

Universal Structure of RC Oscillator and the Chaos Generation

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Abstract: This contribution is about a general structure of autonomous RC oscillator from the viewpoint of possible generation of chaotic waveforms. It is shown that one universal analog circuit can directly represent several strange state space attractors, which are topologically distinct. The so-called dynamical systems of class C and class F have been used as the mathematical models. It is well known that these systems can exhibit strange behavior if the corresponding internal parameters are carefully chosen. The numerical integration and Pspice simulations of several examples serve as an experimental verification of individual circuit configurations in time domain.

Key-Words: autonomous oscillator, chaos, strange attractors, simulation

1 Introduction

There are many practical applications which are based on the chaotic oscillators, like a modulation techniques, masking procedures, random bit generators or coders. From this point of view, the different ways leading to a practical implementation of such an electronic circuits seems to be usefull. In the most publications [1] [2] [3] the authors start with the mathematical model together with the numerical values of system parameters. The design procedures can be based on the integrator block schematics or classical circuit synthesis. The inverse approach, i.e. to analyze the existing electronic circuit from the viewpoint of chaos generation is also the common topic of many scientific articles [4] [5]. In this task, an engineer is restricted to the computer programs.

The aim of this paper is quite different. We focus our attention to the general class of dynamical systems, which can be expressed in compact matrix form as

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{b} h(\mathbf{w}^T \mathbf{x}), \quad (1)$$

where $\mathbf{x} \in \mathcal{R}^3$ are state variables, \mathbf{A} is a square matrix and \mathbf{b} , \mathbf{w} are column vectors. It is well known that a chaotic system must be nonlinear. This property brings the scalar saturation-type function

$$h(\mathbf{w}^T \mathbf{x}) = 0.5 \left(\left| \mathbf{w}^T \mathbf{x} + 1 \right| - \left| \mathbf{w}^T \mathbf{x} - 1 \right| \right), \quad (2)$$

and separates entire state space by two parallel surfaces into three regions. From (1) and (2) it is evident that the dynamical motion is affine in each region, making a brief analysis possible and quite simple. First step of the qualitative analysis is determination of the fixed point. For chaos, there are just three equilibria each located in distinct state space region. The system of three first-order differential equation (1) covers the extensive group of autonomous deterministic dynamical systems with symmetrical vector fields, which can be found in many different scientific fields.

2 Oscillator Configuration

The complete circuit for the modeling of chaos is shown in Fig.1. It is a fully analog oscillator consisting of three linear capacitors, nine linear resistors, three summing amplifiers and a network with piecewise-linear transfer characteristics. Starting with the differential equations describing this system, a short program in Mathcad has been writteln for automatic computation of the circuit element values. Keeping the normalized values of the capacitors constant, i.e. $c_1 = \pm 1$, $c_2 = \pm 1$, $c_3 = \pm 1$, it is not hard to prove that the values of the resistors r_{bi} ($i=1,2,3$) are uniquely given by the elements of the state vector \mathbf{b} . Similarly, the elements of the vector \mathbf{w} are directly the summing coefficients at the input of the nonlinear three-port (with reference ground node), which involves two diode limiters connected in cascade. Summing amplifiers S-1, S-2 and S-3 with two high-impedance inputs are realized by an inverting, noninverting or differential amplifier depending on the values of coefficients s_{nm} . The essential task is to keep the current flowing inside each functinal block low, otherwise we have a nonzero error terms formed into column vector Ψ in the set of differential equation (3), which can result into oscillator disfunction. Fortunately, we have a certain degree of freedom in the design of summing amplifiers. Thus, there is no need to insert a voltage buffer in the front of each amplifier. So final oscillator stands uncomplicated, having two or three integrated circuits, for example AD713 (four operational amplifiers in a single package).

To date, using the presented circuit, we model the so-called Chua's equations [6], the famous first and second canonical equivalent [7], corresponding reference model [8], dynamical systems with state matrices in the elemental canonical form [9], systems optimized from the viewpoint of eigenvalue sensitivities [10] as well as overdamped oscillators [11].

The extension to another dimension for a chance of hyperchaos is also very simple and intuitive. In spite of this, the main advantage of our conception is in its capability to model also nonlinear oscillator, i.e. the dynamical system described by a single third-order differential equation. The publication [12] shows that this is not possible when using a classical circuit synthesis. Although the most straightforward approach to solve this problem is based on the cascade connection of three inverting integrators with operational amplifiers, the final structure of universal RC oscillator has (after calculation of circuit elements) the same complexity. Note that individual circuit elements have a several different representations in terms of the entries \mathbf{A}_{ij} and \mathbf{b}_k . So we can choose the mapping which results into

minimum number of negative values. Tab.1 shows the real values of circuit elements after impedance and frequency denormalization. This listing covers only a very small part of the entire scale of possible dynamical behavior. In each row (some strange attractor) there are few negative values of resistors or capacitors, but each of them represents a grounded two-port and we can implement it as described in [13]. It is much more harder to practically realize floating negative resistor (two opamps needed) than grounded ones (consists of one opamp only). Since the nonlinearity is representing by a three-port network with slope $A_0=1$ in the inner segment of the vector field and $A_1=0$ in both outer segments, each negative impedance converter must work in the linear regime away from the saturation.

