

## Parallel manipulator robots design and simulation

SAMIR LAHOUAR  
 Laboratoire de Mécanique  
 des Solides  
 UMR: 6610 CNRS  
 Bd Pierre et Marie Curie BP  
 30179, 86962 Futuroscope  
 chasseneuil Cedx  
 FRANCE

SAID ZEGHLOUL  
 Laboratoire de Mécanique  
 des Solides  
 UMR: 6610 CNRS  
 Bd Pierre et Marie Curie BP  
 30179, 86962 Futuroscope  
 chasseneuil Cedx  
 FRANCE

LOTFI ROMDHANE  
 Laboratoire de Génie  
 Mécanique  
 Ecole Nationale d'Ingénieurs  
 Monastir 5019  
 TUNISIA

*Abstract:* - Parallel manipulator robots have complex kinematics and present singular positions within their workspace. For these reasons, in most software simulating parallel robots, each kinematic model should be given in advance by users or programmers. In this paper we present a new tool used to design and to simulate parallel manipulator robots. Explicit kinematic equations are generated automatically depending on the architecture of the robot. These equations are used to simulate and detect singularities while planning robot's movement.

*Key-Words:* - Parallel manipulator robots- simulation- singularity detection.

### 1 Introduction

Since the 60's, serial robots have taken a great place in industry. They are used in many industrial applications and in many domains replacing men in hard tasks and hostile environments. However, in the last 20 years, researchers have been interested in another type of robots: parallel robots. Different structures of parallel manipulators have been studied (see, for example [1-7]). Currently, more than 100 different architectures have been proposed and probably not all of them have been discovered. A review of most known architectures is given by Merlet [8].

Due to their high accuracy and their ability to carry high loads, parallel robots are used in many domains such as fine positioning devices, motion generators, ultra fast pick and place tasks. They are also finding their way in the field of machine tools and medical applications. However, the closed nature of kinematic chains forming parallel manipulators makes their workspace limited and their kinematic model complex and architecture dependent. In addition, singularities may appear within the workspace. Singularities are dangerous and thus should be detected and avoided while planning parallel manipulator's motion. This problem is very important and is widely studied in literature, see for instance [9-11] among many others. Moreover much work is carried out to study the dynamics of parallel mechanisms [12-14].

Design and simulation are important issues in studying mechanisms. The design is based on the topology and the geometry of the mechanism. Topology is the way that the joints, links and actuators are arranged, while geometry deals with the dimensions of

links and the locations of the joints. Simulation consists of showing the behavior of a mechanism given its topology and its geometry.

There is a real lack of software permitting to model and simulate mechanisms with closed chains. Commercial packages like ADAMS, CATIA, DADS, SolidWorks... can be used, but the problem with most of them is that they use absolute coordinates as they are easier to program, they can not deal with over constrained mechanisms and they are black boxes and no adding nor modification are allowed.

In this paper, we present our work on parallel manipulator robots modeling and simulation. Our goal is to develop a tool able to model any parallel manipulator with any combination of revolute and linear joints and able to simulate the motion of many robots evolving together in their environment. Based on the design of each parallel manipulator robot, explicit geometrical and kinematical equations are automatically generated. These equations are used to perform possible motions and to detect singularities.

In the next section, we present how to model parallel manipulator robots by presenting some methods used to model them.

Section 3 shows how geometric and kinematic equations are generated and how they are used to simulate motion.

Finally, we present an example of a 3-SPS robot which is a spatial parallel manipulator composed of three legs. Each leg has two spherical joints and a prismatic joint. We give the corresponding generated equations to the 3-SPS robot.

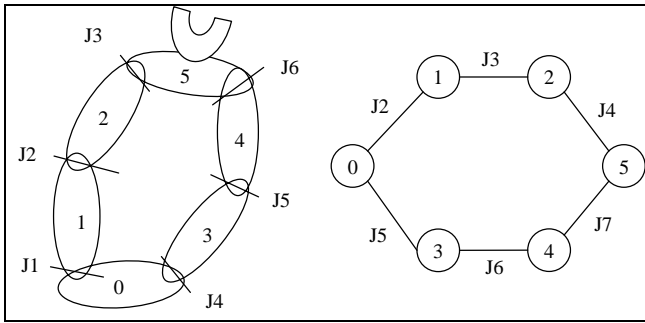


Fig. 1. A parallel robot and its corresponding graph

## 2 Modeling parallel manipulator robots

Parallel manipulator robots as any multibody mechanism are made of links and joints. The structure of the robot can be presented by a graph as shown in fig. 1.

