Berth Allocation with Service Priorities: A Linear Reformulation

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Abstract: **-** The berth-allocation problem (BAP) aims to optimally schedule and assign ships to berthing areas along a quay**.** The vessels arrive at the port over a period of time and normally request and compete for early service start and departure. Berth allocation policies with service priority are important in terminal operations and are applicable in situations involving various vessel sizes, different handling volumes and different service strings. In this paper the discrete and dynamic BAP is formulated as a linear MIP problem with linear constraints, with the objective to minimize the weighted total service time. A heuristic is also presented for large instances of the problem.

Keywords: Container Terminals; Berth Allocation; Service Priority, Heuristic, Cargo Handling, Container Transportation

1 Introduction

The berth-allocation problem (BAP) aims to optimally schedule and assign ships to berthing areas along a quay. The vessels arrive at the port over a period of time and normally request service and departure within a specified time window. These time windows are determined through contractual agreements between the port operator and the carrier. Based on these contractual agreements different ships receive different service priorities varying from berthing upon arrival, to guaranteed service time window and/or guaranteed service productivity. Earliness or lateness of a ship's handling operations completion time implies benefits or costs to both the port operator and the ocean carrier. Berth allocation policies with service priority are important in terminal operations and are applicable in situations involving various vessel sizes, different handling volumes and different service strings. This paper attempts to provide some insight on the BAP, through reformulating an existing problem formulation and proposing a solution heuristic. The original problem is known as the BAP with service priorities and it is a berth scheduling policy introduced by Imai et al. (2003), as a linear MIP. The paper is organized as follows: The next section presents a literature review of existing published studies on the BAP. The problem is presented and formulated in Section 3, and a small instance is used to compare results with the formulation presented by Imai et al. (2003). Section 4 presents a heuristic solution method for solving large instances of the problem, while the final section concludes the paper.

2 Literature Review

Several papers have appeared in the literature dealing with the BAP. One of the first papers presented in this area was by Lai and Shih (1992). The authors assumed that a wharf is represented as a continuous line that could be partitioned into several sections, to each of which only one vessel could be allocated at any specific time. A heuristic algorithm was developed considering a first-come-first-served (FCFS) rule. Brown et al. (1994, 1997) addressed the BAP in naval ports. They identified the optimal set of ship-to-berth assignments that maximizes the sum of benefits for ships while in port. Imai et al. (1997) first introduced the idea that for high port throughput, optimal ship-to-berth assignments should not be based on the First Come First Served (FCFS) rule. However, their formulation may result in some ships' dissatisfaction regarding order of service. Lim (1998) addressed the continuous BAP with the objective of minimizing the maximum amount of quay space used at any time with the assumption that once a ship is berthed, it will not be moved to any place else along the quay before it departs. He also assumed that every ship is berthed upon arrival at the port. Li et al. (1998) formulated the Static Berth Allocation Problem (SBAP) as a scheduling problem with a single processor through which multiple jobs can be processed simultaneously. The objective was the minimization of the make-span. Similar to Li et al. (1998), Guan et al. (2002) considered the berth allocation problem as a multiprocessor task scheduling. They developed a heuristic to minimize the total weighted completion time of ship service and performed worst-case analysis. Weights were assigned to each job depending on the vessel's size. Imai et al. (2001) addressed the Dynamic Berth Allocation Problem (DBAP) with the objective to minimize the sum of a ship's waiting and handling time. Handling time was assumed deterministic and dependent on the berth. In the same context Nishimura et al. (2001) addressed the same problem but for a public berth system. In this paper the authors extended the work done by Imai et al. (2001) to include physical restrictions (water-depth and quay length). They also dropped the assumption that each berth can handle one ship at a time. Service priority relied on the FCFS rule. The objective was to minimize service time (including waiting time). Imai et al. (2003) modified and extended the discrete DBAP formulation of Imai et al (2001) and Nishimura (2001) in order to include service priority constraints. The objective was to minimize the total service time while differentiating priorities to ships by variation of their service time in the solution. Imai et al. (2005) extended their previous work by solving the DBAP in a continuous berth space with the objective of minimizing the total completion time.

