Practical Aspects of Predictive Control Algorithms

A.W.ORDYS, J.M.KARWATZKI, A.CURLEY
Automotive Control Centre
Kingston University
London
UK

Abstract: This paper presents some algorithms for Model Based Predictive Control. The algorithms use state-space models, which have recently become increasingly attractive to researchers. There are, however, a number of issues which should be taken into consideration when programming a state-space predictive controller. The algorithm will depend on the type of the model, the assumptions about disturbances acting on the system, the objectives of the controller and the form of the performance index. This article investigates these different cases and provides practical guidelines on how a predictive control algorithm should be constructed. A simple example illustrates the difficulties and shows how they may be overcome to enable successful implementation of MBPC in state space.

Keywords: Model Based Predictive Control, state space systems, Kalman filter, MATLAB.

1 Introduction

Model Based Predictive Control (MBPC) algorithms originated from the process industry. The first reported applications have been in oil refineries with the use of models in the form of Finite Impulse Response or Finite Step Response [5]. Since that time, Model Predictive Control has been continuously gaining popularity. Many industrial applications have been reported ([13],[14],[17]) and other forms of model description have been proposed. Among those, the discrete-time transfer function model, and the resulting Generalized Predictive Controller, is especially well known ([3],[4]). In recent years, Model Predictive Control is being considered for fast systems, with applications in aerospace, metal processing, automotive and defense areas. In parallel with this, state-space models are more commonly used and are becoming a standard for this type of control. State space is a convenient form of description, especially for optimal design of multivariable stochastic systems. When designing state space predictive controllers the problem formulation must accommodate the system properties, often related to the type of the model which is used, and the desired features of the closed loop system, which, for fast systems, may be more demanding than in traditional applications of predictive control. This article shows, in a practical way, how to build the algorithms including a method of determining state estimates, and a technique for obtaining optimal control. The rest of the paper is organized as follows: Section 2 presents the state-space model of the system. Section 3 discusses the state estimation, using a Kalman filter. Section 4 discusses the construction of a state-space predictive control algorithm. Section 5 provides the conclusions from this work.

2 System Description

Consider the system model in sampled data state space form:

$$\begin{aligned} x_{t+1} &= A\cdot x_t + B\cdot u_t + G\cdot v_t \\ y_t &= D\cdot x_t + w_t \end{aligned} \tag{1}$$

where:

x_t is a vector of system states. (The initial state of the system x_0 is assumed to have Gaussian distribution with the mean value:

$$\bar{x}_0 = \mathrm{E}\{x_0\} \tag{2}$$

and the co-variance matrix:

$$\mathrm{E}\left\{\left(x_o - \mathrm{E}\{x_0\}\right)\left(x_o - \mathrm{E}\{x_0\}\right)^T\right\} = X_0\ ) \tag{3}$$

u_t is a vector of control signals,

y_t is a vector of output signals,

v_t and w_t are vectors of disturbances ([7]), assumed to be Gaussian white noises with zero mean value and the co-variance matrix:

$$\mathrm{E}\left\{\begin{bmatrix} v_t \\ w_t \end{bmatrix}\begin{bmatrix} v_t^T & w_t^T \end{bmatrix}\right\} = \begin{bmatrix} V & Z \\ Z^T & W \end{bmatrix} \tag{4}$$

Correlation between state and output disturbances can occur as a result of converting from a

polynomial description (CARMA or Box-Jenkins) into a state space description.
A, B, G, D are constant matrices.
As a practical example take the case of an under-damped, second order, non-minimum phase system:

$$x_{t+1} = \begin{bmatrix} 1.94 & -0.98 \\ 1.00 & 0 \end{bmatrix} x_t + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u_t + \begin{bmatrix} 1.0 \\ 0.65 \end{bmatrix} v_t$$

$$y_t = \begin{bmatrix} -0.16 & 0.24 \end{bmatrix} x_t + w_t$$

with the initial state co-variance matrix:

$X_0 = \begin{bmatrix} 0.8 & 0 \\ 0 & 1.3 \end{bmatrix}$ and the noise co-variance matrix:

$$\mathrm{E}\left\{ \begin{bmatrix} v_t \\ w_t \end{bmatrix} \begin{bmatrix} v_t^T & w_t^T \end{bmatrix} \right\} = \begin{bmatrix} 1.4 & 0.015 \\ 0.015 & 0.7 \end{bmatrix}$$

The state and output noises are correlated. The next section shows how to de-couple these noises to enable further development of the MBPC algorithms.

