

# Exploring Computer Algebraic Systems' Potential in e&bLearning: an Interactive Digital Control course<sup>\*</sup>

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**Abstract:** - The use of Computer Algebraic Systems (CAS) adds a powerful tool to Engineering education. Their symbolic and numeric computation capabilities, the visualization capacity of their graphic outputs together with programming and text editing features allow Engineering teachers to design electronic documents suitable for distance as well as blended learning. This paper explores the main CAS systems' potentials and the material's requirements for e&bLearning. Those potentials have been exploited in the creation of a Digital Control course, designed for distance or blended instruction of postgraduate students.

**Key-Words:** - Computer Algebraic Systems (CAS), eLearning, bLearning, educational research, engineering education, digital control learning

## 1 Introduction

During the last two decades, Computing Algebraic Systems are becoming very popular within the education community, as publications in educational conferences and journals shows. In the beginning, the use of CAS systems was mainly oriented to mathematics learning but their huge potential in Engineering Education was soon revealed [6]. On the other hand, distance and self-training courses using eLearning tools and blended courses taking advantage of computational aids are turning into a must in educational environments. This assertion is even more evident when we are talking about postgraduate courses or complementary material.

Computer Algebraic Systems are enforcing the development and adaptation of educational methods and making possible the elaboration of efficient and flexible distance materials. CAS systems change dramatically and modify the attitudes of both teachers and students improving problem solving abilities by means of automated symbolic and numeric calculations; understanding and focusing on the essential steps of the problem without getting lost in the details; and producing nicely readable, easily accessible and always up to date documents.

It is widely accepted that visualization enhances insight –a picture is worth thousand words–, engagement and satisfaction among students. Another important fact to bear in mind is that interactivity augments comprehension, giving the user the

opportunity to explore and experiment with the concepts.

In the second part, a Digital Control course implemented as a collection of Maple *worksheets* is presented. It has been designed as a component of a postgraduate control course as well as complementary material for interested students of the undergraduate complex analysis course. It is an example of how Computer Algebraic Systems' potential can be exploited for creating an e&bLearning instrument.

## 2 Valuable CAS features in Engineering e&bLearning

Engineering learning must take advantage of Computer Algebraic Systems capabilities. Symbolic algebra computations, visualizations of interactive graphics, simulation, high level programming language as well as numerical computation and text formatting features open wide possibilities in the design of electronic instructional materials. The simplicity and user-interface functionality together with a nice appearance of the documents produced by systems like Maple or Mathematica provide a great environment to enhance the user's insight and capabilities.

Documents created with these systems might be used both for eLearning in an on line course and for

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bLearning. In this case, the instructor can utilize the materials in a demonstrative way during the lectures and as additional resources for the learners.

We will expose in detail the effect of those features in engineering e&bLearning

### 2.1 Symbolic manipulation

Problem solving in Engineering can be regarded as a four steps process. The first task is to state the problem in mathematical terms; the second step is to symbolically manipulate the equations in the desired way; the third task is to do the numerical calculations and the fourth to analyze the obtained solution. It is also important to repeat the process with different problem solving methods in order to compare their effectiveness.

Pedagogical considerations make it important to keep calculations symbolic as long as possible as symbolic solutions provide deeper insight and are important for students to better analyze the obtained results. Because of this, frequently, engineering courses spend a long time improving the students' skills in symbolic manipulation and it turns out to be the main focusing point, hiding the major goals: understanding the underlying concepts, modeling the problem and analyzing the results. CAS systems' power in symbolic manipulation can overcome this problem and facilitate learner's insight, analysis and experimentation.

### 2.2 Programming language

The best way to understand a solving method is to express it by means of an automated task i.e. to create a program that performs the method. The full and flexible programming language included in CAS systems provides an excellent tool for implementing those automated tasks.

This programming language is also useful for the instructor, who can, in selected cases, simplify the student task in order to clarify the concepts. Furthermore, the usage, analysis and re-programming of these procedures also will help the method's understanding, serving as a medium for interactive learning of how to design, implement and test it.