Guan and Cheung (2004) presented a berth allocation model that allows multiple vessels to moor at a berth, considers vessel arrival time and optimizes the total weighted flow time. Following the idea by Imai et al. (2003) they apply a weight coefficient to each ship. They develop a tree procedure and a heuristic that combines the tree procedure with the heuristic in Guan et al. (2002). Kim and Moon (2003) studied the continuous SBAP and formulated a MIP model and used simulated annealing to find near optimal solutions. The objective was to minimize delays and handling cost by non-optimal locations of the ships' berthing. Unlike Lim (1998), Imai et al (2001), Brown et al. (1994, 1997) and Lai and Shi (1992), Park and Kim (2002) consider the continuous BAP with the objective of estimating the berthing time and location by minimizing the total waiting and service time and the deviation from the preferred berthing location. Park and Kim (2003), extend their previous work to combine the BAP with consideration of quay crane capacities. Their study determined the optimal start times of ship services and associated mooring locations while at the same time determines the optimal assignment of quay cranes to ships. The handling time was considered independent from the berthing location of the ship. Lee et al. (2006) following the work of Park and Kim (2003) presented a method for scheduling berth and quay cranes, which are critical resources in container ports. A bi-level programming model with the objective of minimizing the sum of total completion time of all the vessels and the completion time for all the quay cranes is formulated by considering various practical constraints such as interference between the quay cranes. Cordeau et al. (2005) considered the discrete case of the DBAP and provided two formulations: a formulation similar to Imai et al. (2001) and a Multi Depot Vehicle Routing Problem with Time Windows formulation. To avoid simplifications contrary to Park and Kim (2003) the authors did not solve the BAP and the Quay Crane Assignment Problem (QCAP) simultaneously. The objective was the minimization of the total (weighted) service time for all ships, defined as the time elapsed between the arrival in the harbor and the completion of handling. Imai et al., (2006) addressed the berth allocation problem at a multi-user container terminal with indented berths for fast handling. A new integer linear programming formulation was presented, which was then extended to model the berth allocation problem at a terminal with indented berths, where both mega-containerships and feeder ships are to be served for higher berth productivity. Wang and Lim (2006) solve the DBAP by minimizing un-allocation, position and delay costs, using a Stochastic Beam Search Heuristic.

3 Model Formulation

As mentioned earlier, the berth scheduling policy modeled in this paper was originally proposed by Imai et al. (2003). In the same manner the BAP presented in this paper assumes only one long wharf at a multi-user terminal. Considering a variety of ship sizes, in terms of ship length, a number of ship location combination alternatives at the wharf are possible. However, for simplicity in the solution procedure, the wharf is virtually divided into several blocks, a practice adopted in major container ports, and in the BAP we obtain a set of assignments of ships to those blocks that are hereafter referred to as berths. We also assume that each berth can service one ship at a time and that there are no physical and/or technical restrictions such as the relationship between ship draft and effective quay water depth. Furthermore, as with most papers presented in the literature, the ship handling time is assumed dependent on the berth where it is assigned, since it is related to the routing distance and the time of the landside transfer operations.

In formulating the BAP we define the following variables: $\boldsymbol{i}=(1,\ldots\ldots,\boldsymbol{I}) \in \boldsymbol{B}$ set of berths, $\boldsymbol{j}=(1,\ldots.,\boldsymbol{T}) \in \boldsymbol{V}$ set of ships, $\boldsymbol{k}=(1,\ldots.,\boldsymbol{T}) \in \boldsymbol{U}$(is this O or U? see formulations below) set of service orders, $\boldsymbol{S_i}$=time when i berth becomes idle, $\boldsymbol{A_j}$=arrival time, $\boldsymbol{C_{ij}}$= handling time of ship j at berth i, $\boldsymbol{X_{ijk}}$=1 if ship j is serviced at berth i with $(k\text{-}1)$ successors, $\boldsymbol{y_{ijk}}$= idle time of berth i between departure of ship j and its immediate predecessor. The original problem formulated by Imai et al. (2003) is shown in equations 1 through 5.