### 2.1 Links

Each link has a position and an orientation in the 3D space. In order to determine the position and the orientation of the link we can use either absolute coordinates or joint coordinates.

Where joint coordinates correspond to joints in the mechanism and absolute coordinates correspond to position and orientation with respect to reference frame.

We propose three kinds of links (see fig. 2): base link, serial link and mobile link.

#### 2.1.1 Base link

We define the base link as the first link of a robot. Its position and orientation are given with respect to the environment using absolute coordinates. The base link is unique for each robot and is used to place the robot.

#### 2.1.2 Serial link

A serial link is connected to another link (called father link) according to the joint between both links. The position and orientation of a serial link depends on the joint coordinates and on the position and orientation of the father link.

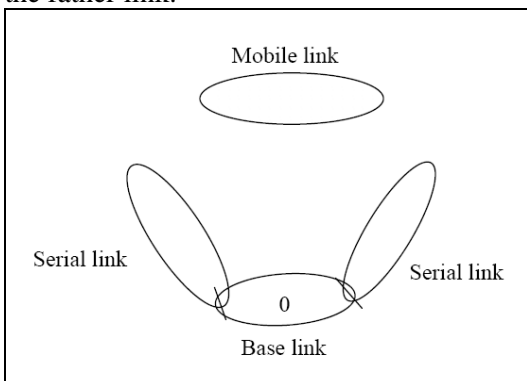


Fig. 2. Link types

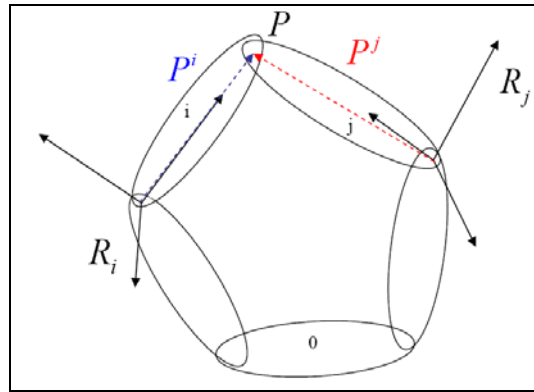


Fig. 3. Coincidence constraint between two points

Supported joints are revolute joints, linear joints, spherical joints and universal joints.

#### 2.1.3 Mobile link

A mobile link is positioned using absolute coordinates with respect to the base link.

### 2.2 Constraints

Constraints are used to restrict motion between two different links. In design, constraints are either used to define a joint between two links or to weld two links. We show in the next section how constraints are used to replace joints and how with a judicious use we can generate explicit geometrical model of the multibody mechanism.

Supported constraints are: coincidence between two points, parallelism between two vectors and coincidence between two links.

#### 2.2.1 Coincidence between two points

The coincidence between two points (see fig. 3) is given by the equation driven from a point P belonging simultaneously to two links i and j. This equation is written in this manner

$${}^0\mathbf{T}_i \mathbf{P}^i = {}^0\mathbf{T}_j \mathbf{P}^j \tag{1}$$

Where  ${}^0\mathbf{T}_i$  the homogeneous matrix of the frame is  $R_i$  attached to link i and  $\mathbf{P}^i$  is the homogeneous point vector of coordinates of point P in the frame  $R_i$ .

$${}^i\mathbf{T}_j = \left[ \begin{array}{ccc|c} & & & \\ & {}^i\mathbf{A}_j & & \mathbf{O}_{ij} \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \tag{2}$$

