Path Design and Control for Collision Avoidance System of Cars

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Abstract: - The paper deals with design and realization problems of collision avoidance systems (CAS) of cars (ground vehicles). CAS emergency path design is based on the principle of elastic band with improved reaction forces for road borders and static obstacles allowing quick computation of the force equilibrium. The CAS path (reference signal) is smoothed and realized using receding horizon control (RHC). The car can be modelled by full (non-affine) or simplified (input affine) nonlinear models. Receding horizon control uses linearization along the path within the horizon and analytical optimization of a quadratic criterion with end-constraint. Differential geometric approach (DGA), known from control literature for the simplified nonlinear model, has been used for control initialization in the first horizon. For state estimation Kalman filters and measurements of two antenna GPS and Inertial Navigation System (INS) are used. The software validity test was performed using thorough simulation. A stand-alone software has been been developed using the C Compiler of MATLAB R2006a satisfying real time expectations.

Key-Words: - Collision avoidance; Lie-algebra; Receding horizon control; GPS/INS; Stand-alone software

1 Introduction

In a Collision Avoidance System (CAS) the path design has to be performed online in the presence of static and dynamic obstacles. The path information can be converted to reference signals for the low level control subsystems based differential geometry (Lie-algebra), optimal predictive control etc. Examples for static and dynamic obstacles may be the lost payload of a truck and an oncoming vehicle in the other lane. To solve the full problem sensor fusion of environmental information produced by radar, laser and computer vision systems is preferred, but this is not part of the paper. The sensory information can be used to initiate an elastic band which is the basis for emergency path design [1].

The stability properties of the path can be divided into 5 categories called Characteristic Velocity Stability Indicator (CVSI) which may be interpreted as understeering, neutral steering, oversteering, high oversteering and braking away [2]. If the generation of the drivable trajectory cannot be finished within a given time limit $T_{\text{max}}$, or if the estimated CVSI cannot be accepted because wrong stability properties, then emergency braking follows without steering. Otherwise the developed path will be performed by using an appropriate control method.

The speed of the computation of path design depends on the complexity of the situation and the number of iterations needed to find the force-equilibrium of the elastic band.

The control of cars is usually based on classical control methods (PID etc.), or advanced methods using Lie-algebra results of nonlinear control [7], optimal control [8] or predictive control [8], [9], [10]. Nonlinear control assumes known dynamic model of the car. Based on a simplified (input affine) nonlinear model Freud and Mayr [6] derived a control method for cars using differential geometric approach (DGA). Although the paper concentrates on receding horizon control (RHC), the DGA solution of Freud and Mayr is used for initialization of the control sequence in the first horizon and for comparison.

For state estimation Kalman filters and measurements of two antenna GPS and Inertial Navigation System (INS) are typical approaches [11], [12].

An important question is the quick development and test of the full software. For this purpose MATLAB and Optimization Toolbox is a useful developing environment. Stand-alone software can be generated using the C Compiler of MATLAB R2006a.

The paper is organized as follows. The elastic band model for path design and some modified potential functions are given in Section 2. The nonlinear car model and its simplified version are presented in Section 3. Control based on DGA approach is
summarized in Section 4. RHC control based on linearization along the CAS path in the horizon is presented in Section 5. State estimation using GPS/INS sensory information and Kalman filters is discussed in Section 6. Results of simulation tests are reported in Section 7. Conclusions are given in Section 8.

2 Path design using elastic band

Typical approaches for path design are constrained optimization using splines [4] and the elastic band [3] originated from Khatib. Because of real time considerations the paper prefers path design using elastic band with some modifications of the results in [1].

It is assumed that satisfactory sensory information is present about the path section and the static and dynamic obstacles. The sensory information can be used to initiate an elastic band which is the basis for CAS emergency path design.

