Urban Traffic Control in Game Theoretic Framework

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Abstract: In this study, a game theoretic solution is proposed for urban traffic control. The concept relies on the idea that an urban network equipped by traffic lights in junctions can be considered as a multi-agent scenario. Every agent is a junctions and each of them has the right to make an own decision on the distribution of the green time lengths in its crossing. Any decision made by a junction intends to minimize the number of vehicles within its incoming road-links but any distribution of the leaving vehicles caused by a decision sets back the receiving junctions. The technique proposed in this paper converts the conflict situation arisen into a game theoretic problem. A suboptimal noncooperative solution for green time distribution of the junctions is also provided through an illustrative example of simple traffic network.

Key–Words: Urban traffic, Optimal control, Game theory, Nash Equilibrium

1 Introduction

The growing traffic of urban areas requires efficient control system to avoid heavy congestion problems. A most common tool for control applies traffic lights in the junctions (also used the synonym term crossing). The goal of traffic control, based on the measurement of most relevant traffic properties, consists of finding a green time distribution for traffic lights used in the same junction such that the overall behavior of junctions assists to pass as many vehicles through the network as possible.

The literature of developed traffic modeling and proposed control strategies are growing fast, but they still meet a lot of challenges. Some promising approaches apply cell-transformation model which is a discrete approximation to the hydrodynamic model of traffic flow [1], [2]. In general, the computational complexity allows often only heuristic-based or soft computing methods as successful representations of control strategies. In [1], genetic algorithm is applied for optimization, but fuzzy experts [3] and knowledge based methods [4] were also successfully tested in applications. Some recent works reported different techniques, as well. Control strategy on the base of stochastic system modeling is able to release incident-induced traffic congestion in [5]. The methodology in [6] organizes traffic flow into arterial structure which is especially useful to establish green corridors in traffic network [7]. The store-and-forward model described in [8] relies on state-space representation often preferred in control engineering. The control strategy discussed in this paper applies optimal LQ control.

Each junction in the network has a right to make own decision, hence any junction (agent or player, all of them are considered synonyms in this paper) corresponds to an agent in the environment. Interaction among agents are realized by the control strategies they carry out through the chosen green time distribution, as decision. It is possible to define a game in many multi-agent applications. Games assign cost function to every agent for every combination of agents' decisions. The game is cooperative if agents are cooperating for a common goal usually on the base of a command arriving from a higher level supervisor [9]. In many cases, however, the game is noncooperative [10], mainly if agents are opponents in their goal (zero-sum games) or they pursue different goals (nonzero-sum games) or there is a common goal but there is also an individual goal for each agent [11]. In order to find an optimal decision, different types of equilibrium points have been elaborated. Ones of the most widely used equilibrium point in noncooperative games is the Nash equilibrium point. If there exist a hierarchy among agents, Stackelberg strategies also lead to optimal solution.

This paper shows how the urban traffic control problem should be converted to game theoretic problem. As a result, a suboptimal game theoretic solution is also provided to the problem. An illustrative example of traffic control demonstrates the efficiency of the proposed solution.

The paper is organized, as follows. Section 2 describes a traffic model applied in the simulation test. Section 3 presents the proposed control algorithm. The simulation results on a regular traffic network (inspired by many North American cities) with size 5×5 are illustrated in Section 4. Finally, Section 5 draws some conclusions.

2 The Traffic Model

For the traffic modeling, the Store-and-Forward model is used. The main notations and the concept are borrowed from [8], however, it contains some minor changes in the notation, assumptions and interpretation to fit the model to game theoretic description.

In the model, the urban network is realized by a graph having edges and nodes. The edges represent road-links, the nodes represent junctions. Consider a junction j. Let I_j be a set containing the incoming road links of j. Similarly, let O_j be a set containing the outgoing road links of j. The model are based on the following assumptions:
(ASF1): The vehicles pass every road link in a constant time. If the inflow is higher than the outflow at the end of the road link, the vehicles are stored (at the end of the road link). For each outcome link, a separated lane is designated from the incoming links of the junctions
(ASF2): Junctions assure at least a minimal green time from their any incoming road link to to their every outgoing road links. The minimal green time of jth junction from wth incoming road link to ith outgoing road link is denoted by $g^j_{w,i,min}$.
(ASF3): The cycle time C_j and the total lost time L_j of junction j are given. In addition, $C_j = C$ for all j.
(ASF4): The relative offsets of cycles between any two junctions are fixed (and consistent to others).
(ASF5): The saturation flows S_z, $z \in I_j$ are known for every junction.
(ASF6): The turning rates $t_{z,w}$, $z \in I_j$, $w \in O_j$ are known or estimated for every junction.
(ASF7): The junctions are arranged in a matrix structure. Every junction has 4 incoming road link and 4 outgoing link road(This often occurs in many North American cities.)
(ASF8): Road links are able to accept new vehicles from their source link without congestion. Based on the assumptions above, one writes that