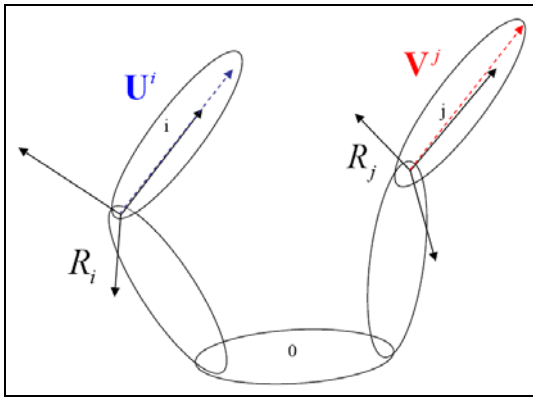


Fig. 4. Parallelism constraint between two lines

Where  ${}^iA_j$  is the orientation matrix of frame  $j$  with respect to frame  $i$ . While  $O_{ij}$  are the coordinates of  $O_j$  the origin of frame  $R_i$  with respect to frame  $R_j$ .

$$P^i = \begin{pmatrix} x^i & y^i & z^i & 1 \end{pmatrix}^T \quad (3)$$

Where  $x^i$ ,  $y^i$  and  $z^i$  are coordinates of  $P$  in frame  $R_i$ .

### 2.2.2 Parallelism between two vectors

Parallelism constraint can be used to make two lines or two planes of two different links parallel (see fig. 4 and fig. 5).

The equation corresponding to this constraint is written as

$${}^0T_i v^i = {}^0T_j v^j \quad (4)$$

$v^i$  is the homogeneous vector given by:

$$v^i = \begin{pmatrix} v_x^i & v_y^i & v_z^i & 0 \end{pmatrix} \quad (5)$$

Where  $v_x^i$ ,  $v_y^i$  and  $v_z^i$  are the coordinates of vector  $v$  in the frame  $R_i$ .

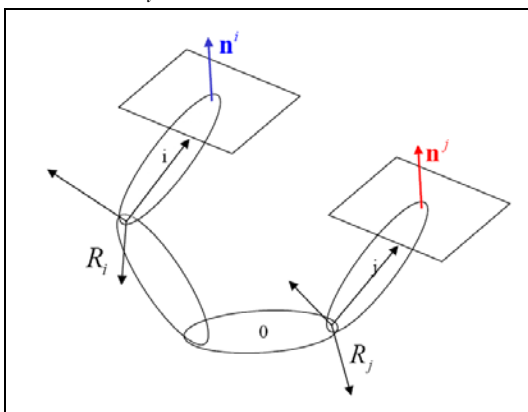


Fig. 5. Parallelism constraint between two planes

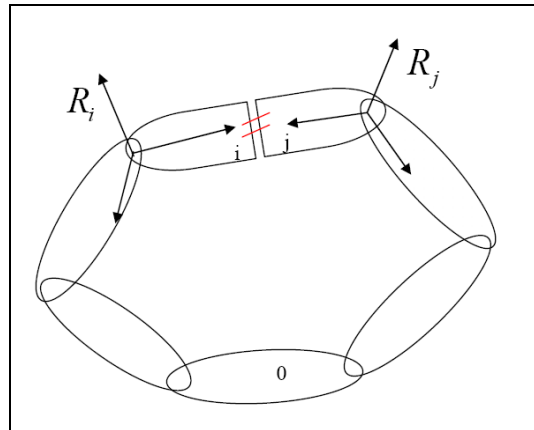


Fig. 6. Coincidence constraint between two links

### 2.2.3 Coincidence between two links

Two frames  $R_i$  and  $R_j$  are attached to the same body (see fig. 6) if

$${}^iT_j = T_c \quad (6)$$

Where  $T_c$  is a constant homogeneous matrix.

$${}^iT_j = {}^iT_0 {}^0T_j = ({}^0T_i)^T {}^0T_j \quad (7)$$

## 3 Generating closure equations

We propose two methods to design parallel robots. Then we show a possible use of our method in order to design serial manipulators and to generate automatically their kinematics.

### 3.1 First method

As shown in fig. 7, a parallel manipulator is made of a base platform, a mobile platform and serial legs. The idea is to replace a non actuated joint in each leg by the corresponding constraints. These joints are called closure constraints.