### 2.3 Visualization and Interactive Graphics

Interactive graphical output is a very important facility that CAS systems give the engineering educators. It provides a visualization tool with extraordinary learning possibilities as users have analytical as well as visual concepts at their disposal.

It is widely accepted that visualization enhances

comprehension, engagement and satisfaction among students: an image speaks itself. Visualization technologies can be used to illustrate many engineering concepts, as can be noticed in fig. 1.

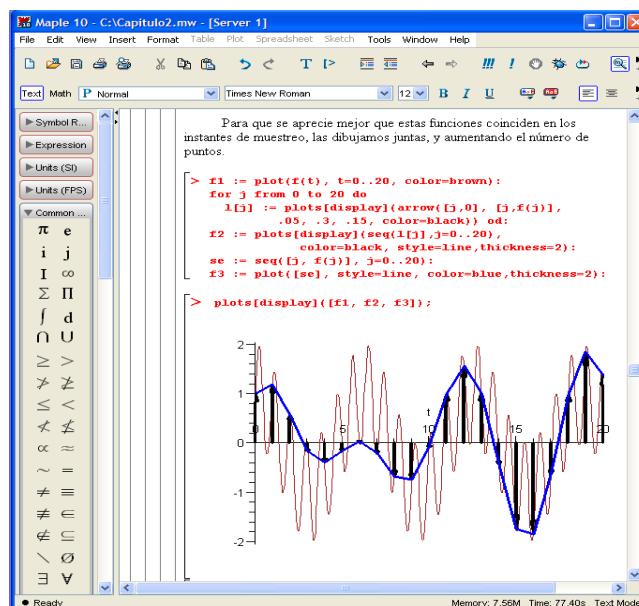


Fig. 1: Digital Control course screenshot

According to [3], learners who are actively engaged with the visualization have consistently outperformed those who passively viewed visualizations. The capabilities of CAS' graphical production provide powerful insight of concepts, methods and results. In this sense, instructors should take advantage of their graphics interactivity proposing data and methods' modifications to turn out the students in the mayor figures of their own learning process.

### 2.4 Simulation

The automation of tasks provided by procedures created with CAS programming languages along with their interactive graphical resources, allow instructors as well as students to create their own simulations, promoting additional options exploration and enhancing concepts and processes comprehension. Furthermore, their symbolic and numerical computation features, permit the simulation of complex engineering tasks, much closer to reality than simplified ones. In addition, the system provides the capability to compare different methods' solutions and diverse environment situations.

### 2.5 Text formatting

CAS systems like Maple or Mathematica combines their mathematical capabilities with well designed

editing resources, improving their value as eLearning tools. They present interfaces that are book-like content windows that hold interactive text and graphics. They also support conventional mathematical notation inside the text as well as in the outputs producing interactive books of mixed explanations and code. Users can navigate through these books by reading the text, using the hyperlinks, experimenting with the pre-programmed examples, modifying parameters and solving proposed problems.

## 2.6 Tutorial

The features listed above make available the feasibility of creating interactive tutorial documents that facilitate the student to learn individually, making CAS systems impressive tools for e&bLearning. Computing power provides user with the opportunity for open experimentation beyond the exercises requirements.

One of the principal elements when creating a tutorial document is the possibility of including readable text using standard mathematical notation among the executable and graphic parts. This characteristic is included in most CAS systems.

In addition, the material can be organized by a clustering mechanism that lets the user open and collapse parts of the document and navigate inside it by means of bookmarks and hyperlinks.

## 3 Requirements for eLearning materials using CAS

In the design of electronic instructional materials, should be kept in mind the importance of appearance, simplicity and user-interface functionality to the success of such materials. Easy of use as well as adaptability to the variety of interactive mechanisms used in educational applications are key issues (see [1], [2]).

The requirements an eLearning document ought to fulfil are quite related with those required by a visualization tool (see [7], [8]). Among them, the followings should be considered:

- Easy of use: the students' effort should be directed to learn the concepts, not the system.
- Intuitive and visual: good graphic examples and activities will be enormously helpful for a better understanding.
- Interactive: facilitating and encouraging users to modify inputs and procedures, to explore and interact with the document and to create their own examples and solutions.