In the actual state of the vehicle an elastic band will be started from the origin \( r_0 \) of the vehicle coordinate system. The elastic band consists of \( N \) springs, the connection points \( r_i \) of the springs are the nodes. The spring constant and the initial length of the \( i \)-th spring are \( k_i \) and \( l_{0i} \), respectively. The goal is to find the force equilibrium of the system using iteration. In every iteration step the reaching time \( t_i \) will also be determined based on spline technique and the longitudinal velocity of the vehicle. If the path belonging to the force equilibrium is stabilized then the vehicle can move along the path taking into consideration the reaching time. The elastic band is characterized by the information:

\[
(r_0, t_0) \rightarrow (r_i, t_i) \rightarrow \frac{k_i}{d_{ij}} \rightarrow (r_{i+1}, t_{i+1}) \rightarrow \cdots \rightarrow (r_N, t_N)
\]

Obstacles are modelled by safety circles. The center and diameter of the safety circle \( O_j \) are \( r^{O_j} \) and \( d_j \), respectively. The diameter \( d_j \) consists of the sum of the diameter of the obstacle and the width of the vehicle. If the elastic band of the vehicle reaches the safety circle of a node then new elastic bands will be generated from the existing ones so that typically more than one elastic bands are iterated in order to find force equilibrium. After reaching the force equilibrium, for each equilibrium the time-parametrized trajectory, curvature and lateral acceleration are determined based on spline technique. For the vehicle that path will be chosen for which the lateral acceleration is the smallest.

The path design is based on the internal potentials of the springs, the external potentials of the left \((B_l)\) and right \((B_r)\) borders of the road and the external potentials of the moving obstacles. Forces at each node are given by the directional derivatives of the corresponding potential fields. During the computation of the influence of moving obstacle the velocity of the moving obstacle should also be taken into consideration.

2.1 Internal and external forces

For every node \( r_i \) as internal potential the potential energy of the spring section can be chosen (its negative gradient is the force acting on the node):

\[
V_i^{\text{int}} = \frac{1}{2} k_i (|r_{i+1} - r_i| - l_{0i})^2
\]

(1)

\[
V_i^{\text{int}} = \sum_{i=0}^{N-1} V_i^{\text{int}} = \sum_{i=0}^{N-1} \frac{1}{2} k_i (|r_{i+1} - r_i| - l_{0i})^2
\]

(2)

The external potential of the borders of the road should decrease logarithmically approaching the middle of the road. Along the borders of the road reference points \( r^{B_{jl}}, q \in \{l, r\} \) will be chosen (\( l \) stands for left, \( r \) for right border). For every node \( r_i \) of the elastic band the nearest points \( r_{jl}^{B_i} \) and \( r_{jr}^{B_i} \) along the borders will be chosen which determine the force \( F_{jl}^{B_i} \) acting on the node. Without obstacles it can be expected that assuming road width \( b \), two lanes, and force equilibrium \( F_{jl}^{B_i} = F_{jr}^{B_i} \), the ratio \( k^{B_l}/k^{B_r} = 0.75b/0.25b = 3 \) should be fulfilled between the friction constants of the borders. The forces suggested in [1] are not able to place back the elastic band in the middle of the road without obstacles and act in different way at left and right lane boundaries. Hence our choice was

\[
F_{jl}^{B_i} = M^B \exp[-\frac{1}{2} \left( \frac{r_{jl}^{B_i} - r_i}{\sigma^{B_l}} \right)^2] \frac{r_i - r_{jl}^{B_i}}{|r_i - r_{jl}^{B_i}|} \]

(3)

\[
\sigma^{B_l} = k^{B_l} / \sqrt{2 \ln(M^B/m^B)}
\]

with \( M^B = 2, m^B = 0.05 \).

The shape (curvature etc.) of the elastic band in force equilibrium is significantly influenced through the forces generated by the static obstacles. It has been established that the suggestions in [1] cannot guarantee satisfactory path curvature and centripetal acceleration. In order to assure slowly decreasing repulsive force our choice was

\[
F_{jl}^{O_i} = k^{O_i} \frac{d^{O_i}}{2} \frac{r_i - r^{O_i}}{|r_i - r^{O_i}|} \]

(4)
A suitable parameter is \( k^{O_i} = 3 \) if \( d^{O_i} \) is the diameter of the static obstacle.