$$\frac{d\mathbf{u}}{dt} = \begin{pmatrix} -d_1(g_1 + g_{s1} + g_{b1}) & d_1 g_{s1} s_{12} & d_1 g_{s1} s_{13} \\ d_2 g_{s2} s_{21} & -d_2(g_2 + g_{s2} + g_{b2}) & d_2 g_{s2} s_{23} \\ d_3 g_{s3} s_{31} & d_3 g_{s3} s_{32} & -d_3(g_3 + g_{s3} + g_{b3}) \end{pmatrix} \mathbf{u} + \frac{1}{2} \begin{pmatrix} d_1 g_{b1} \\ d_1 g_{b1} \\ d_1 g_{b1} \end{pmatrix} (|\mathbf{w}^T \mathbf{u} + 1| - |\mathbf{w}^T \mathbf{u} - 1|) + \Psi \quad (3)$$

where $\mathbf{u}=(u_1 \ u_2 \ u_3)^T$ are voltages across capacitors and $d_i=1/c_i$ ($i=1,2,3$).

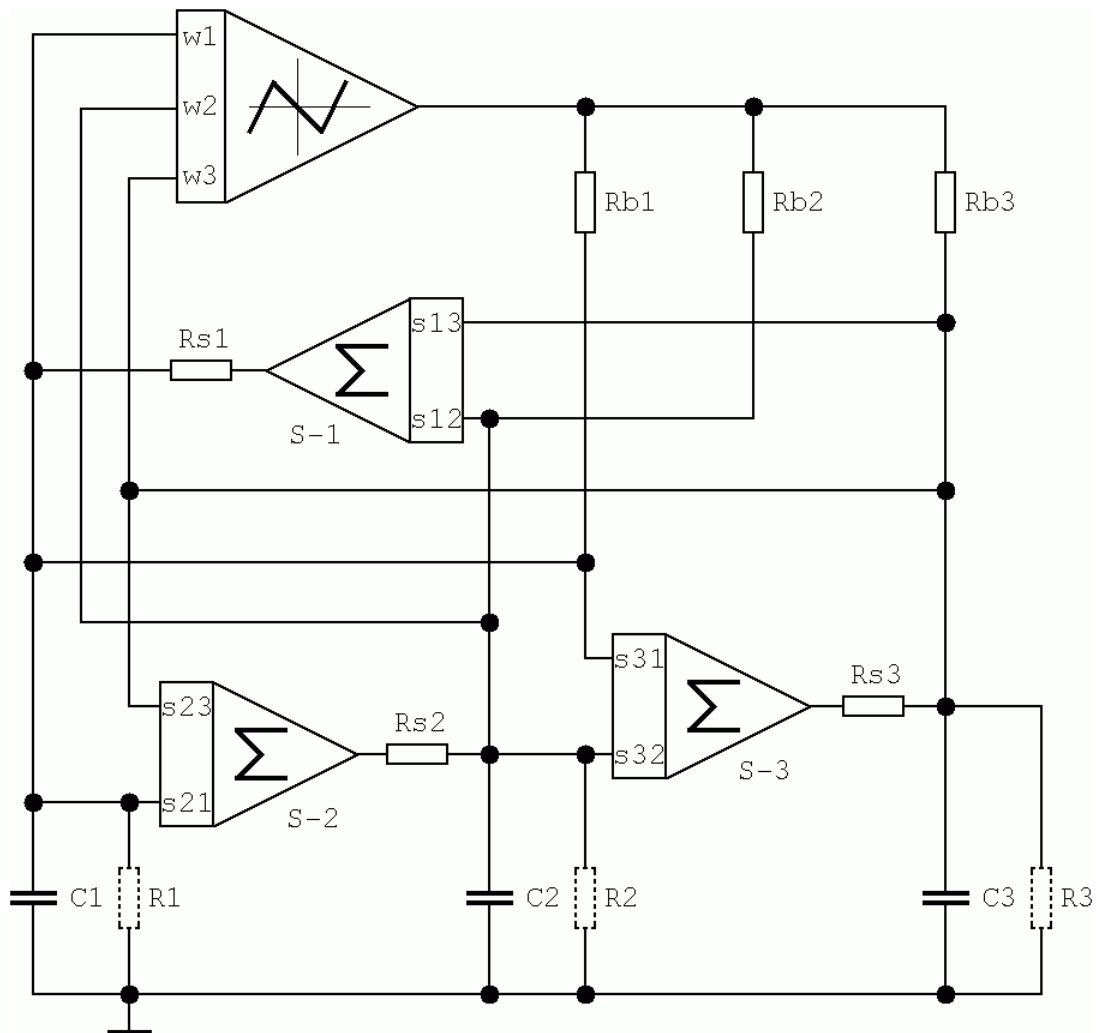


Fig.1: General structure of RC oscillator.