2.1 Systems Requiring Integral Action

Predictive control algorithms usually require integral action to eliminate steady state errors. Integral action can be conveniently introduced, at the model definition stage, by the following substitution([12]):

$$\chi_{t+1} = \begin{bmatrix} x_{t+1} \\ u_t \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x_t \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} \Delta u_t + \begin{bmatrix} G \\ 0 \end{bmatrix} v_t \tag{5}$$

$$y_t = \begin{bmatrix} D & 0 \end{bmatrix} \chi_t + w_t \tag{6}$$

where: $\Delta u_t = u_t - u_{t-1}$ is a new control signal.

Remark: For systems with so-called direct feed through a more general state space formulation can used:

$$\begin{aligned} x_{t+1} &= A \cdot x_t + B \cdot u_t + G \cdot v_t \\ y_t &= D \cdot x_t + C u_t + w_t \end{aligned} \tag{7}$$

In this case, the state vector is extended ([10]) as follows:

$$\chi_{t+1} = \begin{bmatrix} x_{t+1} \\ u_{t+1} \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} x_t \\ u_t \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta u_{t+1} + \begin{bmatrix} G \\ 0 \end{bmatrix} v_t \tag{8}$$

$$y_t = \begin{bmatrix} D & C \end{bmatrix} \chi_t + w_t \tag{9}$$

where: $\Delta u_{t+1} = u_{t+1} - u_t$ is a new control signal.

3 State Estimation Procedures

3.1 Correlated State and Output Noise

If the disturbances in the state space formulation are correlated, i.e. Z in equation (4) is non-zero, a transformation is applied which removes the correlation. This then enables a straightforward use of Kalman filter equations. Following the idea from [1], denote:

$$\zeta_t = w_t - Z V^{-1} v_t \tag{10}$$

Then:

$$\mathrm{E}\left\{ \begin{bmatrix} v_t \\ \zeta_t \end{bmatrix} \begin{bmatrix} v_t^T & \zeta_t^T \end{bmatrix} \right\} = \begin{bmatrix} V & 0 \\ 0 & W - Z V^{-1} Z^T \end{bmatrix} \tag{11}$$

i.e. the disturbances v_t and ζ_t are uncorrelated. Substituting (10) into the output equation in (1) yields:

$$\begin{aligned} x_{t+1} &= A \cdot x_t + B \cdot u_t + G \cdot v_t \\ y_t &= D \cdot x_t + \zeta_t + Z V^{-1} v_t \end{aligned} \tag{12}$$

The state is now expanded by adding the noise signal v_t:

$$\chi_{t+1} = \begin{bmatrix} x_{t+1} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} A & G \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_t \\ v_t \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_t + \begin{bmatrix} 0 \\ I \end{bmatrix} v_{t+1} \tag{13}$$

$$y_t = \begin{bmatrix} D & Z V^{-1} \end{bmatrix} \begin{bmatrix} x_t \\ v_t \end{bmatrix} + \zeta_t \tag{14}$$

Equations (13) and (14) describe the system with uncorrelated state and output disturbances. The system initial state has a Gaussian distribution with the mean value:

$$\bar{\chi}_0 = \begin{bmatrix} \mathrm{E}\{x_0\} \\ 0 \end{bmatrix}$$

and the co-variance matrix:

$$\mathrm{E}\left\{ \left(\chi_o - \mathrm{E}\{\chi_0\}\right)\left(\chi_o - \mathrm{E}\{\chi_0\}\right)^T \right\} = \begin{bmatrix} X_0 & 0 \\ 0 & V \end{bmatrix}$$

