A Modified Method for Blind Source Separation

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Abstract: - Blind source separation is an important but highly challenging technology in astronomy, physics, chemistry, life science, medical science, earth science, and applied sciences. Independent Component Analysis (ICA) employed technologies in applied computer science for blind source separation. In the separation of blind sources under multiple sensors, it can estimate approximately the types of signal. This study proposed a modified ICA algorithm which can estimate the actual phase and amplitude and retrieve the signals separated by blind source separation to its original state. This method has great potential for application in many different fields.

Key-Words: - Computer Science; Blind Source Separation; Independent Component Analysis

1 Introduction

Independent Component Analysis is a technology that incorporates statistics, computer science, and digital signal processing. It can estimate the original signal from the mixed signals being measured. To separate mixed signals of diverse sources is a difficult task. ICA is one of the approaches in the studies of signal separation. The early studies of blind source separation are represented by Herault et al [1]. They proposed a novel neural network learning algorithm [2]. Based on the feedback neural network, this learning algorithm can separate mixed independent signals and accomplish the goal of blind source separation by means of selecting odd nonlinear function to establish Hebb training. This paper immediately drew the attention among the scientists in the studies of neural network and signal processing. In 1994, Comon introduced the concept of independent component analysis and proposed cost functions and the uncertainty of signal retrieval, and so forth [3]. In 1995, Bell and Sejnowski published another landmark study [4],[5]. Their study has three major contributions. First, it is the first time the neural network with sigmoid nonlinear function was employed to cancel the high-order statistic correlation in the measured signals. Second, the study established contrastive function on the principle of information maximization, thereby incorporating ICA and information theories. Third, it developed a line iteration learning algorithm (Infomax algorithm), which successfully separated ten mixed voice signals. However, the algorithm requires matrix inverse and the convergence speed is slow. The effectiveness of the algorithm is affected by the ways of mixture in the original signals. It can only separate super Gaussian signals. Despite these disadvantages, the study led to increasing research interest in ICA. In 1999, Hyvärinen proposed a fast iteration algorithm called FastICA, with greatly increased convergence speed [6]-[9]. The major applications of ICA may be divided into two types. One is InfomaxICA proposed by Lee in 1998 [4],[5]. The other is FastICA by Hyvärinen in 1999. The latter is based on artificial neural network learning algorithm. After derivation, it may be completed by fixed-point algorithm with a faster convergence speed.

In recent years, there have been broad applications of ICA in diverse fields [10]-[24]. In addition to the processing of acoustic and imaging signals, applications of ICA are also found in feature extraction, financial data, and telecommunication. One of the major applications of ICA in biomedical research is the analysis of Electroencephalogram (EEG). ICA was proposed by Makeig et al. in 2002. They analyzed signals of electroencephalography (EEG) with ICA to understand the correlation between brain activities and finger movement [24]. EEG records electrical potentials of brain activities by means of sensors installed in various positions on the epicranium. The electrical potential is composed of basic components of brain activities and some noises [2],[23]. If these components are independent, ICA is able to retrieve the specific information that interests us regarding brain activities.
However, ICA is not without disadvantages. During estimation, aliasing and errors may occur. The estimate results may have opposite phase and unequal amplitude, leading to aliasing after the original signals are retrieved. This study proposed an modified ICA, which can estimate the actual phase and amplitude, allowing precise retrieval of the original signals.

2 Cocktail-party problem

2.1 Traditional ICA

Cocktail-party problem is the most famous example of ICA application [4]-[6], [9]. In a cocktail party, as shown in Figure 1, there are four different positions $s_1$, $s_2$, $s_3$, and $s_4$, and four original signals. $s_i(t)$ is the sound of a police car; $s_2(t)$, the rock music; $s_3(t)$, the classical music; and $s_4(t)$, the speaking voices. All the original signals are supposed to satisfy statistical independence. In traditional ICA, the signals are supposed to be in non-Gaussian probability distribution, with certain exception for some of the components. The modified ICA can use either Gaussian or non-Gaussian probability distributed signals and therefore has broader application, as most signals in our daily life are in Gaussian distribution. When the four sound sources occur at the same time, we can use four microphones are used to record sounds at different positions in the meeting.

In the above equation, $A$ is unknown, which makes the retrieval of $s(t) = [s_1(t), s_2(t), s_3(t), s_4(t)]$ difficult. However, ICA can retrieve the related $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$ signals into statistically independent signals. The estimated $W$ matrix is called unmixing matrix. ICA aims to estimate $W^{-1}A$ and $W$ matrix and retrieving the statistically independent signals. The equation for signal retrieval may be shown as the following:

\[
\begin{align*}
x_1(t) &= a_{11}s_1(t) + a_{12}s_2(t) + a_{13}s_3(t) + a_{14}s_4(t) \\
x_2(t) &= a_{21}s_1(t) + a_{22}s_2(t) + a_{23}s_3(t) + a_{24}s_4(t) \\
x_3(t) &= a_{31}s_1(t) + a_{32}s_2(t) + a_{33}s_3(t) + a_{34}s_4(t) \\
x_4(t) &= a_{41}s_1(t) + a_{42}s_2(t) + a_{43}s_3(t) + a_{44}s_4(t)
\end{align*}
\]

(1)

Fig. 1. Cocktail-party problem.

