h}_i^{\max} > 0$, for all $i \in I$, $\gamma_f \in (0, 1)$, $\eta_b \in (0, 0.5)$, $\overline{\alpha}^{\min} > 0$.

1. *Initialize*. Initialize the filter (using (10)) and set $k \leftarrow 0$.

2. *Check convergence*. Stop if the relative measures of primal and dual infeasibilities are less or equal to 10^{-4}.

3. *Compute search direction*. Compute the search direction $\mathbf{\Delta}_k$ from the linear system (5).

4. *Backtracking line search.*

   4.1 *Initialize line search.* Compute the longest step size $\overline{\alpha}_k^{\max}$ to ensure positivity of slack and dual variables. Set $l \leftarrow 0$, $\overline{\alpha}_{k,l} = \overline{\alpha}_k^{\max}$.

   4.2 *Compute new trial point.* If the trial step size becomes too small, i.e., $\overline{\alpha}_{k,l} < \overline{\alpha}^{\min}$, go to step 8. Otherwise, compute the trial point $\boldsymbol{u}_k(\overline{\alpha}_{k,l})$.

   4.3 *Check acceptability to the filter*. If $\left(\overline{h}_1(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l})), \ldots, \overline{h}_{2m+2n}(\boldsymbol{u}_k^1(\overline{\alpha}_{k,l}))\right) \in \overline{F}_k$, reject the trial step size and go to step 4.6.

   4.4 *Check sufficient decrease with respect to current iterate*. If (7) holds, accept the trial step and go to step 5. Otherwise go to step 4.6.

   4.5 *Check sufficient decrease with respect to current iterate*. If $\overline{\alpha}_{k,l}$ is a b_μ-step size ((8) holds) and the Armijo condition (9) for the b_μ function holds, accept the trial step and go to step 5.

   4.6 *Choose new trial step size*. Set $\overline{\alpha}_{k,l+1} = \overline{\alpha}_{k,l}/2$, $l \leftarrow l+1$, and go back to step 4.2.

5. *Accept trial point.* Set $\overline{\alpha}_k \leftarrow \overline{\alpha}_{k,l}$ and $\boldsymbol{u}_k \leftarrow \boldsymbol{u}_k(\overline{\alpha}_k)$.

6. *Augment the filter if necessary*. If k is not a b_μ-type iteration, augment the filter using (11). Otherwise, leave the filter unchanged.

7. *Continue with next iteration*. Increase the iteration counter $k \leftarrow k+1$ and go back to step 2.

8. *Reset the filter.* Reset the filter using (10) and continue with the regular iteration in step 7.

4 Numerical experiments

We tested this interior point framework with a multidimensional filter line search technique on 50 small constrained problems from the Hock and Schittkowski (HS) collection [7]. The tests were done in double precision arithmetic with a Pentium 4 and Fortran 90.

Table 1: Numerical results

	Filter Method			Merit Function	
Problem	N_{it}	N_{feas}	N_{b_μ}	N_{it}	N_{mf}
HS1	24	27	26	24	27
HS2	42	44	25	42	44
HS3	1	2	0	1	2
HS4	5	6	2	5	6
HS5	12	120	119	9	26
HS6	8	9	1	8	9
HS10	(13)	(14)	(0)	(13)	(14)
HS11	8	23	15	8	9
HS12	15	32	17	15	16
HS14	8	9	0	8	9
HS15	(9)	(10)	(2)	(18)	(20)
HS16	(11)	(12)	(1)	(11)	(12)
HS17	5	6	2	7	10
HS18	11	12	0	12	76
HS19	25	26	0	31	48
HS20	(22)	(23)	1	(22)	(25)
HS21	5	6	0	5	6
HS23	26	29	4	31	78
HS24	15	16	11	16	19
HS27	16	17	0	17	70
HS28	6	7	0	6	7
HS30	8	9	0	8	9
HS31	13	24	14	13	14
HS32	8	9	2	8	34
HS33	10	11	2	10	11

Some of the chosen values for the constants are similar to the ones used in [12]: $\overline{h}_i^{\min} = 10^{-4}\max\left\{1, |\overline{h}_i(\boldsymbol{u}_0^1)|\right\}$, $\overline{h}_i^{\max} = 10^4\max\left\{1, |\overline{h}_i(\boldsymbol{u}_0^1)|\right\}$, for all $i \in I$, $\gamma_f = 10^{-5}$, $\eta_b = 10^{-4}$, $\overline{\alpha}^{\min} = 10^{-12}$.