For our practical second order example the noise de-coupling routine leads to the following state space representation:

$$x_{t+1} = \begin{bmatrix} 1.94 & -0.98 & 1 \\ 1 & 0 & 0.65 \\ 0 & 0 & 0 \end{bmatrix} x_t + \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} u_t + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_t$$

$$y_t = \begin{bmatrix} -0.157 & 0.236 & 0.011 \end{bmatrix} x_t + \xi_t$$

with: $\mathrm{E}\left\{\xi_t^2\right\} = 0.699$

3.2 Standard Kalman Filter

Assuming known distribution of the initial state of the system (equation (3)) the estimate of the system state can be calculated using standard Kalman filter equations:

$$\begin{aligned} \hat{x}_{t+1/t+1} &= A\hat{x}_{t/t} + Bu_t + K_{t+1}\left[y_{t+1} - D\left(A\hat{x}_{t/t} + Bu_t\right)\right] = \\ &= \hat{x}_{t+1/t} + K_{t+1}\left[y_{t+1} - D\hat{x}_{t+1/t}\right] \end{aligned} \tag{15}$$

$$K_{t+1} = P_{t+1/t} D^T \left(D P_{t+1/t} D^T + W \right)^{-1} \quad (16)$$

$$P_{t+1/t} = A P_{t/t} A^T + G V G^T \quad (17)$$

$$P_{t+1/t+1} = P_{t+1/t} - P_{t+1/t} D^T \left(D P_{t+1/t} D^T + W \right)^{-1} D P_{t+1/t} \quad (18)$$

or:

$$P_{t+1/t} = G V G^T + A\left[P_{t/t-1} - P_{t/t-1} D^T \left(D P_{t/t-1} D^T + W \right)^{-1} D P_{t/t-1} \right] A^T \quad (19)$$

with the initial conditions (assuming that the first available measurement is $\boldsymbol{y_0}$):

$$\hat{x}_{0/-1} = \mathrm{E}\{x_0\} \quad (20)$$

$$P_{0/-1} = X_0 \quad (21)$$

or (equivalent to the above):

$$K_0 = X_0 D^T \left(D X_0 D^T + W \right)^{-1} \quad (22)$$

$$P_{0/0} = X_0 - X_0 D^T \left(D X_0 D^T + W \right)^{-1} D X_0 \quad (23)$$

$$x_0 = \mathrm{E}\{x_0\} + K_0 \left[y_0 - D\mathrm{E}\{x_0\} \right] \quad (24)$$

3.3 Steady-state Kalman Filter

When the iterations described by (17) and (18) reach the steady state the Kalman filter becomes stationary and it is then described by the equations:

$$\hat{x}_{t+1/t+1} = A\hat{x}_{t/t} + Bu_t + K\left[y_{t+1} - D\left(A\hat{x}_{t/t} + Bu_t \right) \right] = (A - KDA)\hat{x}_{t/t} + (B - KDB)u_t + K y_{t+1} \quad (25)$$

$$K = P_{pred} D^T \left(D P_{pred} D^T + W \right)^{-1} \quad (26)$$

and $\boldsymbol{P_{pred}}$ is a steady-state solution of the Kalman filter prediction equation, i.e. it fulfills:

$$P_{pred} = G V G^T + A\left[P_{pred} - P_{pred} D^T \left(D P_{pred} D^T + W \right)^{-1} D P_{pred} \right] A^T \quad (27)$$

with the initial conditions given by (21).

For our practical example, the steady-state Kalman filter equation is obtained as follows:

$$\hat{x}_{t+1} = \begin{bmatrix} 2.098 & -1.33 & 1.01 \\ 1.15 & -0.35 & 0.66 \\ 0.001 & 0.003 & 0 \end{bmatrix} \hat{x}_t + \begin{bmatrix} 0.679 \\ 0.180 \\ 0.001 \end{bmatrix} u_t + \begin{bmatrix} 2.283 \\ 2.288 \\ 0.017 \end{bmatrix} y_{t+1}$$

3.4 Kalman Filter with Integral Action

If the model used in the controller design requires integral action, as described by equations (5) and (6) certain simplifications to the Kalman filter equations are possible. The initial conditions can be set-up as follows:

$$\hat{\chi}_{0/-1} = \begin{bmatrix} \hat{x}_{0/-1} \\ \hat{u}_{-1/-2} \end{bmatrix} = \begin{bmatrix} \mathrm{E}\{x_0\} \\ u_{-1} \end{bmatrix} \quad (28)$$

$\boldsymbol{u_{-1}}$ is here the known (constant) level of the control signal before the predictive controller is put into action.

$$P_{0/-1} = \begin{bmatrix} X_0 & 0 \\ 0 & 0 \end{bmatrix} \quad (29)$$

Using the definitions of block matrices as in equations (5) and (6) and denoting those block matrices by $\breve{A}, \breve{B}, \breve{G}, \breve{D}$, the iterations of the Kalman filter Riccati equation as described by (17) and
(18) will become:

$$\breve{P}_{t+1/t} = \begin{bmatrix} P_{t+1/t} & 0 \\ 0 & 0 \end{bmatrix} \quad (30)$$

where $P_{t+1/t}$ is updated using the prediction co-variance equation
(19).

