Signal Processing in Measurement Systems

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Abstract: - Most technical measuring and technological equipment, instruments and systems, such as linear and circular scales, measuring transducers, numerical controlled machine tools, etc. have some kind of information – measuring systems that give the information about the position of the part of the machine or instrument, measuring information, etc. The information – measuring systems are calibrated against the reference standards of measure comparing their accuracy at some pitch of calibration, for example, at the beginning, middle point and the end of the stroke or at every tenth of the stroke in the range of measurement. It depends on the written standards and methodical documentation of these machines or instruments.

Key-Words: - information, measurement, control, assessment, entropy.

1 Introduction

The measurement result is usually presented as an evaluation of the systematic (mean) value of the measurand within the estimated distribution limits that depend on the standard deviation given the appropriate level of confidence. This is a commonly accepted form of presentation of such a measurement result. Uncertainty analysis is based on classical statistical principles, and it assigns the measurand a standard uncertainty at effective degrees of freedom allowing the measurand to be estimated inside the confidence region set for a chosen probability. The established standards and investigations in this field [1, 2] contain the main rules for expressing the uncertainty of measurement. It is absolutely important approach in determining the real value of the measured value, performing the instrument accuracy assessment [3] and assuring the quality assessment at all levels of industrial production.

The sampling strategy in measurement [4, 5] is widely discussed in literature. Wunderli [5] discusses a practical approach in the expression of the result of a measurement and its uncertainty. It is noted that uncertainty expresses a dispersion of the values that could be reasonably attributed to the measurand. The author agrees that sampling is critical for decision-making process, but, in his opinion, it is not a part of the measurement uncertainty. Nevertheless, the discussion in the article shows that further procedures, such as verification and traceability need to operate with representative sampling. Sladek and Krawczyk [6] have highlighted the problem that exists due to the great amount of information that is present in the calibration of the total volume of a coordinate measuring machine (CMM). The authors show that it is technically difficult and economically demanding to calibrate the enormous number of points available, e.g. the 324 000 steps of information of the CMM in the measuring volume arising from six rotary axes. Suitable sampling of measuring points on the surface of industrial parts was shown to be a very important task, as was the sampling of the points in the CMM volume during its calibration.

An important branch of measurement theory is information theory [7, 8] where the information entropy of measuring devices and instruments is assessed and evaluated. They complement each other; nevertheless, no real proposals have been made for joining them together in the expression of the results of measurement. Many authors discuss the task of uncertainty monitoring and sampling; nevertheless, there is no general approach given to solve this task.

The task is to add to the measurement result expressed by systematic and random errors (*uncertainty*) an assessment that includes the information entropy of the measurement, i.e. showing the quantity of measurement information that was evaluated out of the total available information (*indeterminacy*). An idea to include the sampling procedure into the equation for the measurand is especially important bearing in mind modern information-measuring systems consisting of translational transducers that can combine a wide range of data or measurement values reaching hundreds of thousands of numerical values including some intelligent functions. This is also important in view of traceability of measurement as it can clearly indicate which part of the information was assessed from the total volume of the information.

2 Problem Formulation

The quantity of information can be evaluated by joining it with the general expression of the measurement result, i.e., expressing the systematic part of the result, the uncertainty of the assessment, and adding to it the quantity of information entropy that shows the indeterminacy of the result. This measure can be used to define mutual information, i.e., the amount of information one random variable contains about another [7]. The expression of the measurement result (the measurand) is widely [1,2] accepted as $X = \overline{x} \pm \varepsilon, P$, where X is the result of measurement; $\overline{x}$ is the systematic part of the measurement result, ε is the uncertainty of measurement expressed further as $\varepsilon = tS/\sqrt{n}$, where t is the Student coefficient, S is the estimate of standard deviation, n is number of trials, and P is the probability selected according to the chosen level of confidence. The systematic part of this expression is used to assess or verify the preestablished performance of measurement procedure and to design corrective actions if necessary to improve this procedure [9]. The whole measurement procedure further is estimated by determining the measurement uncertainty. The full information entropy according to [7, 8] is expressed as:

$$H = \log_a \left(\frac{1}{p_i} \right) = -\sum_{i=1}^{n} p_i \log_a p_i \text{ ; } p_i = \frac{1}{n} \tag{1}$$

or , for an analogue function,

$$H = -\int_{t_1}^{t_2} p(t) \ln p(t) dt \tag{2}$$

where p_i – probability of the appearance of a message; n – probability sample, a – logarithm basis. Information conveyed by measurement process in case of equal probability would be: $I = H_0 - H_1$; i.e., the difference between the information existing before the measurement H_0 and after performing the measurement (receiving information) H_1. This evaluation is widely used in signal processing, communications, economics and financial operations, evaluation of stock exchange operations, etc. Mutual entropy shows a measure of dependency between two variables. For expression of mutual information such mathematical expression is used [7, 8]:

$$I(X,Y) = H(X) - H(X/Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

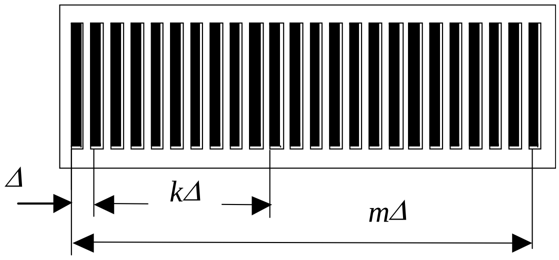

Fig. 1. An example of the available pitch of calibration of linear raster scales

Only a small part of the information (about its accuracy) contained in information - measuring systems can be determined during calibration process. For example, in Fig. 1 the raster scales are shown, where *Δ* - a pitch of the raster scale, *kΔ* - a pitch of the accuracy calibration, *mΔ* - the total number of strokes in the scale. For the circular scale, the calibration by using the polygon can be performed for only 8 of the strokes of the scale. Conventional measurement formulas show the measurand after the calibration without indication, which part of the scale is measured. This disadvantage can be lessened using the information entropy expressions.

It is evident that most information would be received if all possible points could be calibrated. However, that is impossible from a technical point of view. For presenting data, it is very useful to give the real situation of the measurement result as the outcome of the measuring process.

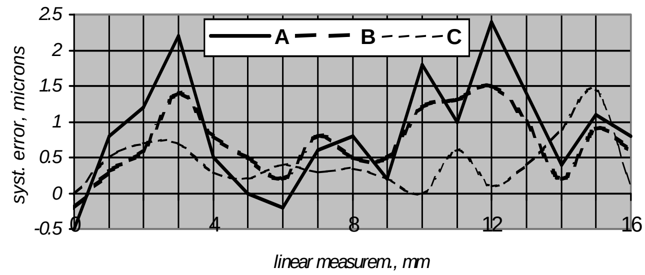

Fig. 2. Examples of discrete (A) and analogue (B,C) functions having different bias features for entropy evaluation

Fig. 2 shows a different type of error distribution in linear measurements. The graph indicated by **A** shows sharp changes in accuracy in contrary to the graphs **B** and **C**. So, it would be of great importance to indicate, which number of points are measured during the calibration process. The part of the analogue graph **C** in the interval (4-13) mm shows a minimal changes, and this part of the scale or the transducer could be used for more precise measurements.

3 Problem Solution

The digital output of photoelectric rotary and translational transducers have the last digit equal to the value of 0.1 μm or 0.1” (of arc). The measuring range is equal to 10 … 30 m, and a full or several rotations of the shaft in the circular case. The value in arc seconds of one revolution is 1,296,000”, i.e. the same number of discrete values in the display unit. It will increase to ten times this number if the indication is at every 0.1 of the value. The measurement results indicated in the display unit can be proved by metrological means (calibrated) only at every increment of 1/100, 1/1000 or even a smaller portion of total information. A suitable theoretical approach to provide more information in this area would be information entropy.