$$\begin{aligned} \sum_{w \in I_j} \sum_{i \in O_j} g^j_{w,i} + L_j &= C \\ g^j_{w,i} &\geq g^j_{w,i,min} \quad \forall j \end{aligned} \tag{1}$$

where $g^j_{w,i}$ is the effective green time of junction j from incoming road link w to outgoing link i. Note that $i, w \in \{1, \ldots, 4\}$. Of course, the expression (1) generates an inequality constraint when cycle times are set up at traffic lights. Considering a road-link z between junction M and junction N ($z \in I_N, z \in O_M$) as shown in Figure 1, the discrete dynamics is given by

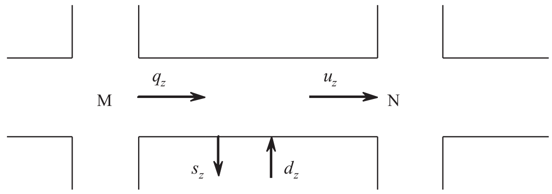

Figure 1: The schematics of a road link

$$x_z(k+1) = x_z(k) + T\left[q_z(k) - s_z(k) + d_z(k)u_z(k)\right]$$

where x_z is the number of vehicles within link z, q_z is its inflow, u_z is its outflow in the time period $[kT, (k+1)T]$, $k = 1, 2, \ldots$ with control time interval T. The additional terms d_z and s_z denote the demand and the exit flow, respectively. In the most cases, there is a strong relation between the demand and the exit flow described as $s_z(k) = t_{z,0} q_z(k)$. The equation (2) is described now as

$$x_z(k+1) = x_z(k) + T\left[(1 - t_{z,0})q_z(k) + d_z(k)u_z(k)\right]$$

(ASF9): The length of control time interval is at least C.
If x_z is sufficiently high, then (ASF8) and (ASF9) imply that the average value of the outflow is

$$u_z(k) = \frac{S_z G_z(k)}{C} \tag{2}$$

where the effective green time G_z of the road link z is

$$G_z(k) = \sum_{i \in O_j} g^j_{z,i}(k) \quad z \in I_j \tag{3}$$

Exploiting that

$$q_z(k) = \sum_{w \in I_M} t_{w,z} u_w(k) \tag{4}$$

the final form of discrete dynamics related to road link z is

$$\begin{aligned} & x_z(k+1) = x_z(k) \\ + \; & T\left[(1 - t_{z,0}) \sum_{w \in I_M} t_{w,z} \frac{S_w \sum_{i \in O_M} g^M_{w,i}}{C} \right. \\ - \; & \left. \frac{S_w \sum_{i \in O_N} g^N_{w,i}}{C}\right], \quad z \in O_M, z \in I_N \end{aligned} \tag{5}$$

Considering the equations (5) together for every road link in the network, one arrives the nonlinear discrete state equation of the urban traffic network. Note that [8] focuses on the system dynamics around the average green time values and applies linear LQ controller. In the next section, we propose a method to solve the problem in game theoretic framework.

3 The Control Algorithm

This section proposes a game theoretic solution for urban traffic problem using Nash equilibrium point. The idea of the concept is that urban traffic control can be considered as a multi-agent game theoretic problem in which each junction tries to minimize the number of vehicles on its incoming road links (local task with high priority) and taking a solidarity to an extent with its play-mate junctions, it also tries to help them (global task with lower priority). The decisions of the junctions (players) reflect a behavior in the green time distributions from incoming links to outgoing links. For example, if an incoming road link of junction j contains significantly more vehicles than other incoming road links, then junction j endeavors to decrease the load of this link by increasing the length of green times from this road link. Depending on the turning ratios, it increases the load on the incoming road link of some neighboring junctions which generates a conflict situation.