The Kalman filter gain is given by:

$$\breve{K}_{t+1} = \breve{P}_{t+1/t} \breve{D}^T \left(\breve{D} \breve{P}_{t+1/t} \breve{D}^T + W_t \right)^{-1} = \begin{bmatrix} K_{t+1} \\ 0 \end{bmatrix} = \begin{bmatrix} P_{t+1/t} D^T \left(D P_{t+1/t} D^T + W_t \right)^{-1} \\ 0 \end{bmatrix} \quad (31)$$

Then, the state estimate update is given by:

$$\hat{\chi}_{t+1/t+1} = \breve{A}\hat{\chi}_{t/t} + \breve{B}\Delta u_t + \breve{K}_{t+1}\left[y_{t+1} - \breve{D}\left(\breve{A}\hat{\chi}_{t/t} + \breve{B}\Delta u_t \right) \right] = \begin{bmatrix} \hat{x}_{t+1/t+1} \\ \hat{u}_{t/t} \end{bmatrix} = \begin{bmatrix} A\hat{x}_{t/t} + Bu_t + K_{t+1}\left[y_{t+1} - D\left(A\hat{x}_{t/t} + Bu_t \right) \right] \\ u_t = u_{t-1} + \Delta u_t \end{bmatrix} \quad (32)$$

Therefore, the calculations of the Kalman filter can be performed for the original system, i.e. the system described by equations (1) or (13) and (14).

4 State Space Predictive Control Algorithms

Many early state space predictive control algorithms presented in the literature, e.g. [8], were a transformation into state space of the Generalized Predictive Controller of Clarke et al [3]. Later on, the research on stability and robustness resulted in many modifications and new algorithms, e.g. [14], [16]. Many authors considered terminal constraints or a penalty on the final state [2], [6]. This would guarantee the stability of the predictive controller by linking it to the design procedure of Linear

Quadratic Gaussian control, which has well established and well understood stability and robustness properties. This section will outline two algorithms of predictive control: the state-space version of Generalized Predictive Controller (GPC) and the Dynamic Performance Predictive Controller (DPC) [11]. The latter combines the features of predictive and LQG approaches.

4.1 State-Space Generalized Predictive Controller (GPC)

Following [8], the k-step prediction of the output signal may be calculated from the relationship:

$$\hat{y}_{(t+k)/t} = \mathrm{E}\{y_{t+k}|t\} = DA^k\hat{x}_{t/t} + \sum_{j=1}^{k} DA^{k-j}Bu_{t+j-1} \quad (33)$$

The performance index to be minimized is defined as follows:

$$J_t = \mathrm{E}\left\{\sum_{j=0}^{N}\left[\left(y_{t+j+1} - r_{t+j+1}\right)^T\left(y_{t+j+1} - r_{t+j+1}\right) + +\lambda \cdot u_{t+j}{}^T \cdot u_{t+j}\right]\right\} \quad (34)$$

where: r_{t+j+1} represents a vector of reference (set point) signals, $\lambda \geq 0$ is a control weighting factor. Denoting:

$$R_{t,N} = \begin{bmatrix} r_{t+1} \\ r_{t+2} \\ \vdots \\ r_{t+N+1} \end{bmatrix}, \quad U_{t,N} = \begin{bmatrix} u_t \\ u_{t+1} \\ \vdots \\ u_{t+N} \end{bmatrix} \quad (35)$$

where $R_{t,N}$ is a block vector of N+1 future reference signals, $U_{t,N}$ is a block vector of N+1 future control signals. The performance index, neglecting a constant (control independent) term, can be expressed in a static (vector) form:

$$J_t = \left(\hat{Y}_{t,N} - R_{t,N}\right)^T\left(\hat{Y}_{t,N} - R_{t,N}\right) + \lambda \cdot U_{t,N}{}^T \cdot U_{t,N} \quad (36)$$