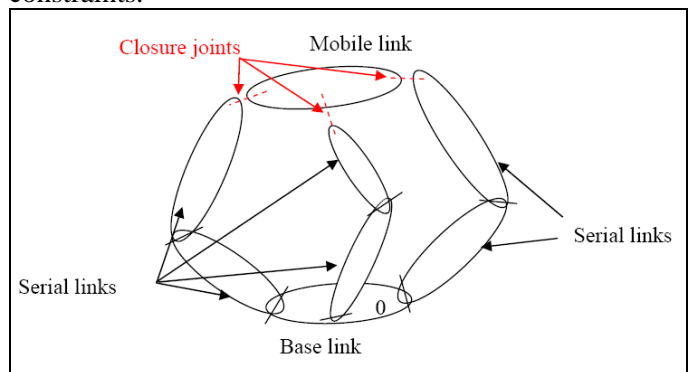


Fig. 7. First modeling method

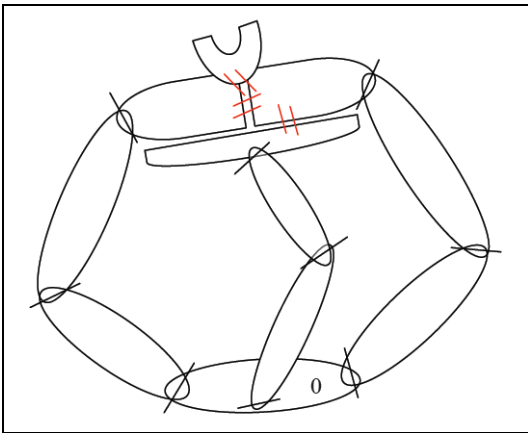


Fig. 8. Second modeling method

The base platform is modeled using a base link while the mobile platform is modeled by a mobile link. Legs are modeled using serial links.

By replacing only non actuated joints by constraints, we keep the actuated joint coordinates in the generated equations. Mobile link has 6 absolute coordinates among which 3 are used for positioning and 3 used for orientation. These coordinates are attached to the mobile platform which is the end effector of the manipulator, so they are called the output variables.

Constraints between mobile links and some serial links give equations involving operational variables and joint coordinates.

### 3.2 Second method

To model parallel manipulators we can consider that the mobile platform is a part of each leg as shown in fig. 8. Then by applying constraints of coincidence between the links representing the mobile platform we get closure constraints that need to be satisfied to generate a valid model of the manipulator.

This method is used in order to generate equations with all joint coordinates. If in addition operational variables are needed in the generated equations, a mobile link coincident with one of the links representing the mobile platform is added. The same method is used in the next paragraph in order to generate geometrical model of serial manipulators.

### 3.3 Geometric model of serial manipulators

Geometrical model is a set of equations between output variables and joint coordinates. The idea is to model the manipulator with serial joints and to use a mobile link attached to the last link with a coincidence constraint as shown in fig. 9.

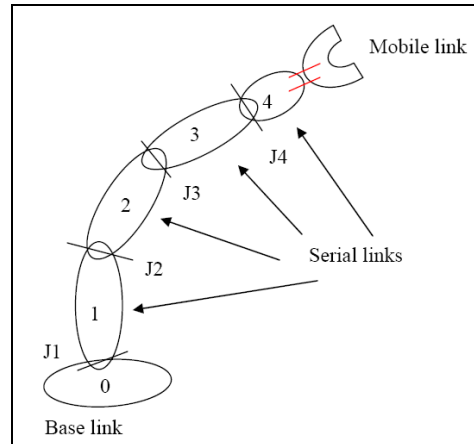


Fig. 9. Modeling serial manipulators

## 4 Equation analysis and interpretation

The number of equations depends on the constraints used to model the mechanism. A constraint of coincidence between two points gives 3 equations, which is equivalent to a constraint of parallelism between two vectors. While a constraint of coincidence between two links gives 6 equations.

The variables used to describe the motion of the mechanism depend on the links used to build the model. A mobile link has six variables. The number of variables given by a serial link depends on the nature of its corresponding joint. For a revolute and a linear joint there is only one variable while for spherical joint there are three variables and for universal joint there are two variables.

We call  $X$  the vector of variables. This vector includes all the variables of the system. It contains active and passive joints as well as output variables. The equations can be written as

$$F(X) = 0 \quad (8)$$

Where  $F$  is a vector function.