(LP1):
$$\sum_{i}\sum_{j}\sum_{k}\{C_{ij}+S_i-A_j+\sum_{m\neq j}\sum_{h<k}C_{im}X_{imh}\}a_jX_{ijk} + \sum_{i}\sum_{j}\sum_{k}(y_{ijk}+\sum_{m\neq j}\sum_{h<k}y_{imh})a_j \quad ,(1)$$

Subject to: $\sum_{i\in B}\sum_{k\in U}X_{ijk}=1, \forall j\in V$, (2)

$$\sum_{j\in V}X_{ijk}\leq 1, \forall i\in B, k\in U \text{ , (3)}$$

$$\sum_{m\neq j\in T}\sum_{h<k\in O}(C_{im}X_{imh}+y_{imh})+y_{ijk}-(A_j-S_i)X_{ijk}\geq 0, \text{ , (4)}$$

$\forall i\in B, j,\in T, k\in O$

$X_{ijk}\in\{0,1\}$, Integer, $y_{ijk}\geq 0$ Positive (decision variables), (5) where a_j is a weight for ship j.

The objective function seeks to minimize the weighted service time. Constraints *(2)* ensure that ships must be serviced once; constraints *(3)* that each berth services one ship at a time; and constraints *(4)* that each ship is serviced after its arrival. For further explanations of the objective function and the constraints the reader is referred to Imai et al. (2003). The resulting formulation is non-linear (MINLP). MINLP problems are precisely so difficult to solve, because they combine all the difficulties of both of their subclasses: the combinatorial nature of mixed integer programs (MIP) and the difficulty in solving nonconvex (and even convex) nonlinear programs (NLP). Because subclasses MIP and NLP are among the class of theoretically difficult problems (NP-complete), so it is not surprising that solving MINLP can be very challenging. Imai et al. (2003) reduced the problem to a Lagrangian relaxation problem in order to look into the availability of the subgradient optimization. Although the subgradient method was adaptable to this problem, enormous computational effort was expected because the relaxed problem was a quadratic assignment problem which was NP-hard. Therefore, they eventually employed a GA based

heuristic algorithm, an approach widely utilized for complicated combinatorial problems.
In order to avoid these issues, this paper presents a reformulation of the problem as a linear problem, as shown in equations 6 through 11.

(LP2): $\min \sum_i \sum_j \sum_k -a_j DT_{ijk}$, (6)

Subject to: $\sum_i \sum_j \sum_k X_{ijk} = 1, \forall j$, (7)

$\sum_{j \in V} X_{ijk} \le 1, \forall i \in B, k \in U$, (8)

$$\sum_{m \neq j \in T} \sum_{h<k \in O} (C_{im} X_{imh} + y_{imh}) + y_{ijk} - (A_j - S_i) X_{ijk} \ge 0, \quad \forall i \in B, j \in T, k \in O \quad (9)$$

$$DT_{ijk} \le M(1 - X_{ijk}) - (C_{ij} + S_i - A_j) X_{ijk,} - y_{ijk} - \sum_{j \neq m \in T} \sum_{h<k \in O} (C_{im} X_{imh}) - \sum_{j \neq m \in T} \sum_{h<k \in O} y_{imh}, \quad \forall i \in B, j \in T, k \in O \quad (10)$$

$X_{ijk} \in \{0,1\}$, Integer, $y_{ijk} \ge 0$, $DT_{ijk} \le 0$, (11), where DT_{ijk} is an auxiliary variable.