Thus, by finding the stationary point, the vector of optimal control signals becomes:

$$U_{t,N} = \left(S_N{}^T \cdot S_N + \lambda \cdot \mathrm{I}\right)^{-1} S_N{}^T\left(R_{t,N} - F_{t,N}\right) \quad (37)$$

where the following matrix and vector are defined:

$$S_N = \begin{bmatrix} DB & O & \cdots & O \\ DAB & DB & & \vdots \\ \vdots & \vdots & \ddots & O \\ DA^NB & DA^{N-1}B & \cdots & DB \end{bmatrix}, \quad \Phi_N = \begin{bmatrix} D \\ DA \\ \vdots \\ DA^N \end{bmatrix} \quad (38)$$

$$F_{t,N} = \begin{bmatrix} D \\ D \cdot A \\ \vdots \\ D \cdot A^N \end{bmatrix} \cdot A \cdot \hat{x}_{t/t} = \Phi_N \cdot A \cdot \hat{x}_{t/t} \quad (39)$$

4.2 Dynamic Performance Predictive Controller (DPC)

This is a solution combining properties of the GPC and LQ problem. A more detailed derivation can be found in [11]. The optimal control is given by the formula:

$$U_{t,N} = -\left(\lambda\mathrm{I} + S_N{}^T S_N + \beta^T\tilde{H}^1\beta\right)^{-1} \times \left[\left(S_N{}^T\Phi_N A + \beta^T\tilde{H}^1 A\right)\hat{x}_t + \left(\beta^T\tilde{H}^2\Theta_N - S_N{}^T\right)R_{t,N}\right] \quad (40)$$

where $\tilde{H}^1$ and $\tilde{H}^2$ represent the steady-state (algebraic) solutions of the coupled Ricatti equations:

$$\tilde{H}_j^1 = A^T\left(\Phi_N{}^T\Phi_N + \tilde{H}_{j+1}^1\right)A - A^T\left(\Phi_N{}^T S_N + \tilde{H}_{j+1}^1\beta\right) \times \left(\lambda\mathrm{I} + S_N{}^T S_N + \beta^T\tilde{H}_{j+1}^1\beta\right)^{-1}\left(S_N{}^T\Phi_N + \beta^T\tilde{H}_{j+1}^1\right)A \quad (41)$$

$$\tilde{H}_j^2 = -A^T\Phi_N{}^T + A^T\tilde{H}_{j+1}^2\Theta_N - A^T\left(\Phi^T S_N + \tilde{H}_{j+1}^1\beta\right) \times \left(\lambda\mathrm{I} + S_N{}^T S_N + \beta^T\tilde{H}_{j+1}^1\beta\right)^{-1}\left(\beta^T\tilde{H}_{j+1}^2\Theta_N - S_N{}^T\right) \quad (42)$$

Θ_N is a transition matrix for the reference signal, i.e. it is assumed that:

$$R_{t+1,N} = \Theta_N \cdot R_{t,N} \quad (43)$$

4.3 State Space Formulation of the Controller

The equations of the predictive control and of the Kalman filter can be combined together, leading to a state space formulation of the controller. The optimal control is described by equation (37) or (40):

$$U_{t,N} = -M_X\hat{\chi}_t + M_R R_{t,N} \quad (44)$$

where, for the system with integral action, the vector $\hat{\chi}_t$ comprises the state estimate and the control signal equation (32)..

For our practical example, the system is further expanded, to accommodate integral action (equations (5) and (6)). The controller tuning parameters are selected as:

$N_1 = 1,\ N_2 = 50,\ N_u = 3,\ \lambda = 0.1$

Therefore, for this example M_x has dimension (3x4) and M_R has dimension (3x50).