For the more exact discussion, let J denote the set of junctions. In this case the number of players are $\gamma = |J|$. Let $G^1_{i_1}$ denote the decision of the first player, let $G^2_{i_2}$ denote the decision of the second player etc., where $G^j_{i_j}$ usually change somehow the green time distribution

$$g^j(k) = \Big(g^j_{1,1}(k), \dots, g^j_{1,|O_j|}(k), \dots, g^j_{|I_j|,1}(k), \dots, g^j_{|I_j|,|O_j|}(k)\Big) \tag{6}$$

of junction j. Note that (ASF7) implies that the number of incoming and outgoing road links of junction j is $|I_j| = |O_j| = 4$. Numerous combinations of functions are allowed to define on $g^j(k)$, as decision set. Let $X^n(G^1_{i_1}, \dots, G^\gamma_{i_\gamma})$ be the cost of nth player and let $X(G^1_{i_1}, \dots, G^\gamma_{i_\gamma})$ be the cost vector including the cost of all players. Then, a decision vector $(G^{1*}_{i_1}, \dots, G^\gamma_{i_\gamma}*)$ is said to be a Nash equilibrium strategy in the kth control time period, if the inequalities

$$\begin{aligned} X^1(k, G^{1*}_{i_1}, G^{2*}_{i_1} \dots, G^{\gamma*}_{i_\gamma}) &\leq X^1(k, G^1_{i_1}, G^{2*}_{i_1} \dots, G^{\gamma*}_{i_\gamma}) \\ &\vdots \\ X^\gamma(k, G^{1*}_{i_1}, G^{2*}_{i_1} \dots, G^{\gamma*}_{i_\gamma}) &\leq X^\gamma(k, G^{1*}_{i_1}, \dots, G^{\gamma-1*}_{i_{\gamma-1}} G^\gamma_{i_\gamma}) \end{aligned} \tag{7}$$

are satisfied. It is easy to realize that if the number of decisions over $g^j(k)$ and/or the size of network i.e. the number of players γ are increasing then it is not possible to find a solution of (7) in real time. In order to overcome these problems, our method organizes junctions into groups and operate only with few decisions.

A group includes at most 4 members as depicted in Figure 2. As the figure shows, it is possible the groups to contain different number of junctions. Every group defines a subgame solved parallel to other subgames. The concept relies on the idea that the effect of a junction's decision to the cost of another junction is decreasing if the distance between the junctions increases.

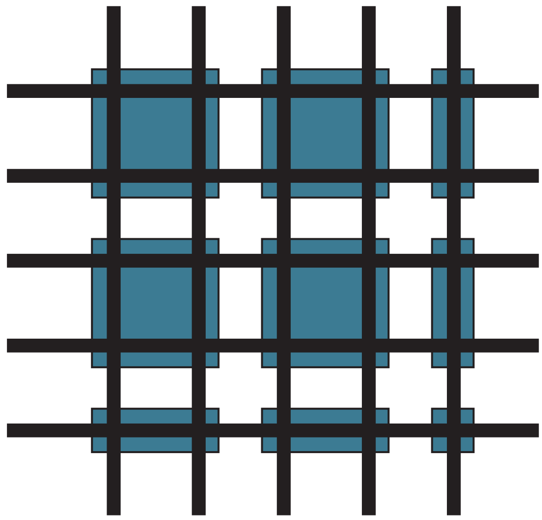

Figure 2: The grouping of junctions into subgames

Decisions are restricted to 4 different choices in the proposed technique, each choice prefers exactly one incoming road link of the junction at the other incoming road links' expense. The total green time of the junctions (and the cycle time) is maintained at constant value with this strategy during the whole traffic control. Of course, it is possible to choose other functions over $g^j(k)$.

The algorithm of urban traffic control on game theoretic basis requires some additional notations. Let Z be the set of all road links. Let $H(j,f)$ be the (an arbitrary ordered) set of junctions having a minimal distance f from junction j. Distance f is measured by road links, therefore it is an integer. Denote $H_p(j,f)$

the pth element of $H(j,f)$ and let $|H(j,f)|$ be the number of elements in $H(j,f)$. The proposed algorithm for urban traffic control is the following.