The potential function of the moving obstacle \( O_j \) acts for every node \( i \). During the evaluation of the potential function it should be taken into consideration that the node position \( r_i \) will be reached at time instant \( t_i \). Hence the position \( r^{O_i}(t_i) \) of the moving obstacle has to be extrapolated from the initial position \( r^{O_i}(t_0) \) and assumed velocity \( v^{O_i}(t_0) \) by using linear extrapolation. The force acting on node \( i \) was the same as in [1]:

\[
F_{i}^{O} = k^{O_i} \exp \left( \frac{d^{O_i}}{2} \right) \left[ r_i - r^{O_i}(t_i) \right] \quad (5)
\]

### 2.2 Solution of force equilibrium problem

Let \( M \) be the number of static and moving obstacles. Then the force equilibrium condition has the form:

\[
F_{i}^{sum} = F_{i}^{u} + F_{i}^{R} + \sum_{j=1}^{M} F_{i}^{O_j} = 0 \quad (6)
\]

The equilibrium should be satisfied for each node. Hence, introducing the notation

\[
x = (r_1, r_2, \ldots, r_N)^T
\]

\[
f(x) = \left( (F_1^{sum})^T, (F_2^{sum})^T, \ldots, (F_N^{sum})^T \right)^T
\]

the numerical problem is to solve the nonlinear system of equations \( f(x) = 0 \). We used \( \text{fsolve} \) of the Optimization Toolbox supplementing to it beside \( f(x) \) also the Jacobian (derivative) of \( f(x) \).

The path design was tested for a problem similar to that in [1] (41 nodes; static obstacle of diameter 2.5m; moving obstacle of diameter 3.5m, initial distance 120m, assumed velocity 15m/s; average velocity of own car 20m/s). The computation of the force equilibrium using \( \text{fsolve} \) needed approximately 1s.

The elastic band is the abstract form of the CAS emergency path which has to be converted to reference signals for the control. It is useful to compute also the higher order derivatives of the path because they can support the test of the total control system including state estimation using GPS/INS and Kalman filters. Moreover the higher order derivatives are also needed for CVSI computations. In the developed program first the elastic band was approximated by linear sections and the time instants belonging to the nodes were determined assuming known average velocity of the own car. Then based on the time distribution the kinematic variables have been determined by using third order polynomial approximations between the nodes.

The approximation was performed using the spline technique of MATLAB (\text{spline}, \text{ppval}, \text{unmkpp}, \text{mkpp}). Based on the derivatives the kinematic variables to the CAS reference path can be determined as follows:

\[
v = (\dot{x}^2 + \dot{y}^2)^{1/2} \quad \dot{v} = \frac{\dot{x} \ddot{x} + \dot{y} \ddot{y}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}
\]

\[
\psi = \arctan(\dot{y} / \dot{x}) \quad \dot{\psi} = \frac{\ddot{y} \dot{x} - \ddot{x} \dot{y}}{\dot{x}^2 + \dot{y}^2}
\]

\[
\kappa = \frac{\dddot{y} - \dot{y} \dddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}}
\]

where \( v \) is the velocity, \( \psi \) is the orientation, and \( \kappa \) is the curvature.

### 3 Nonlinear dynamic model of the car

Two nonlinear dynamic models have been used based on the bicycle model of vehicles which differs in the approximation of trigonometrical functions [5]. Denote \( \beta \) the side slip angle, \( \nu_G \) the velocity, \( X,Y \) the position, and \( \delta_w \) the steering angle, respectively. Front and rear longitudinal forces are denoted by \( F_{IF} \) and \( F_{IR} \), cornering stiffnesses by \( c_F \) and \( c_R \). It is useful to introduce front and rear side forces \( S_v \) and \( S_h \) according to

\[
S_h = c_R (-\beta - \frac{l_g \dot{\psi}}{v_G}), \quad S_v = c_F (\delta_w - \beta - \frac{l_F \dot{\psi}}{v_G}) \quad (9)
\]

The front longitudinal force is assumed to be \( F_{IF} = 0 \). The air resistance disturbance \( T(x) \) is not considered during control. Control input and state are \( u = (S_v, F_{IR})^T \) and \( x = (\beta, \nu, \dot{\psi}, v_G, X, Y)^T \), respectively.

#### 3.1 Full nonlinear model of ground vehicles

Ground vehicles can usually be modelled with satisfactory precision by the state equations

\[
\dot{\beta} = -\dot{\psi} + \frac{1}{m_v v_G} \{ F_{IF} \sin(\delta_w - \beta) - (F_{IR} - T) \sin(\beta) + c_F (\delta_w - \beta - \frac{l_F \dot{\psi}}{v_G}) \cos(\delta_w - \beta) + c_R (-\beta + \frac{l_g \dot{\psi}}{v_G}) \cos(\beta) \}
\]
3.1 Approximated nonlinear vehicle model