For the DPC controller, the matrix M_x is:

$$M_x = \begin{bmatrix} 1.39 & -1.14 & 0.59 & 0.90 \\ -0.84 & 0.375 & -0.45 & 0.16 \\ -0.57 & 0.77 & -0.17 & -0.07 \end{bmatrix}$$

Combining equations (4) and (5) the vector $\boldsymbol{U_{t,N}}$ in this case is expressed as follows:

$$U_{t,N} = \begin{bmatrix} \Delta u_t \\ \Delta u_{t+1} \\ \vdots \\ \Delta u_{t+N} \end{bmatrix} = \begin{bmatrix} u_t - u_{t-1} \\ u_{t+1} - u_t \\ \vdots \\ u_{t+N} - u_{t+N-1} \end{bmatrix} \quad (45)$$

Applying the receding horizon strategy, from equation (44) only the first n_u rows are selected (n_u is here the dimension of the control vector $\boldsymbol{u_t}$), corresponding to calculating only Δu_t from the control vector $\boldsymbol{U_{t,N}}$:

$$\begin{aligned} \Delta u_t &= u_t - u_{t-1} = \gamma \hat{\chi}_{t/t} + \gamma_R R_{t,N} = \\ &= \gamma_x \hat{x}_{t/t} + \gamma_u u_{t-1} + \gamma_R R_{t,N} \end{aligned} \quad (46)$$

Substituting the steady-state state estimate from the Kalman filter equation (32) obtains:

$$\begin{aligned} & u_t - u_{t-1} = \gamma_x \hat{x}_{t/t} + \gamma_u u_{t-1} + \gamma_R R_{t,N} = \\ & = \gamma_x \left\{ A\hat{x}_{t-1/t-1} + Bu_{t-1} + K\left[y_t - D\left(A\hat{x}_{t-1/t-1} + Bu_{t-1} \right) \right] \right\} + \\ & + \gamma_u u_{t-1} + \gamma_R R_{t,N} \end{aligned} \quad (47)$$

After shifting the equation one step forward:

$$\begin{aligned} u_{t+1} &= \gamma_x (\mathrm{I} - KD) A\hat{x}_{t/t} + \left[\gamma_u + \mathrm{I} + \gamma_x (\mathrm{I} - KD) \right] u_t + \\ & + \gamma_x K y_{t+1} + \gamma_R R_{t+1,N} \end{aligned} \quad (48)$$

This, combined with the state estimate equation (32) gives the controller description in the state space form:

$$\begin{bmatrix} \hat{x}_{t+1/t+1} \\ u_{t+1} \end{bmatrix} = \\ = \left[\begin{array}{c|c} (\mathrm{I} - KD)A & (\mathrm{I} - KD)B \\ \hline \gamma_x (\mathrm{I} - KD)A & \gamma_u + \mathrm{I} + \gamma_x (\mathrm{I} - KD)B \end{array} \right] \begin{bmatrix} \hat{x}_{t/t} \\ u_t \end{bmatrix} + \quad (49) \\ + \left[\begin{array}{c|c} K & 0 \\ \hline \gamma_x K & \gamma_R \end{array} \right] \begin{bmatrix} y_{t+1} \\ R_{t+1,N} \end{bmatrix}$$

For our practical example, with the receding horizon control strategy, the first row of M_x , corresponding to the current control action is implemented. Finally, the closed loop poles can be found as:

$p_{1,2} = 0.873 \pm 0.203i$ - the poles of the Kalman filter

$$\left. \begin{aligned} p_3 &= 0.635 \\ p_{4,5} &= 0.355 \pm 0.161i \end{aligned} \right\} \text{- the remaining poles.}$$

4.4 The MBPC Design Sequence

The original system is defined, i.e. the matrices: $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{G}$, $\boldsymbol{D}$ are provided, the initial state co-variance matrix X_0 and the noise co-variance matrix consisting of $\boldsymbol{V}$, $\boldsymbol{W}$, $\boldsymbol{Z}$ are given.

If the noises are correlated, the state space equations are expanded by adding the noise (equations (13) and (14)).

The Kalman filter gains $\boldsymbol{A_{K2}}$, $\boldsymbol{B_{K2}}$, $\boldsymbol{K}$ (equations (25) and (26)) and the prediction error covariance matrix (equation (27)) are calculated:

$\hat{x}_{t+1/t+1} = A_{K2}\hat{x}_{t/t} + B_{K2}u_t + Ky_{t+1}$

If integral action is needed, the state space equations are further expanded using equations (5) and (6).

The controller tuning parameters $\boldsymbol{N_1}$, $\boldsymbol{N_2}$, $\boldsymbol{N_u}$, $\boldsymbol{\lambda}$ are entered.