### 4.1 Extracting the kinematical model

In order to obtain the kinematic model we only have to derive the function  $F$  with respect to time, which yields:

$$[E_k] \dot{X} = 0 \quad (9)$$

$\dot{X}$  is the derivative of vector  $X$  with respect to time. The mobility of the system is given by

$$m = N - \text{rank}([E_k]) \quad (10)$$

Where  $N$  is the dimension of the vector  $X$ .

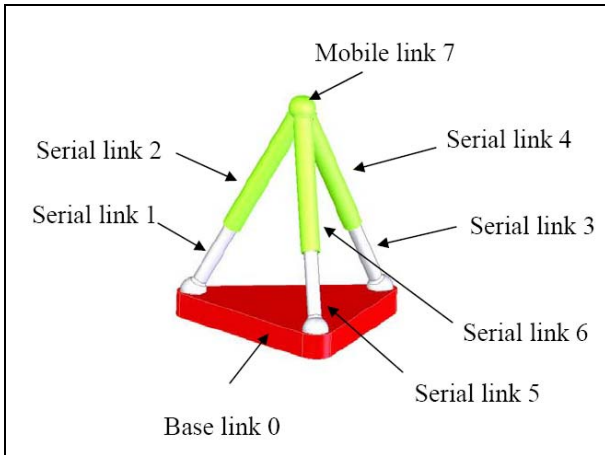


Fig. 10. Modeling the 3SPS Manipulator

We generate automatically an explicit expression of the matrix  $[E_k]$  depending on the geometry of the robot. The variables are divided into active, passive and operational variables. By rearranging equation (9) we can write

$$[M]\dot{q}_a + [N]\dot{q}_p + [K]\dot{X}_{op} = 0 \tag{11}$$

Where  $\dot{q}_a$  is the vector of active joints,  $\dot{X}_{op}$  the vector of output variables while  $\dot{q}_p$  is the vector of the remaining variables, we call them passive variables.

$[M]$  and  $[K]$  should be full rank matrices otherwise an actuated variable could be changed without moving the mobile platform or an output variable could be changed while actuators are locked, which means that the model does not completely describe the behavior of the robot.

$[M]$ ,  $[N]$  and  $[K]$  are evaluated at each step while moving the parallel manipulator.

### 4.2 Moving the parallel manipulator robot

There are two ways to control the robot. In one hand the user controls the active variables and in the other hand the user controls the output variables. In order to do this, it is necessary to eliminate the matrix  $N$  from equation (11). This operation consists in eliminating passive variables in the model. Consequently we obtain a relationship between active and output variables.

To do so, a matrix  $[O]$  is computed. The columns of the matrix  $[O]$  are generated in the null space of matrix  $[N]$  using Gram-Schmidt orthonormalization. We have:

$$[O]^T [N] = 0 \tag{12}$$

By applying the matrix  $[O]^T$  to equation (11) we obtain the following form of the kinetic model.

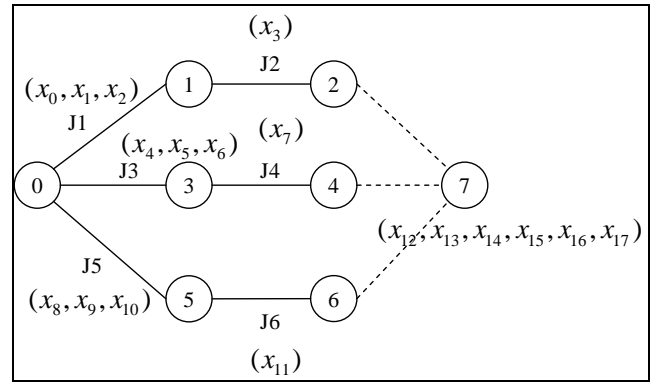


Fig. 11. 3SPS Manipulator graph

$$[B]\dot{q}_a + [A]\dot{X}_{op} = 0 \tag{13}$$

Where  $[A] = [O]^T [K]$  and  $[B] = [O]^T [M]$

Once  $\dot{q}_a$  and  $\dot{X}_{op}$  are known,  $\dot{q}_p$  are computed by solving equation (11).