The information received after the calibration, i.e. the determination of the accuracy of part of the scale (Fig. 1) will be $H_1 = \log_a b$, where $b = m/k$ is the number of calibrated strokes in the scale. These strokes were measured c times each for the statistical evaluation. Then the reduction in the information uncertainty (indeterminacy) due to the information received (1, 2) will be:

$$I = H_0 - H_1 = \log_a m - \log_a b \qquad (3)$$

then $\log_a b = \log_a m - I$; and $b = a^{(\log_a m - I)} = m \times a^{-I}$

Since the total number of measurements is $n = bc$ (each calibration measurement is performed c times), the expression for the measurement result at a given probability and the reduction in information indeterminacy becomes:

$$X = \overline{X} \pm \frac{t \cdot S}{\sqrt{a^{-I} mc}} \;, P, I(H_0, H_1) \qquad (4)$$

Using Formula (3) we can consider two limiting cases: when all available strokes are calibrated and no strokes are calibrated. In the first case as b becomes equal to m, I becomes zero, and the formula above (4) is transformed into

$$X = \overline{X} \pm \frac{t \cdot S}{\sqrt{a^0 mc}} = \overline{X} \pm \frac{t \cdot S}{\sqrt{mc}}, P, 0\,. \qquad (5)$$

In the second case, if no strokes are calibrated, the result is undefined, since no additional information is present. It means that the information–measuring system remains absolutely undefined.

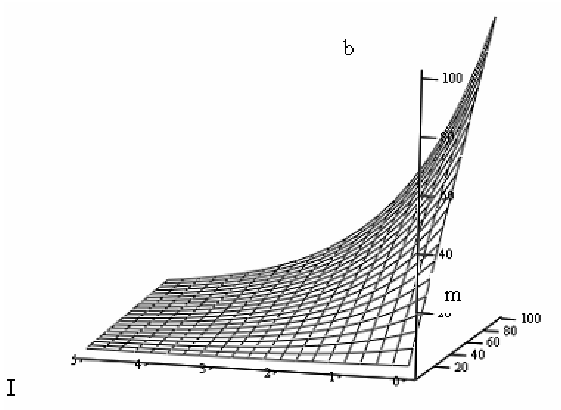

Fig. 3. The relationship between the total and assessed number of strokes with the information quantity of the scale, where m is the number of strokes on the scale, b is the number of the assessed strokes on the scale and I is the quantity of information

In Fig 3 the relationship between the total number of strokes on the scale *m*, its information *I* and the number of the strokes position already assessed on the scale *b* is displayed. It is assumed that the base of logarithm is 2, and this base is used in Formula (3). It can be seen that the reduction in uncertainty is the greatest when additional assessed information is added to a relatively small number of already determined information (for example, the strokes of calibrated accuracy). The slow change in *I* on the right edge of the graph suggests that after a certain point little is gained by determining yet more information. From the graphs it is evident that the information quantity of the data from the scale increases as the proportion of calibrated strokes to the total number of strokes decreases and vice versa. Hence, Formula (3) confirms a logical proposition that the information quantity of the scale is inversely proportional to the number of calibrated strokes. Sensibly again, it can be seen that the information entropy approaches zero as the number of calibrated strokes approaches the number of total strokes in the scale. As the uncertainty of the scale becomes less and less so does its information differential.

It means that the measurement result is determined with the *uncertainty* assessed by probability level *P* and with the *indeterminacy* of the result assessed by the entropy $I(H_1, H_0)$ of evaluation of that part of all the data in question. The measured part of the scale or transducer can be used [10] in such a way as not to contribute the extremes of the systematic error.

A technical implementation of this approach can be photoelectric, optical and electromechanical transducers are used for control and measurement of the strokes of machines and instruments [9]. However, they experience significant systematic errors when being applied in these machine systems with long displacement strokes. A technical solution incorporating the aforementioned advantage of lessening the indeterminacy of the measurement is through the use of multiple indicator heads with a short part of the scale which can be calibrated at very small steps with high accuracy. The indicator heads of a measurement system must be placed in the path of a moving machine part in such a way that the measurement be limited only to the most precise part of the measuring system (scale), and the reading of the information be performed serially by using the heads placed along this path.

4 Conclusion

This new approach to the evaluation of measurement data gives full information on the measurement process performed and the quantity of data assessed during this process.

The systematic error, uncertainty and entropy evaluation during the measurement process permit to evaluate the accuracy of information-measuring systems more exactly and to improve accuracy of those systems by technical means.

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