- Clear and concise explanations of theoretical concepts using standard mathematical notation.
- Exploitation of system's potential on symbolic and numerical computations to increase examples' complexity.
- Sequential organization, with segmented cells structure that lets the user open and collapse alternatively those segments.
- Nicely formatted appearance including mathematical expressions, modifiable source code, graphics and text explanations in a single document.
- Use of hypertext and bookmarks for an easier navigation.

## 4 An Interactive Digital Control course implemented in Maple for e&bLearning

The Digital Control course presented here has been designed as a component of a postgraduate control course as well as complementary material for interested students of the undergraduate complex analysis course. It has been implemented in Maple because this Computer Algebraic System allows both symbolic and numeric manipulation, presents good graphical features and the text editor produces nicely readable and easily navigable *worksheets*. These *worksheets* include text, mathematical notation, interactive graphics and source code that can be executed and modified by the user. Although Matlab has also good graphical features and is better suited for engineering numeric computations, the symbolic capabilities and the text editing possibility together with mathematical notation have forced the choice. This course could also have been implemented in Mathematica notebooks. The election has been, in this case, imposed by license availability.

This course is available in the Applied Mathematics Department Website [9] and it is intended to be part of a more extensive course in control theory. The course presents a comprehensive study on Discrete Time Control Systems and Z-Transform applications. In a similar way a Continuous Time Control Systems course is projected including hyperlinks between both courses to facilitate the comparison between corresponding concepts.

### 4.1 Course's structure

The course contains eight chapters, each one comprising several sections. Every chapter starts with an introductory section, presenting the corresponding

subject. Next sections expound the chapter's topics and show the theoretical concepts as well as some examples and practical applications. Finally, an exercises section with solutions and a references section are provided. Examples, exercises and practical applications have been implemented on the CAS Maple.

There are many hyperlinks connecting related parts or referenced topics. Those hyperlinks allow the user to navigate easily along the different chapters. The course includes also a glossary *worksheet* containing hyperlinks to most concepts so that every definition or property can be found without difficulty, just looking for it at the mentioned glossary *worksheet*. There are also section bookmarks to facilitate the student the navigation inside the chapter. To find the bookmarks, it's necessary to activate them in View/Bookmarks. An index worksheet is also provided containing hyperlinks to every chapter and chapter's section.

While course is being completed, the CAS system becomes more useful and valuable, as both numerical and symbolic computations grow to be more complex. The obtained results happen to be more appealing as higher level concepts are involved.

At exercises' section, the user must solve the proposed exercises, comparing the results with those prepared by the instructor. In some exercises functions and procedures should be designed by the student. The needed Maple's programming language skills should be increased gradually by means of the study of the implemented procedures presented in the examples.

#### 4.1.1 Theoretical part

This course's theoretical contents correspond to usual classic Discrete Time Control Systems topics. So that, basic references used are classical control systems' books [4], [5]. However, course's presentation, implemented on Maple's *worksheets* presents an innovative approach to the subject and uses the whole visualization and interactive potentials of Computer Algebraic System Maple.

The course comprises eight chapters entitled: "Z-Transform", "Discretization of a continuous signal", "Difference Equation solution by means of Z-Transform", "Stability. Jury's Test", "Transient Response Analysis", "Dead Beat Control Design", "Modified Z-Transform in obtaining the response between consecutive sampling instants", "Frequency Response Analysis"

#### 4.1.2 Exercises

This section is included in every course's chapter. It consists of a set of exercise and is aimed to check students' learning level.

When drilling the exercise section down, the exercise's statement, together with a solving zone is displayed (see fig.2). Once the exercise has been worked out, the student may open the instructor's solution subsection. There, the obtained result can be verified. To solve the exercises, sometimes, functions or procedures designed in previous chapters or sections should be used; sometimes one or several package's functions are needed. Usually, in these cases, a hyperlink shows the path to find and execute such code or a suggestion indicates the package's necessity.