Since the sounds are all mixed, it is not possible to distinguish each individual’s words from the signals received. The distance between the microphone and each original signal may be represented by the $A$ matrix: $4 \times 4$, where $A$ should satisfied the conditions of full-rank matrix, also called mixing matrix. The signals represented as $s(t) = [s_1, s_2, s_3, s_4]$ entered the $A$ matrix. Through the $A$ matrix, the four signals interfere and mixed with one another, producing a mixed $x(t)$, which contained four signals $x_1(t), x_2(t), x_3(t),$ and $x_4(t)$. The signals received by the microphones may be represented as the following:

\[
\begin{align*}
u_1(t) &= w_{11}x_1(t) + w_{12}x_2(t) + w_{13}x_3(t) + w_{14}x_4(t) \\
u_2(t) &= w_{21}x_1(t) + w_{22}x_2(t) + w_{23}x_3(t) + w_{24}x_4(t) \\
u_3(t) &= w_{31}x_1(t) + w_{32}x_2(t) + w_{33}x_3(t) + w_{34}x_4(t) \\
u_4(t) &= w_{41}x_1(t) + w_{42}x_2(t) + w_{43}x_3(t) + w_{44}x_4(t)
\end{align*}
\]

(2)

Statistical independence can be measured by entropy. Statistical independence can be measured by entropy. Entropy is the basic concept of information theory [3], [7], [9]. It indicates the degree of indeterminacy of random variables. In other words, the more unpredictable and unstable a variable is, the greater the entropy and the greater its statistical independence. The following equation defines the entropy $H$ of a binomial random variable. It can be extended to continuous random variable and random vectors. While the random vector is $y$, the density distribution is $p(y)$:

\[
H(y) = -\int p(y) \log p(y) dy .
\]

(3)

After measuring the statistical entropy, the greatest entropy may be measured by mutual information, as shown in the following equation:
\[ I(y_1, y_2, y_3, y_4) = H(y_1) + H(y_2) + H(y_3) + H(y_4) - H(y_1, y_2, y_3, y_4). \]  

When the output entropy \( H(y_1, y_2, y_3, y_4) \) is at its greatest value, the mutual information \( I(y_1, y_2, y_3, y_4) \) between the outputs is the smallest. When \( I(y_1, y_2, y_3, y_4) = 0 \), \( y_1, y_2, y_3 \) and \( y_4 \) are statistically independent. The relation between mutual information and the \( W \) value, uncovered by the scholarly efforts from related fields, may be represented as the following

\[
\Delta W \propto \frac{\partial H(y)}{\partial W} = (W^T)^{-1} \left( \frac{\partial p(u)}{\partial u} \frac{\hat{u} - u}{p(u)} \right) x^T
\]

\( \Delta W \) is defined as the modified \( W \), \( u = [u_1, u_2, u_3] \), and \( p(u) = \frac{\partial y}{\partial u} \). However, the learning rule is too complicated as it involves the operation of inverse matrix. In the studies of Amari et al., Cardoso, and Laheld in 1996 [24], the learning rule was multiplied by \( W^T_W \), resulting in its rescale. In consequence, the learning rule is changed into

\[
\Delta W \propto \frac{\partial H(y)}{\partial W} W^T W = [I + \varphi(u)u^T]W
\]

\( \varphi(u) \) is defined as the \( \frac{\partial p(u)}{\partial u} \). The maximum information method is to use estimate ascent algorithm, adjusting \( W \) by means of continuous iteration and achieving the greatest \( H(y) \). The adjustment is presented as such

\[
W_{p+1} = W_p + l\Delta W
\]

where \( p \) is the times of iteration and \( l \) stands for the learning rate. \( W \) will be renewed continuously according to the partial differential equation of the maximized function \( H(y) \) in relation to \( W \). \( H(y) \) will increase until the greatest value is identified. So the best \( W \) may be estimated, enabling the separation of original signals from the mixed signals. However, the estimated original signals may be distorted due to opposite phase and unequal amplitude. Figure 2 shows the estimation of the unmixing matrix \( W \) in a cocktail-party problem.

Fig. 2. Estimation of the unmixing matrix \( W \) in a cocktail-party problem.

### 2.2 Modified ICA

Since the traditional ICA has the problems of opposite phase and unequal amplitude, the study uses gradient estimation algorithm to adjust gain and solve these problems. The positive and negative values of gain can be used to adjust opposite phase and its amount, to adjust unequal amplitude. The proposed method is to add automatically adjustable gain to the traditional ICA. A received signal is employed as the main input while various signals separated by traditional ICA are the reference input. In our proposed method, different gains are multiplied by various reference input and sum together. The sum is compared with the original signals. When the correct gain is selected, the two signals are identical. The gain is adjusted with gradient estimation algorithm, one of the available methods to identify the optimal parameter. The proposed method is presented in Figure 3.