Algorithm (Urban Traffic control in game theoretic framework)
Input: $g^j_{w,z,min}$, L_j, C, S_z, $x_z(0)$, T, $t_{z,0}$, $t_{w,z}$, $d_z(0)$, $g^j_{w,z}$, $\forall j \in J, \forall z \in O_j, \forall w \in I_j$.
Output: $g^j_{w,z}(k)$, $\forall j \in J, \forall z \in O_j, \forall w \in I_j$.
Steps:
Step 1) Initialization.
Δg the quantum of the change in green time. $R > 0$ The radius (measured in edges) in which a junction considers the cost of other junctions, as well.

Step 2) Measure the characteristic of the actual traffic at kth control time interval: S_z, $t_{w,z}$, $d_z(k)$.

Step 3) Compute the potential decisions of each junction:

$$G^j_i = \Big(\delta^j_{1,1}(k), \ldots, \delta^j_{1,|O_j|}(k), \ldots, \delta^j_{|I_j|,1}(k), \ldots, \delta^j_{|I_j|,|O_j|}(k)\Big) \quad (8)$$

Decision i, $i = 1, \ldots, |I_j|$ of junction j, $j \in J$ satisfies that $\delta^j_{p,w}(k) = (|I_j| - 1)\Delta g$, if $p = i$, else $\delta^j_{p,w}(k) = -\Delta g$. If $g^j_{p,w}(k) + \delta^j_{p,w}(k) < g^j_{p,w,min}$ then G^j_i is set to a zero vector. Note that (ASF7) makes $|I_j| = 4$ fix for $\forall j$. It means that each decision prefers only one incoming road link increasing its green time by $3\Delta g$ while the green times from other incoming road link of junction j are decreasing by $-\Delta g$. During this operation the total green time in junction j does not change and does not change the cycle time either. The potential green time from incoming road link p to outgoing road link w at junction j after decision G^j_i is

$$\hat{g}^j_{p,w}(k) = g^j_{p,w}(k) + \delta^j_{p,w}(k). \quad (9)$$

Step 4) Compute to every decision of every junction $t \in J$:

$$X^t(k, G^1_{i_1}, \ldots, G^\gamma_{i_\gamma}) = \sum_{h=0}^{R} \frac{1}{h+1} \sum_{p=1}^{|H(t,h)|} \sum_{z \in I_{H_p(t,h)}} x_z(k, G^1_{i_1}, \ldots, G^\gamma_{i_\gamma}) \quad (10)$$

where $x_z(k)$ is computed by considering (5)

$$x_z(k+1) = x_z(k) + T\left[(1 - t_{z,0}) \sum_{w \in I_M} t_{w,z} \frac{S_w \sum_{i \in O_M} \hat{g}^M_{w,i}}{C} - \frac{S_w \sum_{i \in O_N} \hat{g}^N_{w,i}}{C}\right], \quad z \in O_M, z \in I_N \quad (11)$$

Step 5) Build the normal form of the game. In order to achieve normal form one should evaluate the vector vector function

$$X(G^1_{i_1}, \ldots, G^\gamma_{i_\gamma}) = \Big(X^1(G^1_{i_1}, \ldots, G^\gamma_{i_\gamma})), \ldots, X^\gamma(G^1_{i_1}, \ldots, G^\gamma_{i_\gamma})\Big) \quad (12)$$

for every combination of decisions. The vectors should be arranged in a matrix with dimension $|I_1| \times |I_2| \ldots |I_\gamma|$.

Step 6) Find a Nash equilibrium point of the game by solving (7). If more than one Nash equilibrium exist, agents select one by a known strategy. If there is no Nash equilibrium point in pure strategies, a possible alternative is to find a mixed equilibrium point as described in Proposition 3.5 in [10].

Step 7) Modify the green times according to Nash equilibrium point. It means that $g^j_{p,w}(k) = \hat{g}^j_{p,w}(k)$ where $\hat{g}^j_{p,w}(k)$ is selected by $G^{j*}_{i_j}$.

Step 8) Repeat the procedure from Step 2) for the next control time interval.

Nash equilibrium applied in the Algorithm tries to achieve a balanced vehicle load on the road links of a subnetwork. Since groups play subgames parallel, the solution is suboptimal. It is easy to extend the algorithm to dynamic and hierarchical games, however, it threatens the chance of real time realization.

4 Simulation Results

The simulation results on 5×5 sized traffic network, with $T = 60$ sec, $C = 300$ sec, $t_{z,0} = 0.01$, $d_z = 0.01$, $S_z = 1$, $x_z(0) = 30$, $\Delta g = 3$ sec, $g^j_{w,z,min} = 5$ sec are illustrated in Figure 3-Figure 8.