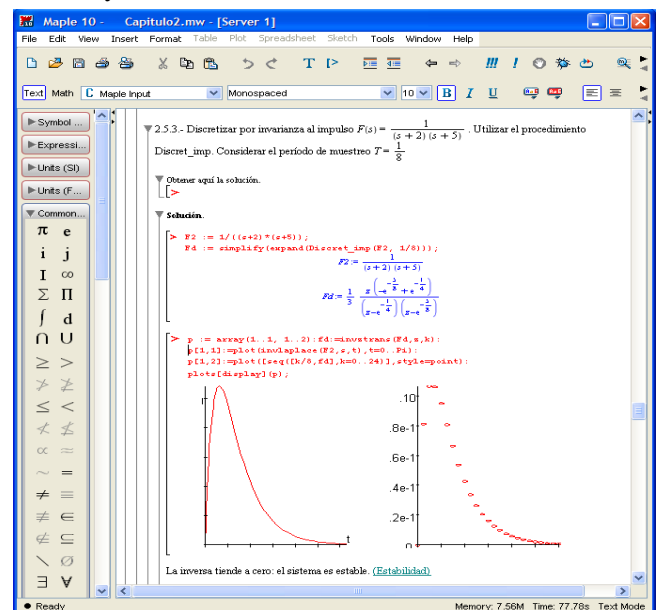


Fig. 2: Exercise segment's structure showing the user's solving zone and the instructor's proposed solution

#### 4.2 Didactical goals

The main didactical goals for this course are:

- Taking maximum advantage of graphic resources offered by Maple in order to improve concepts' insight.
- Using Maple's symbolic and numerical computation capacities to focus student's interest on solving methods and final solutions; allowing the learner to compare solutions obtained by means of different methods as well as with diverse data sets.
- Programming and design of methods and algorithms presented in the theoretical part as best way to enhance comprehension.
- Broadening the user's experimentation possibilities, both by input data modification and by more complex problems' solving.
- Application of exposed concepts and mathematical methods to Discrete Control Systems design.

### 4.3 The problem of software requirement

The problem of eLearning material designed using CAS systems is the requirement for the learner to have the system installed. It is not a simple condition to be fulfilled because affordable student packages usually do not have full functionality and specific commands are not implemented.

In Maple case, a solution for turning student package into functional could be the inclusion of needed specific functions' code inside the *worksheet*.

Another proposed solution, also in Maple case, consists of installing MapleNet in the web server and placing the documents for its web usage. This solution has two main difficulties: once the *worksheet* has been modified, the user can save it to the computer, but it cannot be reviewed unless the system is installed in the computer; another problem is that any modification has to be sent to the Web server to be evaluated and returned to the user, what takes a long time even with extremely simple calculations. The two inconveniences make this solution unfeasible in this situation.

## 5 Conclusions and future work

Usually, science and engineering problems are expressed in math terms. The solution to most of these problems needs a great deal of numerical computations together with complex symbolic and algebraic manipulations. Furthermore, the solution is often theoretically known, but difficult to be obtained, due to the complexity of required calculations. Use of CAS systems like Maple or Mathematica, adds a powerful tool to engineering education, even more powerful when designing eLearning materials.

The capital contributions of Computer Algebraic Systems to engineering e&bLearning are:

- Change in the teacher's as well as the student's perception of what is most important in science.
- Enhancement of concepts and processes comprehension, skipping the often tedious computation parts.
- Revision of subjects' objectives and contents, as CAS systems add more time and possibilities.
- Improvement of experimentation and simulation opportunities, increasing interactivity.
- Potentiality for dealing with much more complex problems, usually closer to reality.
- Use of graphics as a way to facilitate theory's insight.
- Augment of students' engagement, understanding and satisfaction.

As mentioned above, a Continuous Time Control

Systems course is projected as a needed introduction for the course presented here. It will include hyperlinks between both courses to facilitate the comparison between corresponding concepts.

Computer Algebraic Systems can be used for creating and using engineering didactic documents, replacing from word processors to numerical simulation tools. They are a most exciting tool for e&bLearning proposes, although all their possibilities have not yet been exploited.

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