We exercised the algorithm using a symmetric positive definite quasi-Newton BFGS approximation to the matrix $\mathbf{H}$. In the first iteration, the iterative process uses a positive definite approximation to $\nabla^2 F(\boldsymbol{x}_0)$, except on those problems where singularity or ill-conditioning was encountered on that matrix and the identity matrix was used instead.

Tables 1 and 2 summarize the results under the "Filter Method" indication. Results inside parenthesis were obtained with the identity matrix in the first iteration. We compare our interior point multidimensional filter line search method with a similar interior point method based on a l_2 merit function approach ("Merit Function" in the tables).

These tables report on values of the number of iterations (N_{it}) needed to achieve a solution according

Table 2: Numerical results (cont.)

	Filter Method			Merit Function	
Problem	N_{it}	N_{feas}	N_{b_μ}	N_{it}	N_{mf}
HS34	10	11	2	10	12
HS35	2	3	0	2	3
HS36	(9)	(10)	(2)	(9)	(10)
HS37	(14)	(71)	(70)	(13)	(14)
HS38	33	95	94	25	38
HS41	(26)	(27)	(21)	(9)	(11)
HS42	12	13	0	12	13
HS43	12	13	2	12	13
HS44	19	20	1	19	20
HS45	4	5	2	4	5
HS46	15	37	22	15	194
HS48	7	8	1	7	8
HS49	14	15	7	14	15
HS50	14	15	7	14	64
HS51	6	7	0	6	57
HS52	9	10	0	9	10
HS53	8	9	1	9	14
HS55	11	12	2	11	12
HS60	12	56	46	11	18
HS63	11	26	16	11	12
HS64	(54)	(271)	(232)	(55)	(235)
HS65	9	10	2	9	10
HS76	4	5	0	7	8
HS77	(18)	(35)	(19)	(18)	(48)
HS79	13	14	3	11	65

to the convergence tolerance referred to in Algorithm 1, the number of $\overline{\boldsymbol{h}}$ evaluations (N_{feas}) and the number of b_μ evaluations (N_{b_μ}) in the filter method, and the number of merit function evaluations (N_{mf}) in the merit function approach.

We may observe that the two methods behave similarly on almost 50% of the problems. On the remaining problems, for example, HS5, HS11, HS12, HS31, HS37, HS38, HS41, HS60, HS63 and HS64 the number of function $\overline{\boldsymbol{h}}$ evaluations in the filter method exceeds the number of merit function evaluations in the merit function approach. The inverse situation appears on the problems HS15, HS17, HS18, HS19, HS20, HS23, HS24, HS27, HS32, HS34, HS46, HS50, HS51, HS53, HS76, HS77 and HS79.

Based on the cumulative results illustrated in the Table 3 we observe that although the filter method requires slightly more iterations it gains in the number of barrier function and constraints evaluations. The last column of this table contains the number of $\overline{\boldsymbol{h}}$ evaluations that equals the number N_{mf} in the merit function approach. Note that the l_2 merit function depends on b_μ and $\overline{\boldsymbol{h}}$.

Table 3: Cumulative results

	N_{it}	N_{feas}	N_{b_μ}	N_{mf}	$N_{\overline{h}}$
Filter method	672	1301	796	-	-
Merit Function	670	-	-	1510	1510

Doubtless that a comparison based on CPU time might be more appropriate, since the use of the above measures to compare the algorithms efficiency is not a straight matter. However, as the chosen problems are small, CPU time registering is not an easy task. We intend to go on testing the herein proposed method using larger problems and in this situation CPU measuring will be adopted.

5 Conclusions

We present an interior point multidimensional filter line search method for solving a nonlinear constrained optimization problem. The novelty here is that a certain emphasis is given to primal feasibility measures in order to consider a point to be acceptable. However, whenever a trial point is considered feasible, according to a given tolerance, and the appropriate search direction is descent for the barrier function b_μ, the algorithm imposes on the trial point an Armijo condition in order to be acceptable.

The algorithm was tested with a set of small problems and compared with a similar interior point method with a merit function approach. Future developments will focus on the global convergence analysis of the algorithm. The extension of the multidimensional filter to an interior point method needs an adequate theoretical understanding.

Acknowledgements: The financial support from FCT Project POCTI/MAT/61842/2004 is gratefully acknowledged.

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