Typical green time distributions are illustrated on two junctions with coordinates (2,4) and (3,4). By definition, green times are never set up to a constant values. It is seen from the figures that there is no incoming road link of the illustrated junctions that are absolutely dominating the game. In fact, a dominating incoming road link may occur if it has heavy load permanently in relative to others.

Figure 5 and Figure 6 provide a possible way to compare the traffic control strategy to a constant green time set up. Both strategies start from the same green time values, however the proposed Algorithm allows to adapt to network load by changing green times of the traffic lights. The computation of cost function (10) is carried out for each junction. The cost values of individual junctions with constant green times are shown in Figure 5 and with Nash equilibrium point are shown in Figure 6. It is observed that the cost values are increasing in both cases which comes from the

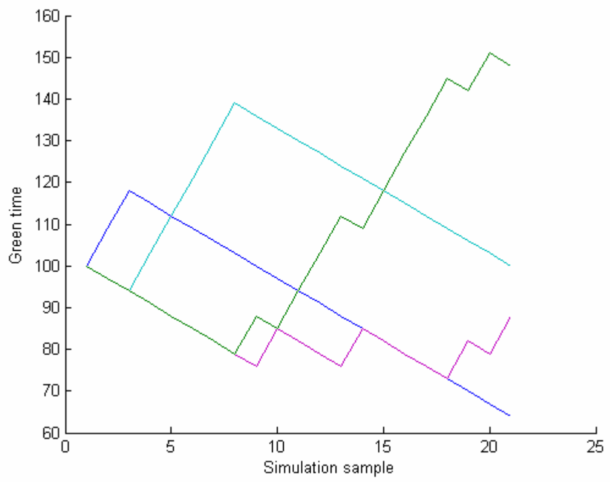

Figure 3: Green times in the junction (2,4)

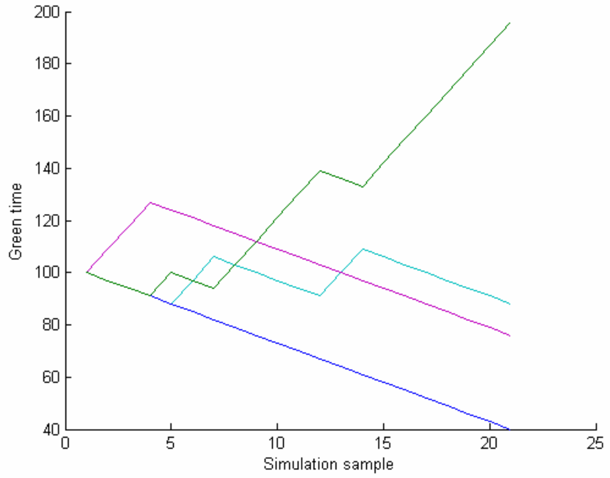

Figure 4: Green times in the junction (3,4)