Using first order Taylor approximation except the position the following input affine nonlinear model can be derived:

\[
\dot{x} = v_G \cos(\psi + \beta)
\]

\[
\dot{\psi} = \frac{1}{I_z} \{ l_F F_{IF} \sin(\delta_w) + l_F c_F (\delta_w - \beta - \frac{l_F \psi}{v_G}) \cos(\delta_w) - l_R c_R (-\beta + l_R \psi) \}
\]

\[
\dot{v}_G = \frac{1}{m_v} \{ l_{FR} \cos(\delta_w - \beta) + (F_{GR} - T) \cos(\beta) \}
\]

\[
-c_F (\delta_w - \beta - \frac{l_F \psi}{v_G}) \sin(\delta_w - \beta) + c_R (-\beta + l_R \psi) \sin(\beta) \}
\]

By introducing the notations

\[
\lambda_i = \lambda_i w_i - \alpha_0 x_i - \alpha_1 x_4 C_{12}
\]

\[
\tilde{y}_2 = \lambda_2 \tilde{w}_2 - \alpha_0 x_6 - \alpha_1 x_4 S_{12}
\]

the closed loop system can be written as

\[
u_1 = -S_{h} + [(C_{12} x_1 - S_{h}) \tilde{y}_1 + (S_{12} x_1 + C_{12}) \tilde{y}_2] m_v
\]

\[
u_2 = T + (C_{12} x_1 + S_{12} \tilde{y}_2) m_v
\]

\[
\tilde{y}_1 = x_4 C_{12}
\]

\[
\tilde{y}_2 = x_4 S_{12}
\]

\[
\tilde{y}_1 = \tilde{x}_4 C_{12} - x_4 S_{12} (\tilde{x}_1 + \tilde{x}_2)
\]

\[
\tilde{x}_1 + \tilde{x}_2 = (-S_{12} \tilde{y}_1 + C_{12} \tilde{y}_2) / \tilde{x}_4
\]

\[
\tilde{x}_4 = C_{12} \tilde{y}_1 + S_{12} \tilde{y}_2
\]

C \equiv A(x) + B(x)u, \quad y = \begin{bmatrix} x_5 \\ x_6 \end{bmatrix}

4 Nonlinear output feedback

Freund and Mayr [6] had shown that the approximated (input affine) nonlinear car model has differential orders \(d_{x_1} = d_{x_2} = 2\) and hence it can be controlled by the control law

\[
u = S^{-1}(x) \left\{ C^*(x) + \Lambda w - M^*(x) \right\}
\]

where

\[
S^{-1}(x) = m_v \begin{bmatrix} C_{12} x_1 - S_{12} & S_{12} x_1 + C_{12} \\ C_{12} & S_{12} \end{bmatrix}
\]

\[
C^*(x) = \begin{bmatrix} \frac{1}{m_v} \left( \frac{1}{T} (S_{12} x_1 + C_{12}) + S_{12} S_{h} \right) \\ \frac{1}{m_v} \left( \frac{1}{T} (C_{12} x_1 - S_{12}) + C_{12} S_{h} \right) \end{bmatrix}
\]

\[
M^*(x) = \begin{bmatrix} \alpha_0 x_5 + \alpha_1 x_4 C_{12} \\ \alpha_0 x_6 + \alpha_1 x_4 S_{12} \end{bmatrix}
\]

\[
S_{12} = \sin(x_1 + x_2)
\]

\[
C_{12} = \cos(x_1 + x_2)
\]

\[
\Lambda = \text{diag}(\lambda_1, \lambda_2)
\]
where the CAS emergency path position functions
\( w_{i,d} = X_d(t) \) and \( w_{2,d} = Y_d(t) \) play the role of reference signals. This DGA control method needs also the first and second derivatives of the reference signals which can be determined based on spline technique. Notice that from the control \( S_v \) and the state variables (or their estimations) the steering angle \( \delta_w \) can also be computed:
\[
\delta_w = \frac{S_v}{c_F} + \frac{l_p}{v_G} \psi
\]

5 Receding horizon control (RHC)

Receding horizon control [9],[10] optimizes a cost function in open loop using the prediction of the system future behavior based on the dynamic model of the system, determines the future optimal control sequence within the horizon, applies the first control in closed loop to the real system, and repeats these steps for the new horizon which is the previous one shifted to the left by the sampling time \( T \). An open question is the stability of the closed loop nonlinear system, but the chance for stability is increasing with increasing the horizon length \( N \) (time \( NT \)).