Lemma I: (LP2) is a linear transformation of (LP1)

Proof: If X_{ijk}=0 then equation 10 is reduced to zero. On the other hand if X_{ijk}=1 then equation 10 is reduced to the following equality:

$$DT_{ijk} = -\{(C_{ij} + S_i - A_j) X_{ijk,} + y_{ijk} + \sum_{j \neq m \in T} \sum_{h<k \in O} (C_{im} X_{imh}) + \sum_{j \neq m \in T} \sum_{h<k \in O} y_{imh}\},$$

which is the finish time of ship *j* serviced at berth *i* with (*k-1*) predecessors. Thus the objective function becomes:

$$\sum_i \sum_j \sum_k a_j \{(C_{ij} + S_i - A_j) X_{ijk}\} + y_{ijk} + \sum_i \sum_j \sum_k \left(\sum_{j \neq m \in T} \sum_{h<k \in O} C_{im} X_{imh} + \sum_{j \neq m \in T} \sum_{h<k \in O} y_{imh} \right)$$

or:

$$\sum_i \sum_j \sum_k (C_{ij} + S_i - A_j + \sum_{j \neq m \in T} \sum_{h<k \in O} (C_{im} X_{imh})) a_j + \sum_i \sum_j \sum_k (y_{ijk} + \sum_{j \neq m \in T} \sum_{h<k \in O} y_{imh}) a_j .$$

This equation is the linear version of the objective function of (LP1).

To verify Lemma I, a number of computational experiments were preformed. Several datasets were generated randomly, but systematically for small to medium instances of the problem (2-5 berths, 5-25 ships, handling time 4-48 hours per ship). All experiments resulted in the same assignment (using LP1 and LP2). Both solution procedures were coded in GAMS[1] on a Precision 670. An example of an instance of 2 berths and 10 ships is presented in table 1.

Table 1. Assignment results for toy problem (2 berths and 10 ships)

	Service Order						
Berth.Ship	S1	S2	S3	S4	S5	S6	S7
Berth1.Ship2		×					
Berth1.Ship5	×						
Berth1.Ship6			×				
Berth1.Ship7					×		
Berth1.Ship9						×	
Berth1.Ship10							×
Berth2.Ship1	×						
Berth2.Ship3		×					
Berth2.Ship4				×			
Berth2.Ship8						×	

Note: *Computational Time=67sec*

4 Proposed Heuristic Solution Procedure

The formulation of the DBAP is a mixed integer program, linear in the constraints and the objective function. It is not known though to be solved in polynomially-bounded time for large instances. This may be solved by a branch and bound algorithm but that approach would be time-consuming. Since in practice frequent changes in estimated ship arrival times are expected, the formulation (LP2) may be required to be solved frequently to obtain a new berth allocation scheme, to cope with these changes in ship arrival times. Consequently the branch and bound algorithm does not seem suitable for solving (LP2) for large instances. This encourages us to develop a heuristic for the problem. The heuristic is based on the concept of online optimization with bounded migration (Sanders et al., 2005).

Heuristic I

Step 0: Sort ships in ascending order of arrival time S={ S_1, S_2, ……, S_n, S_{n+1}, S_j}, where $A_n<A_{n+1}$

Step 1: Select the first n^1 ships N={S_1, S_2, ……, S_n}

[1] http://www.gams.com/

Step 2: Solve LP1 using N

Step 3: Check how many ships from N have finished service before the arrival of ship S_{n+1}. Name this set ND. If ND=empty then include in N all ships that arrive before the finish of the earliest "job" from N and go to step 2 else remove from N ships serviced before the arrival of ship S_{n+1} and add S_{n+1} to N

Step 4: Go to step 2 until N=ND= empty set.

5 Conclusions/Discussion

In this study, we formulated the BAP with ship service priorities. An existing non-linear dynamic berth allocation formulation, proposed in the literature, was reduced to its linear contra part. Service priority considerations reflect practices adopted by port terminal operators and ocean carriers in formulating their contractual service agreements and therefore are closer to port industry strategies followed nowadays in assigning berthing capacity. A number of numerical experiments for small instances were conducted, showing that the new formulation provides the same assignment as the original formulation and at the same time it can be more easily solved. Finally, a heuristic was proposed to cope with large instances of the problem. Future research could focus on experimental analysis of medium to large instances using the proposed heuristic.

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