Fig.2: The plane projections of strange attractors modeled by the universal RC oscillator.

Other shape of piecewise-linear characteristics is also possible, but the mapping between model parameters and circuit element must be recalculated. The variant of RC oscillator without the resistors r_i ($i=1,2,3$) is also possible. Note that we can derive several facts using the equations (3). First of all, if both coefficients $s_{i1}=s_{i2}=0$ then the corresponding summing amplifier S-i can be removed and resistor r_{si} replaced with improved value of the resistor r_i . Also $s_{ik}=s_{ki}=0 \wedge r_{si}=r_{sk}$ leads to the minor change of the oscillator structure in the sense that single inputs of two summing amplifier are interchanged with single resistor between capacitors c_i and c_k .

Fig.2 shows a gallery of the strange attractors using a fourth-order numerical integration of (3) for real-valued circuit elements. The same geometrical structures have been observed in the case of Pspice circuit simulations. Numerical values for individual mathematical models can be found in publications listed in references. First row corresponds to the classical double-scroll attractor, seventh row represents dual double-scroll attractor [14]. Both systems belong to the class C on the contrary to the third and the last row, which expose two different members of class F. Of course, rows of Tab.1 labeled as a) b) etc. are in accordance with rows in Fig.2.

	C_1 [nF]	C_2 [nF]	C_3 [nF]	R_1 [kΩ]	R_2 [kΩ]	R_3 [kΩ]	R_{s1} [kΩ]	R_{s2} [kΩ]	R_{s3} [kΩ]	R_{b1} [kΩ]	R_{b2} [kΩ]	R_{b3} [kΩ]
a)	100	-100	100	-5.6	-1.2	-0.3	11.1	2.4	0.5	0.8	2.4	0.5
	100	100	100	∞	-1	-1	6	1	1	1	∞	∞
	100	-100	100	-0.4	-0.5	-0.5	0.8	0.95	1	0.8	0.95	0.47
b)	100	100	100	-1.4	-0.2	-0.17	2.8	0.44	0.33	0.09	0.44	0.33
	100	100	100	∞	-1	-1	0.11	1	1	1	∞	∞
c)	100	-100	-100	-0.91	-0.6	-13.8	1.8	1.23	27.6	1.2	1.23	27.6
	100	100	100	-0.7	-1	-1	1.41	1	1	1	∞	∞
	100	100	-100	-0.6	-0.87	-0.5	1.2	1.75	1	1.2	1.75	1.45
d)	-100	100	100	-0.32	-0.29	-0.14	0.63	0.57	0.29	0.57	0.57	0.29
	100	100	100	-0.43	-1	-1	0.86	1	1	1	∞	∞
	-100	-100	100	-0.29	-0.24	-0.1	0.57	0.49	0.2	0.57	0.49	0.21
e)	100	100	100	∞	-0.42	-0.5	82	0.85	1	5.6	0.85	1
	100	100	100	∞	-1	-1	1.1	1	1	1	∞	∞
	100	-100	100	-2.8	-0.33	-0.14	5.6	0.66	0.27	5.6	0.66	0.26
f)	100	100	100	-6.25	-0.05	-0.22	12.5	0.1	0.43	3.13	0.1	0.43
	100	100	100	∞	-1	-1	0.7	1	1	1	∞	∞
	100	-100	100	-15.6	-0.46	-0.18	31.3	0.92	0.36	31.3	0.92	0.35
g)	100	100	100	-0.68	-1	-1	1.36	1	1	1	∞	∞
	100	100	-100	-10	-10	-0.28	10	10	0.57	∞	∞	0.57
	100	100	-100	-10	-10	-0.28	10	10	0.57	∞	∞	0.58
h)	-100	100	100	-1.12	-4.32	-4.02	2.23	8.63	8.03	2.18	9.45	4.09
	-100	100	100	-1.12	-0.25	-4.02	2.23	0.5	8.03	2.13	0.5	4.09
	-100	-100	100	-9	-0.25	-40.2	1.79	0.51	80.3	17.6	0.51	40.9
	-100	100	100	-0.9	-0.26	-4.02	1.79	0.51	8.03	1.73	0.51	4.09

Tab.1: Real values of circuit elements for individual strange attractors.

Conclusion

In this article, a novel structure of analog universal RC oscillator working in voltage mode has been discussed. In the case of frequency rescaling into the direction of higher rates, the variant working in current mode should be a solution. Conventional opamps should be replaced with transconductance amplifiers or current feedback amplifiers. This is the topic of our further study.

In addition to the above mentioned advantages, the synthesized circuit is also well suited for the purpose of studying linear topological conjugacy between members of the dynamical systems of class C and F. After minor changes in the piecewise-linear three-port one can easily model a dynamical systems with two-segmented vector fields (absolute value or signum nonlinearity). Multi-scroll chaotic attractors [15] can be also produced.

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