If the nonlinear dynamic model is used for prediction then the optimization is usually a nonlinear optimization problem in real time which is time-critical for quick systems. Hence linearization around the prescribed nominal trajectory may be suggested and optimization of the perturbations using quadratic cost and analytically manageable end-constraint seem to be a good compromise. This is also our concept [8]. However it is a serious problem that the desired CAS emergency path can not easily be converted to the desired control which makes the linearization in the horizon more difficult (and hence nominal is not equal desired).

Denote \( \{x_0, x_1, \ldots, x_N\} \) and \( \{u_0, u_1, \ldots, u_{N-1}\} \) the nominal state and control sequence within the horizon, respectively, \( \hat{x}_0 \) the estimation of the initial state, and let \( \{y_0 = Cx_0, y_1 = Cx_1, \ldots, y_N = Cx_N\} \) be the output sequence belonging to the state sequence. Let \( \{y_{d0}, y_{d1}, \ldots, y_{dN}\} \) be the desired output and \( \{e_0 = y_{d0} - y_0, e_1 = y_{d1} - y_1, \ldots, e_N = y_{dN} - y_N\} \) the error sequence. The nominal control sequence may be the one belonging to DGA control in case of the first horizon, or the shifted previous optimal sequence completed with one new element derived from end-constraint etc. (see Step 1 later on).

Using the perturbations \( \delta x_0 = \hat{x}_0 - x_0, \delta x_1, \ldots, \delta x_N, \delta u_0, \ldots, \delta u_{N-1} \) the full or approximated nonlinear dynamic model can be linearized around the nominal sequences resulting in a linear time-varying (LTV) system \( \delta x_{i+1} = A_i \delta x_i + B_i \delta u_i \). The output errors are \( \delta y_{d0} = C(x_i + \delta x_i) = e_i - \delta y_i \), and the cost function \( J \) can be chosen as a quadratic function penalizing both output errors and large deviations from the nominal control:
\[
J = \frac{1}{2} \sum_{i=0}^{N-1} \| e_i - \delta y_i \|^2 + \frac{1}{2} A \sum_{i=0}^{N-1} \| \delta u_i \|^2 \quad (16)
\]

The state and output perturbations satisfy
\[
\begin{bmatrix}
\delta x_1 \\
\delta x_2 \\
\vdots \\
\delta x_{N-1} \\
\delta x_N
\end{bmatrix} =
\begin{bmatrix}
A_0 & A_1 & A_2 & \cdots & A_{N-1}
\end{bmatrix}
\begin{bmatrix}
\delta x_0 \\
\delta x_1 \\
\vdots \\
\delta x_{N-1}
\end{bmatrix}
\]
\[
\begin{bmatrix}
B_0 \\
A_1 B_0 \\
\vdots \\
A_{N-2} B_0 A_{N-1} B_0 \\
A_{N-1} A_{N-2} B_0 A_{N-1} B_0 A_{N-1} B_{N-2}
\end{bmatrix}
\begin{bmatrix}
\delta u_0 \\
\delta u_1 \\
\vdots \\
\delta u_{N-2}
\end{bmatrix}
\]

or in compact form
\[
\begin{bmatrix}
\delta y_1 \\
\delta y_2 \\
\vdots \\
\delta y_{N-1} \\
\delta y_N
\end{bmatrix} =
\begin{bmatrix}
C A_0 & C A_1 & C A_2 & \cdots & C A_{N-1}
\end{bmatrix}
\begin{bmatrix}
\delta x_0 \\
\delta x_1 \\
\vdots \\
\delta x_{N-1}
\end{bmatrix}
\]
\[
\begin{bmatrix}
C B_0 \\
C A_1 B_0 \\
\vdots \\
C A_{N-2} B_0 A_{N-1} B_0 \\
C A_{N-1} A_{N-2} B_0 A_{N-1} B_0 A_{N-1} B_{N-2}
\end{bmatrix}
\begin{bmatrix}
\delta u_0 \\
\delta u_1 \\
\vdots \\
\delta u_{N-2}
\end{bmatrix}
\]

or in compact form
The discrete time LTV model from the initial state and the nominal $\delta_0$. The following notations simplify the results:

$$L_1 := H^T_1 H_1 + \lambda I, \quad L_\mu := H_2 L^T_1 H^T_2$$

Then it follows from (22) and the end-constraint

$$\partial U = L^{-1}_1 (\bar{m} + H^T_2 \mu - H^T_1 P_1 \delta_0)$$

$$e_N - P_2 \delta_0 - H_2 [L^{-1}_1 (\bar{m} + H^T_2 \mu - H^T_1 P_1 \delta_0)] = 0$$

$$\mu = L^{-1}_0 [e_N - H_2 L^{-1}_1 \bar{m} - (P_2 - H_2 L^{-1}_1 H^T_1 P_1) \delta_0]$$

and therefore

$$\partial U = L^{-1}_1 \{H^T_2 L^T_1 e_N + (I - H^T_2 L^T_1 H_2 L^{-1}_1) \bar{m}$$

$$-(H^T_1 P_1 + H_2 L^{-1}_1 (P_2 - H_2 L^{-1}_1 H^T_1 P_1) \delta_0)\}$$

The closed loop control is $u_0 + \delta_0$, where $u_0$ is the nominal control and $\delta_0 \in \Re^r$ is the first element of the open loop optimal sequence $\partial U$.

In every horizon the following steps are repeated:

**Step 1.** From the initial state $x_0$ and the nominal control $\{u_0, u_1, \ldots, u_{N-1}\}$ the nominal state sequence $\{x_0, x_1, \ldots, x_N\}$ in the horizon is determined by using the approximated nonlinear dynamic model of the car. Here $x_0$ is coming from the shifted previous horizon and can differ from the estimated state $\hat{x}_0$. The desired state sequence is the one computed from the CAS emergency path with zero slide slip angle. The output is assumed to be $y = (X, Y)^T$, hence the desired output sequence can easily be computed from the desired state sequence in the horizon using $C = [e_x, e_y]^T$. The output sequence belonging to the nominal state sequence can also be computed using $C$. The error sequence is the difference between them. In case of the first horizon the nominal control sequence is computed by the DGA method and $x_0$ is initialized from the determined CAS path with zero side slip angle.

**Step 2.** The discrete time LTV model

$$\delta x_{i+1} = A_i \delta x_i + B_i \delta u_i$$

is determined from the approximated nonlinear model $\dot{x} = f(x,u)$ using Euler formula and therefore

$$A_i := I + T \frac{df}{dx}|_{(x,u)}$$

$$B_i := T \frac{df}{du}|_{(x,u)}$$

**Step 3.** The optimal change $\partial U$ of the control sequence is computed by (24) using $\delta_0 = \hat{x}_0 - x_0$ where $\hat{x}_0$ is the estimated state. The optimal control
sequence is $U := U + \delta U$. The first element $u_0$ of it will be applied in closed loop.

**Step 4.** In order to initialize the control sequence for the next horizon the optimal state sequence belonging to the initial state $x_0$ and the optimal control sequence $\{u_0, u_1, \ldots, u_{N-1}\}$ is determined using the approximated nonlinear dynamic model $\dot{x} = f(x, u)$. The result at the end of the transients is the state $x_N$. The unknown new $u_N$ can be determined in three ways: i) $u_N$ is determined by using $x_N$ and the DGA method. ii) $u_N$ is computed such that the difference between $x_{N+1}$ computed from the discrete time nonlinear model by Euler formula and $x_{d,N+1}$ according to the CAS path, that is $f(x_N) + G(x_N)u_N - x_{d,N+1}$, is minimized in LS-sense. iii) The last control signal is simply repeated: $u_N := u_{N-1}$.

**Step 5.** The nominal control sequence for the next horizon is $\{u_1, u_2, \ldots, u_N\}$ which is the augmented optimal control sequence $\{u_0, u_1, \ldots, u_N\}$ shifted by 1 to the left.

It is possible to put integrator into the controller using augmented state $\tilde{x}_i := (\tilde{x}_i^T, \tilde{\delta}u_{i-1}^T)^T$ where $\tilde{x}_i = \tilde{x}_{i-1} + \tilde{\delta}x_i$ and the change of the control $\tilde{\delta}x_i$ has to be optimized. Putting

$$A_i := \begin{bmatrix} A_i & B_i \\ 0 & I \end{bmatrix} \text{ and } B_i := \begin{bmatrix} B_i \\ 1 \end{bmatrix}$$

the earlier results remain valid in the new variables. However in this case $\delta R$ is the optimal change of the control differences and the optimal $\delta U$ is the cumulative sum of $\delta R$.