Fig. 3. The structure of the modified ICA.
algorithm and is therefore the gain. \( y(n) \) is the modified algorithm output, and \( e(n) \) is the errors.

\[
\mathbf{u}(n) = [u_1(n), u_2(n), \ldots, u_L(n)]^T
\]

\[
\theta(n) = [\theta_1(n), \theta_2(n), \ldots, \theta_L(n)]^T
\]

where \( L \) indicates the number of processed signals. The output of gradient estimation algorithm

\[
y(n) = \theta_1(n)u_1(n) + \theta_2(n)u_2(n) + \ldots + \theta_L(n)u_L(n)
\]

where \( u_1(n) \) is estimation of the sound of a police car, \( u_2(t) \) is estimation of the rock music, \( u_3(t) \) is estimation of classical music, and \( u_4(t) \) is estimation of the speaking voices. \( \theta_1(n), \theta_2(n), \ldots, \theta_L(n) \) are parameter vector at time \( n \).

The equation for errors in gradient estimation algorithm

\[
e(n) = x(n) - y(n)
\]

If Eq. 8 is applied to Eq. 9, a renewed equation of the weight function will be produced:

Thus, substituting Eq. 8 into Eq. 9, we may compute the updated value of the parameter vector \( \theta(n+1) \) by using the simple recursive relation

\[
\theta_1(n+1) = \theta_1(n) + \mu [e(n)u_1(n)]
\]

\[
\theta_2(n+1) = \theta_2(n) + \mu [e(n)u_2(n)]
\]

\[
\theta_3(n+1) = \theta_3(n) + \mu [e(n)u_3(n)]
\]

\[
\theta_4(n+1) = \theta_4(n) + \mu [e(n)u_4(n)]
\]

where \( \mu \) is the step-size.

The results of this algorithm are produced by means of the smallest average errors. The advantage of this algorithm is that it utilizes only addition and multiplication. In this gradient algorithm, the parameter \( \mu \) is the step size, which mainly affects the stability and convergence speed of the system.

### 3 Results and Discussion

This study uses CPU-P4 2.0 GHz industrial computer with MATLAB 6.5 legal software. The cocktail-party problem includes four simultaneous sound sources from four different positions \( s_1, s_2, s_3, \) and \( s_4 \), representing individual original signals. When the sounds are mixed, individual sources cannot be distinguished. Figure 4 shows the time domain of the signals received by the microphones. In this system, if the mixed matrix \( A \) is known, the analysis of \( s_1, s_2, s_3, \) and \( s_4 \) is simply the question of solving linear equations. However, the mixed matrix \( A \) is usually unknown in our studies. Since the sources of the signals are often uncertain, it is not possible to determine the mixed matrix produced by the distance and to measure \( s_1, s_2, s_3, \) and \( s_4 \) individually. ICA reorganizes a set of complex data into independent components by means of statistic algorithm. As long as there are sufficient amount of known \( x \) and independent components are actually present, it can estimate \( u \), a value close to the original independent component. In such analysis, the mixed matrix \( A \) is not necessarily required. ICA is able to separate the independent components \( u_1, u_2, u_3, \) and \( u_4 \) as shown in Figure 5. However, the amplitude of the separated signals become smaller, showing distortion of the original signals. We proposed a modified ICA to solve this problem. Using gradient estimation algorithm, the proposed method can estimate the original signals. After the separation by ICA, the automatically adjustable gain is added. Finally the modified ICA retrieves the amplitude. Its time domain is shown in Figure 6. The original signals \( s_1, s_2, s_3, \) and \( s_4 \) is presented in Figure 7. Figure 8 represents the Gaussian distribution of original signals. It can be observed that the retrieved signals are very close to the original. So is the retrieve amplitude. In terms of frequency domain, the retrieved signals are very similar to the original. Unlike the traditional ICA which allows only one signal in Gaussian distribution, all the signals in the modified method are in Gaussian distribution.

**Fig. 4. Time domain of signals received by microphone.**
Fig. 6. Time domain of signals separated by ICA.

Fig. 8. Time domain of signals separated by modified ICA.

Fig. 10. Time domain of original signals.

Fig. 12 Gaussian distribution of original signals.

4 Conclusion

One of the limitations of ICA is that the original signals should be non-Gaussian and only one Gaussian signal is allowed. Otherwise aliasing and errors may occur during estimation. The opposite phase and unequal amplitude in the estimation will lead to the distortion of microphone signals during retrieval. This study proposed a modified ICA, which allows more accurate estimation of phase and amplitude in Gaussian signals and ensure effective retrieval of original signals. This method has promising potential for application in many fields.

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