The parameters Φ_N, $\boldsymbol{S_N}$ (equation (38)), β (equation (40)), $\boldsymbol{L}$ (control weighting matrix λI needed for the solution of the Riccati equations (41), (42)), Θ_N (equation (43)) are calculated:

Remark: for the GPC algorithm, only Φ_N, $\boldsymbol{S_N}$ are needed.

The matrices associated with the state estimate vector and with the reference signals vector from equation (44) are calculated:

The MATLAB functions performing the above are available from the authors.

4.5 Incorporation of Constraints

Many practical systems have physical constraints (input/output) incorporated in them ([2]) and a control design for constrained systems is attractive to control practitioners. When there are constraints in the system the predictive control algorithm cannot be described by analytical equations and, instead, a numerical optimization has to be performed. However, a substantial part of calculations can still be performed analytically. Namely, when writing the algorithms for constrained predictive control, the procedures of defining the system states, incorporating integral action defining the noise signals and estimating the states using a Kalman filter remain unchanged. The optimization problem is represented by a quadratic equation (e.g. (36)) which, in the constrained case, has to be solved numerically. However, unlike the unconstrained case, where the matrices M_X and M_R can be calculated off line, the constrained case requires on line calculations of predictive control action.

5 Conclusions

The article uses a practical example to describe state space implementation of Model Based Predictive Control. The procedure leading from the initial problem formulation to the controller design is explained in a fairly detailed way. In particular, the algorithms to deal with: correlated noises, integral action, simplification of the Kalman filter and state space representation of the controller are presented.

References:

[1] Anderson, B.D.O. and J. Moore, *Optimal Filtering*, Prentice Hall, Engelwood Cliffs, 1979

[2] Bartlett R.A., Biegler L.T., Backstrom J., Gopal V., *Quadratic programming algorithms for large-scale model predictive control*, Journal of Process Control, 2002

[3] Clarke, D.W., C. Mohtadi and P.S. Tuffs, *Generalised Predictive Control - Part 1. The basic algorithm*, Automatica, Vol. 23, No. 2, 1987, pp.137-148,

[4] Clarke, D.W., C. Mohtadi and P.S. Tuffs, *Generalised Predictive Control - Part 2. Extensions and Interpretations*, Automatica, Vol. 23, No. 2, 1987, pp.149-160,

[5] Cutler, C.R. and B.L. Raemaker, *Dynamic matrix control - A computer control algorithm,* Proceedings JACC, San Francisco, USA, 1980

[6] Maciejowski J., Predictive Control with Constraints, Prentice Hall; 1st edition (May 2, 2002) ISBN: 0201398230

[7] Muske K.R., Badgwell T.A., *Disturbance modeling for offset-free linear model predictive control*, - Journal of Process Control, 2002 - Elsevier

[8] Ordys A.W. and David W. Clarke, *A state-space description for GPC controllers*, International Journal of Systems Science, Vol.24, No.9, 1993

[9] Ordys A.W., *Steady-state offset in predictive control*, American Control Conference, San Diego, USA, June 1999

[18]

[10] Ordys A.W. and A.W. Pike*, State-space Generalized Predictive Control incorporating direct through terms*, 37th IEEE Control andDecision Conference, Tampa, Florida, Dec. 1998

[11] Ordys A.W., M. Tomizuka and M. Grimble, *State-space dynamic performance preview predictive controller*, Transactions of ASME, Journal of Dynamic Systems, Measurement and Control, to appear.

[12] Hangstrup M., A.W. Ordys and M.J. Grimble *Dynamic algorithm of LQGPC Predictive Control*, 4th International Symposium on Methods and Models in Automation and Robotics, Miedzyzdroje Aug. 1997

[13] Qin S.J., TA Badgwell T.A., *A survey of industrial model predictive control technology*, Control Engineering Practice, 2003

[14] Richalet J. *Industrial applications of model based predictive control.* Automatica, Vol.29, No.8, 1993, pp.1251-1274,

[15] Rossiter, J.A., Rice, M.J., and Kouvaritakis, B., "*A numerically robust state-space approach to stable predictive control strategies*", Automatica**,** 34, 1998, pp. 65-73

[16] Rossiter J.A., *Model-Based Predictive Control: A Practical Approach*, CRC Press June 27, 2003, ISBN: 0849312914

[17] Sandoz D.J., Desforges M.J., Lennox B., Goulding P.R., *Algorithms for industrial model predictive control,* Computing & Control Engineering Journal, 2000