### 6 State estimation using GPS/INS

It was assumed that the measured signals are supplied by a two antenna GPS system and an INS system containing accelerometers and gyroscopes.

The evaluation of GPS/INS signals is based on the results of Ryu and Gerdes [11]. It is assumed that the GPS/INS system software package provides high level information in form of following signals:

- $V_{GPS}^{m}$: measured velocity of the car in the GPS earth coordinate system.
- $\psi_{GPS}^m$: the orientation of the car in the car fixed coordinate system.
- $a_{x,m}$: longitudinal acceleration of the car in the car fixed coordinate system.
- $a_{y,m}$: transversal acceleration of the car in the car fixed coordinate system.
- $r_m$: the angular velocity of the car in $z$ (yaw) direction in the car fixed coordinate system.

It is assumed that the first antenna is immediately above the INS system and the INS system is located in the COG, otherwise small modifications are needed. The sensors have biases which have also to be estimated. The accuracy of sensors can be characterized by $\sigma_1$ and the bias. The additive Gaussian noise is concentrated into the domain $[-3\sigma, 3\sigma]$ to which comes yet the value of the bias.

For a vehicle moving in the horizontal plane $V_{GPS}^m = (V_{GPS}^1, V_{GPS}^2, 0)^T$ is satisfied, from which the measured value of the side slip angle can be determined and from it the components of $u$:

$$\gamma = \arctan2(V_{GPS}^2, V_{GPS}^1) \Rightarrow \beta_{GPS}^m = \gamma - \psi_m^{GPS}$$

$$u_{\psi,m}^{GPS} = \begin{bmatrix} V_{GPS}^1 \\ \cos(\beta_{GPS}^m) \end{bmatrix} + \text{noise}$$

$$u_{r,m}^{GPS} = \begin{bmatrix} V_{GPS}^2 \\ \sin(\beta_{GPS}^m) \end{bmatrix} + \text{noise}$$

The state estimation can be based on a two stage Kalman filter. The first stage estimates the angular velocity $\dot{\psi} = r$. For this purpose two methods can be suggested:

$$\begin{align}
\begin{bmatrix} \dot{\psi} \\ \dot{r}_{bias} \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ r_{bias} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} r_m \\ \text{noise} \end{bmatrix} \\
\psi_{GPS}^m &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} \psi \\ r_{bias} \end{bmatrix} + \text{noise} \\
\frac{d}{dt} \begin{bmatrix} \psi \\ r_{bias} / s_r \end{bmatrix} &= \begin{bmatrix} 0 & -1/s_r \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ r_{bias} / s_r \end{bmatrix} + \text{noise} \\
\psi_{GPS}^m &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} \psi \\ r_{bias} / s_r \end{bmatrix} + \text{noise}
\end{align}
$$

where $s_r$ is the sensitivity of the gyroscope.

The state estimation is performed by Kalman filter, thus the continuous time models will be converted to discrete time ones:

$$A_d = \begin{bmatrix} 0 & -T \\ 0 & 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} T \\ 0 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$A_d = I_3 + A_d T, \quad B_d = 0_{3 \times 1}, \quad C_d = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

From the estimated values the angular velocity can be computed:
\[
\begin{align*}
  r &= \dot{\hat{\theta}} := -\hat{i}_{bias} + r_m \quad (28a) \\
  \hat{r} &= \dot{\hat{\theta}} := r_m (1/\dot{\hat{s}}_r) - (\hat{i}_{bias}/\dot{\hat{s}}_r) \\
  \end{align*}
\]

This value can be applied in the second Kalman filter which is based on the relation \( a = \dot{u} + \phi \times u \) from which the following continuous time model arises for the estimation of the velocities:

\[
\begin{bmatrix}
  \frac{d}{dt} a_{x,bias} \\
  \frac{d}{dt} a_{y,bias}
\end{bmatrix}
= \begin{bmatrix}
  0 & -1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  u_x \\
  u_y \\
  a_{x,bias} \\
  a_{y,bias}
\end{bmatrix}
+ \begin{bmatrix}
  1 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  a_{x,m} \\
  a_{y,m}
\end{bmatrix}
+ \text{noise} \\
\begin{bmatrix}
  u_{x,GPS} \\
  u_{y,GPS}
\end{bmatrix}
= \begin{bmatrix}
  1 & 0 & 0 & 0 & a_{x,bias} \\
  0 & 1 & 0 & 0 & a_{y,bias}
\end{bmatrix}
\begin{bmatrix}
  u_x \\
  u_y \\
  a_{x,bias} \\
  a_{y,bias}
\end{bmatrix}
+ \text{noise}
\]