In order to move the platform, the explicit form of matrix  $[O]$  is not compulsory since it has usually a complicated form, it is just evaluated at each step. It is the same for matrices  $[A]$  and  $[B]$ .

### 4.3 Singularity detection

While moving the robot, singularities could be detected. They correspond to a loss of rank of one of the matrices  $[N]$ ,  $[A]$  or  $[B]$ .

The rank of these matrices is continuously checked while moving the parallel manipulator.

## 5 Example

We give a simple example of a parallel manipulator robot and we show the generated equations. The studied robot is a 3SPS manipulator shown in fig. 10. This robot is made of three legs with a spherical joint, a linear joint and a second spherical joint, the corresponding graph is given in fig. 11. Joint J1 is a spherical joint it has three revolute variables  $(x_0, x_1, x_2)$ . It is the same for J3 and J5. While J2, J4 and J6 are linear joints, each one has an active variable which are  $x_3$ ,  $x_7$  and  $x_{11}$  respectively. Link 7 is a mobile link, it is oriented using three variables  $(x_{12}, x_{13}, x_{14})$  and it is positioned using variables  $(x_{15}, x_{16}, x_{17})$ .

This robot is modeled with a base link, 6 serial links and a mobile link. The geometrical dimensions are given in fig. 12.

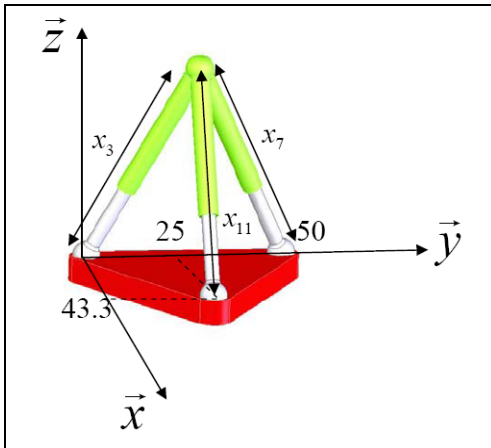


Fig. 12. Dimensions of the 3SPS manipulator

There are three constraints of coincidence between two points defined between each leg and the mobile link. The generated equations are:

$$F(\mathbf{X}) = \begin{pmatrix} x_3 \sin x_1 - x_{15} \\ -x_3 \sin x_0 \cos x_1 - x_{16} \\ x_3 \cos x_0 \cos x_1 - x_{17} \\ x_7 \sin x_5 - x_{15} \\ -x_7 \sin x_4 \cos x_5 + 50 - x_{16} \\ x_7 \cos x_4 \cos x_5 - x_{17} \\ x_{11} \sin x_9 + 43.3 - x_{15} \\ -x_{11} \sin x_8 \cos x_9 + 25 - x_{16} \\ x_{11} \cos x_8 \cos x_9 - x_{17} \end{pmatrix} = 0 \quad (14)$$

with  $\mathbf{X} = (x_0 \dots x_{17})^T$

Thereafter, we use the following notation:  $s_i = \sin x_i$  and  $c_i = \cos x_i$ .

Deriving  $F$  with respect to time gives the matrix  $[E_k]$  that we divide into three matrices  $[M]$ ,  $[N]$  and  $[K]$ .

We choose  $\dot{\mathbf{q}}_a = (\dot{x}_3 \ \dot{x}_7 \ \dot{x}_{11})^T$ ,  $\dot{\mathbf{X}}_{op} = (\dot{x}_{15} \ \dot{x}_{16} \ \dot{x}_{17})^T$  and

$\dot{\mathbf{q}}_p = (\dot{x}_0 \ \dot{x}_1 \ \dot{x}_4 \ \dot{x}_5 \ \dot{x}_8 \ \dot{x}_9)^T$

## 6 Conclusion

We presented a new methodology for modeling parallel manipulator robots and we showed how to generate closure equations automatically which gives the kinematic model. We also proposed the way to eliminate the passive parameters in the model. We also proposed the way to eliminate the passive parameters in the model. This work is the first step of a tool that we are still developing in order to model parallel manipulators in their environments and to simulate and plan their motion avoiding singularities and without collision with obstacles.

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