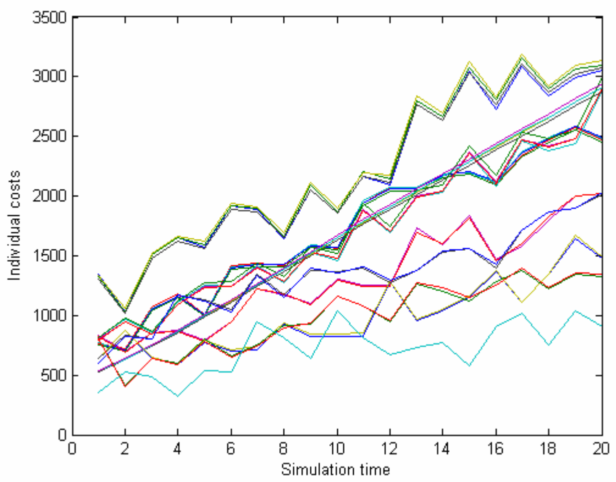

Figure 5: Induvidual costs with constant green times

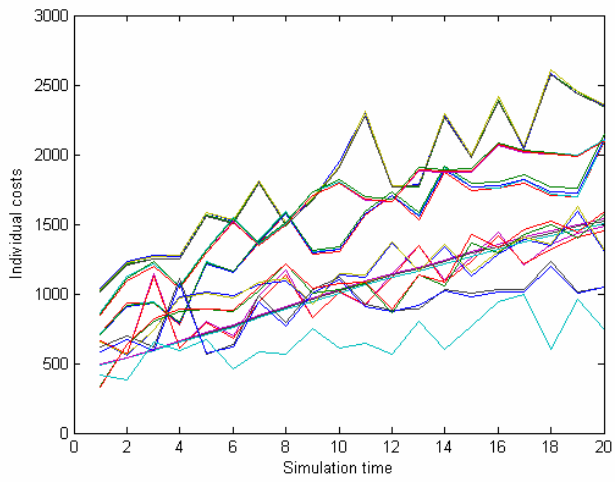

Figure 6: Induvidual costs with Nash strategy

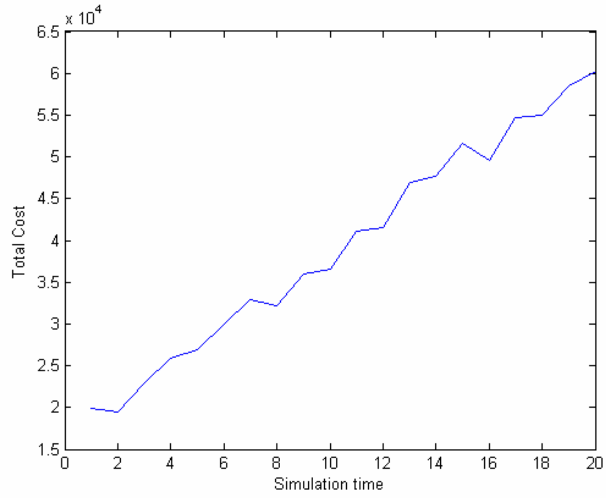

Figure 7: The total cost with constant green times

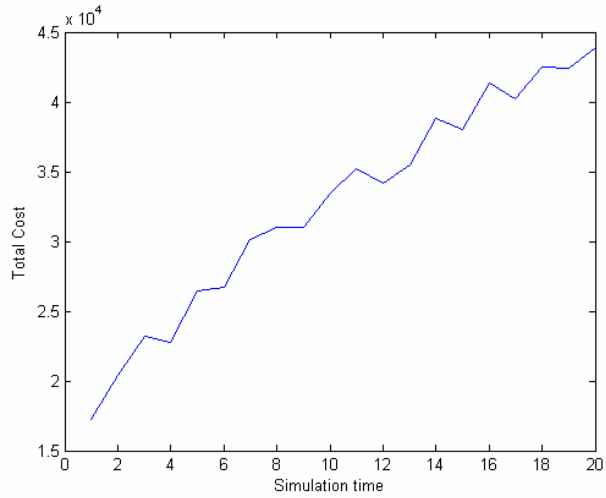

Figure 8: The total cost with Nash strategy

fact that too many vehicles enter into the whole network and the traffic lights cannot clear the road links from the cars. Still, this phenomenon provides better simulation environment to see the differences between the two concepts. Using constant green times for traffic light, relatively many junctions achieve cost values around 3000 while the highest cost values occur for Nash equilibrium strategies are lower than 2500. Of course, it is possible to define other cost functions, too. They may provide other green time distributions in the network. The definition of the cost function depends on the user. It is possible, for example, to define a game in which a priority for vehicles with distinguished signal appears in the decisions. The new scenario can be easily integrated in game theoretic framework.

The efficiency of the whole network can be measured by the total cost of the junctions. The total cost in case of constant green times is depicted in Figure 7, in case of Nash equilibrium strategy is depicted in in Figure 8. As observed from the figures, the strategy of constant green times leads to around 30% worse performance than Nash equilibrium strategy.

The control strategy implemented in Matlab2006a and executed on PC with 3Ghz Pentium processor has been fulfilled the real time requirements. The performance of game theoretic traffic control can be further improved by considering bigger groups and by more sophisticated decisions. Sophisticated methods require increasing number of alternatives in decision, i.e. improvements spoil the chance of real time realization. Similar case occurs if junction plays dynamic games exploiting the effect of decisions in time.

5 Conclusion

A game theoretic framework using Nash equilibrium point has been proposed in the paper. Simulation results were underlying the intuition that a game theoretic strategy is able to outperform constant green times strategies. The solution is computationally expensive and can be realized only if some simplifications are carried out. The most important simplifications appear in bundling junctions into groups and decreasing the number of of decisions. In further studies, we intend to increase the performance by establishing a hierarchy among groups.

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