The model contains \( r \) which is varying in real time hence it is useful that the discrete time form of the model can analytically be found:

\[
\begin{align*}
  A_{d,2} &= \begin{bmatrix}
  C(rT) & -S(rT)/r & S(rT) & -[1-C(rT)]/r & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  -S(rT) & [1-C(rT)] & C(rT) & S(rT) & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0
\end{bmatrix}
  \\
  B_{d,2} &= \begin{bmatrix}
  S(rT)/r & [1-C(rT)]/r & 0 \\
  0 & 0 & 0 \\
  -[1-C(rT)]/r & S(rT)/r & 0 \\
  0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0
\end{bmatrix}
  \\
  C_{d,2} &= \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0
\end{bmatrix}
\end{align*}
\]

where in the system matrices \( C \) and \( S \) stand for \( \cos \) and \( \sin \), respectively. Since \( r \approx 0 \) can not be excluded in practical cases hence limit values of system matrices were analytically determined for software implementation based on the L’Hospital rule. The estimation is performed by the second Kalman filter. From its estimated values \( \hat{\theta}_1, \hat{\theta}_2 \) and the output \( \hat{\psi} \) of the first estimator some other state variables can be estimated:

\[
\hat{\psi}_G = \sqrt{\hat{u}_x^2 + \hat{u}_y^2}, \quad \dot{\hat{\theta}} = \tan(\hat{u}_y, \hat{u}_x) \quad (31)
\]

Since the state variables \( X, Y \) of the position are not supported by absolute measurements hence their values were determined by numerical integration:

\[
\begin{align*}
  \dot{X} &= \dot{\hat{X}} + T \hat{V}_G \cos(\hat{\psi} + \dot{\hat{\theta}}) \\
  \dot{Y} &= \dot{\hat{Y}} + T \hat{V}_G \sin(\hat{\psi} + \dot{\hat{\theta}}) \\
\end{align*}
\]

Kalman filters were implemented in the following form (\( Q \) and \( R \) are the covariances of the noises):

\[
\begin{align*}
  P_{x}(t+1) &= A_d P_x(t) A_d^T + Q \\
  K &= P_x(t) C^T [C P_x(t) C^T + R]^{-1} \\
  P_x(t) &= [I - K C] P_x(t)
\end{align*}
\]

The measurement sampling times were \( T_{INS} = T = 0.01s \) (100Hz), \( T_{GPS,vel} = 0.1s \) (10Hz), \( T_{GPS,att} = 0.2s \) (5Hz).

### 7 Simulation results using RHC

Fig. 1-3 show the RHC control transients (error, controller output, states) realizing the determined CAS emergency path based on the modified elastic band technique and using state estimation by Kalman filters and GPS/INS sensory information. Fig. 1 contains the error transients, Fig. 2 the control signals and Fig. 3 the simulated and estimated states, all for the predictive control method with extra integrator in the controller. The nominal control sequence for the first horizon has been generated by DGA nonlinear output feedback. In the figures all signals are in SI units (m, rad, N, s, m/s etc.). The horizon length and sampling time were \( N = 10 \) and \( T = 0.01s \), respectively.

![Fig. 1. Position and orientation errors using RHC.](image-url)
8 Conclusions
In the paper methods have been presented for improved CAS emergency path design, nonlinear dynamic modelling of cars, control using nonlinear output feedback and predictive control, and state estimation based on GPS/INS sensory information. The nonlinear predictive control problem is solved by using linearization along appropriately chosen nominal control and state sequences, and analytical solution of the minimization of the cost function satisfying end-constraint. RHC control transients show significant improvement with respect to DGA control. The developed algorithms have been implemented in MATLAB R2006a using the Optimization Toolbox. Thorough software validity test was performed for CAS path design, state estimation, DGA and RHC control. The MATLAB software has been converted to stand-alone C program using the MATLAB C compiler. The transients in Fig. 1-3 were generated by using the stand-alone C version. The real time expectations are satisfied for 0.01s sampling time and $N=10$ horizon length.

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References: