FINE TUNED H infinity (H∞) DESIGN SCHEME FOR AN AIR BLOWN GASIFICATION CYCLE UNIT.

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Abstract: - The H∞ is a modern and efficient control method and guarantees closed loop stability. The purpose of this paper is to use a combination of the classical P.I control method with H∞ in order to produce an efficient controller for a real application, which concerns the design of a control scheme for an air blown gasification cycle unit plant (gasifier) for the production of environmentally clean energy. The gasifier is a multivariable six input four output plant. For a successful design there is a need for a proper weighting functions selection. This case study is based upon the data provided for the 100% load operating point of the gasifier.

Key-Words: - H∞, Multivariate Analysis, Loop Shaping, Stability, Robustness, Optimization, P.I.

1 Introduction.

Year by year the pollution of planet earth is increasing and electrical power generation makes a big contribution to it. As a consequence the protection of the environment has become a necessity in every engineering design regarding power generation. Integrated Gasification Combined Cycle (IGCC) power plants are being developed around the world in order to provide environmentally clean and efficient power generation from coal. The plant can be considered as a reactor and its operation is based on a combined gasification with a gas and steam cycle.

The aim of this research is the development of a dynamic simulation model and a controller using the H∞ method for the plant (gasifier). This case study is based upon the data provided for the 100% load operating point of the gasifier. The problem of controlling this plant is that it is numerically ill-conditioned. This means that commercial software packages have a strong chance of failing to provide an accurate solution for the problem because their algorithms cannot handle such numerics.

2 Problem Specification.

Schematically the gasifier unit is described in Figure 1. A brief description of the gasifier is that it is a non-linear, truly multivariable system, having five inputs (coal, limestone, air, steam and char extraction) and four outputs (pressure, temperature, bed mass and gas quality) with a high degree of cross coupling between them.

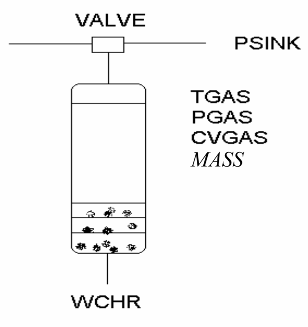

Fig. 1: The Gasifier

The *control inputs* are:
WCHR char extraction flow (kg/s)
WAIR air mass flow (kg/s)
WCOL coal flow (kg/s)
WSTM steam mass flow (kg/s)
WLS limestone mass flow (kg/s)

The disturbance input is PSINK sink pressure (N/m^2) (This represents the pressure upstream of the gas turbine that would vary according to the position of the gas turbine fuel valve.)

The *outputs* to be controlled are:
CVGAS fuel gas calorific value (J/Kg),
MASS bed mass (Kg),
PGAS fuel gas pressure (N/m^2),
TGAS fuel gas temperature (K).

Note that:

1) The output vectors in the state – space model are ordered as given above.

2) Limestone absorbs sulphur in the coal so WLS should be at least set to a fixed ratio of WCOL. Nominally this should be to 1:10 limestone to coal. This leaves effectively 4 degrees of freedom for the control design.

3) The data provided for the linear models provided (100%, 50%, 0% load) are completely open loop meaning that the bedmass controller is added solely for the validation of the simulation.

4) The order of the system under consideration is 25 and no model reduction has been applied to the data provided.

The proposed controller should be able to regulate the outputs bearing in mind that the input and output limits must not be exceeded.

Table 1: Input limits

	Max (Kg/s)	Min (Kg/s)	Rate (Kg/s^2)
WCOL	10	0	0.2
WAIR	0	0	1.0
WSTM	6	0	1.0
WCHR	3.5	0	0.2

For the output limits the following applies:

1) The CV fluctuation should be minimized, but must always be less than ±10 KJ/Kg

2) The pressure fluctuation should be minimized, but must always be less than ± 0.1 bar.

3) The bed mass should fluctuate by less than 5% of the nominal

4) Temperature fluctuation should kept to a minimum, but must always be less than ± 1 C. [12]

2.1 H∞ Optimal Control.

There are many process control problems where significant uncertainties exist in the system models, which therefore require robust control designs. Robust control design procedures enable good performance to be maintained even though significant modeling errors exist in the system description. [1], [2].

If a system has disturbance rejection robustness, the output will not be unduly influenced by the presence of disturbances. However, stability robustness is the most important requirement, since the final closed loop design should be stable despite modeling errors.

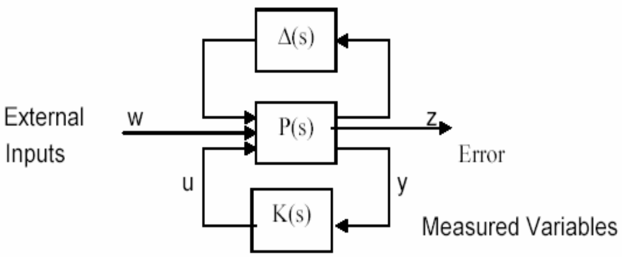

Fig.2: Standard representation of inaccurately known plant under feedback control

The uncertainty inherent in a plant model can be obtained by the representation of Figure 2, where P(s) is the plant to be controlled, K(s) is the designed controller and Δ(s) is the perturbation. It should be noted that the external inputs "w" is a vector of all the signals entering the system, and the "error" (z) is a vector of all the signals required to characterize the behavior of the closed loop system. Both of these vectors may contain elements, which are abstract in the sense that they may be defined mathematically, but do not represent signals that actually exist at any point in the system. u is the vector of control signals, and y is the vector of measured outputs.

If the perturbation Δ(s) is not present the system is more simplified. P(s) is now derived from the *nominal* plant model. However, it may also include weighting functions, which depend on the design problem that is being solved. Suppose that P(s) is partitioned as

$$P(s)=\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \quad (1)$$

so that

$$z = P_{11}w + P_{12}u, \quad (2)$$

$$y = P_{21}w + P_{22}u. \quad (3)$$

Then, u and y can be eliminated using $u=Ky$, to obtain

$$z = [\, P_{11} + P_{12}K(I - P_{22}K)^{-1} P_{21}]w. \quad (4)$$

For convenience, expression (4) can be written as follows: $z = F_1(P,K)w$.

By suitably defining w and z (or, equivalently, P), it is possible to put a number of practical design problems into the form

$$\text{minimise } \|F_1(P,K)\|_\infty$$

where the minimisation is over all realisable controllers K(s) that stabilise the closed loop system, and $\|.\|_\infty$ is defined as $\|G\|\infty = \sup_\omega \overline{\sigma}(G(j\omega))$.

This is known as the H∞ - optimisation problem**.**

2.2 Solution of the H∞ problem.

2.2.1 Equivalence to the model matching problem.

It was shown earlier that the general H∞ could be formulated as minimize $\|F_1(P,K)\|_\infty$ over the stabilising compensator K.

Combining P and J of Figure 3 into a transfer function T and assuming that $P_{22} = G$, the following conclusions can be deducted:

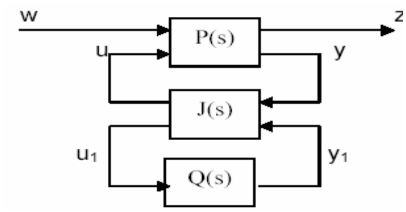

Fig.3: Inaccurately known plant under feedback control

$z = F_l(P,K)w = F_l(T,Q)w =$
$[T_{11} + T_{12}Q(I - T_{22}Q)^{-1}T_{21}]w.$ (5)

Here, $T_{22} = 0$ (for the scope of this research) so the previous expression can be simplified to
$z = [T_{11} + T_{12}QT_{21}]$w. (6)

So it can be now said that all that is needed is to

$$\min_{Q\in\infty} \|T_{11} + T_{21}\|_\infty \quad (7)$$

This is sometimes known as the model - matching problem because to solve it we need to choose Q such that $T_{12}QT_{21}$ "matches" the model – T_{11} as well as possible [3], [4].

2.2.2 Equivalence to Hankel Approximation

From now on $X^*(s)$ will be used to denote $X^T(-s)$. Moreover, the argument (s) will be dropped for convenience reasons. Assuming that the following relationships are valid

$T_{12}\,T^*_{12} = I$ and $T^*_{21}\,T_{21} = I$ (8)
it is easy to show that, if X and Y are all-pass (or inner) and $\sigma(.)$ denotes any singular value of (.), then $\sigma(XAY) = \sigma(A)$, and that, for any $\sigma(A^*) = \sigma(A)$. Hence it can be said that

$$\|T_{11} + T_{12}QT_{21}\|_\infty = \|T_{12}(T^*_{12}T_{11}T^*_{21} + Q)T_{21}\|_\infty =$$
$$\|T^*_{12}T_{11}T^*_{21} + Q\|_\infty = \|T_{21}T^*_{11}T^*_{12} + Q\|_\infty \quad (9)$$

An *all pass transfer function G*(*s*) is one that: G(s) = $G_o(s)*A(s)$, $A(s) = (s-a)/(s+a)$, where $|A(s)| = 1$ while its phase decreases.
Since $Q \in H\infty$ and $T_{ij} \in H\infty$, Q^* has only unstable poles while $T_{21}T^*_{11}T_{12}$ potentially has both stable and unstable poles. It can be shown, by a rather complicated analysis, that $T_{21}T^*_{11}T_{12}$ in fact has only stable poles [5].

If R is defined likewise as
$R = T_{21}T^*_{11}T_{12}$ (10)
then

$$\min_{Q\in H\infty} \|T_{11} + T_{12}QT_{21}\| = \min_{Q\in H\infty} \|R + Q\| \quad (11)$$

which converts the model – matching problem into the problem of approximating a stable transfer function R by an unstable one ($-Q^*$). This is known as the *Hankel approximation problem*, or the *Nehari extension problem*. [6]

2.2.3 Loop Shaping

The loop shaping procedure described here is based on H∞ robust stabilisation combined with classical loop shaping, [9]. It is essentially a two-stage design process. First, the open loop plant is augmented by pre and/or post – compensators to give a "*desired*" shape to the singular values of the open loop frequency response. Then, the resulting shaped plant is robustly stabilised with respect to coprime factor uncertainty using H∞ optimisation. An important advantage is that no problem – dependent uncertainty modelling, or weight selection, is required for this second step.

A multivariable system described by a proper transfer function matrix G(s) can be described by the following state space equations
$\dot{x}(t) = Ax(t) + Bu(t)$
$y(t) = Cx(t) + Du(t)$
if and only if
$G(s) = C(sI-A)^{-1} B + D$ (12)

The matrices N, M ∈ H∞ constitute a right coprime factorisation (RCF) of G if and only if :

$G = NM^{-1}$ (13)
M is invertible, that is $\det(M) \neq 0$
There exists $\tilde{V}, \tilde{U} \in H\infty$ such that
$\tilde{V}M + \tilde{U}N = I$ (14)

An arbitrarily large number of RCF can be generated for a single system G. The normalised right and left coprime factor realisations are

$$\begin{bmatrix} M \\ N \end{bmatrix} \stackrel{s}{=} \left[\begin{array}{c|c} A + BF & BR_1^{-1/2} \\ \hline F & R_1^{-1/2} \\ C + DF & DR_1^{-1/2} \end{array}\right] \quad (15)$$

$$\begin{bmatrix} \tilde{M} & \tilde{N} \end{bmatrix} = \begin{bmatrix} A + HC & H & B + HD \\ R_2^{-1/2}C & R_2^{-1/2} & R_2^{-1/2}D \end{bmatrix} \quad (16)$$

where
$$\begin{aligned} F &= -R_1(D^tC + B^tQ) \\ H &= -(BD^t + PC^t)R_2^{-1} \\ R_1 &= I + D^tD \\ R_2 &= I + DD^t \end{aligned} \quad (17)$$

and where P and Q are the solutions to the following *control and filter Riccati* equations:

$$\Phi^t Q + Q\Phi - QBR_1^{-1}B^t Q + C^t R_2^{-1} C = 0$$
$$\Phi P + P\Phi^t - PC^t R_2^{-1} CP + BR_1^{-1}B^t = 0 \quad (18)$$

where $\Phi = A - BR_1^{-1}D^t C = A - BD^t R_2^{-1} C$

The "*central*" controller of the set of all stabilising controllers for $\gamma > \gamma_{opt} > 0$ is [12]

$$K \stackrel{s}{=} \begin{bmatrix} A + BF + \gamma^2 X^{-1} PC^t (C + DF) & \gamma^2 X^{-1} PC^t \\ B^t Q & -D^t \end{bmatrix} \quad (19)$$

where $X = I + PQ - \gamma^2 I$ (20)

and $\gamma = \sqrt{1 + \lambda_{\max}(P * Q)}$ (21)

It should be noticed that low values of γ, for example up to 5, lead to a desirable closed loop response, since the controlled shaped plant does not change the *'loop shape'* significantly. High values of γ lead to large differences between the shaped and controlled plant, and this may lead to possibly poor robustness and / or performance which indicate a poor open loop shaping strategy. Moreover, for plants with D matrix zero, γ should be larger than [7], [8].

The following assumptions are typically made in H∞ design:

$$\text{If } P(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (22)$$

(A, B_2, C_2) is stabilisable and detectable.

D_{12} and D_{21} have full rank.

$$\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} \quad (23)$$

has full column and row rank for all ω.

$D_{11} = 0$, $D_{22} = 0$. (24)

If the Matlab computing facility is not going to be used, then these conditions must be checked before designing the controller. However, if Matlab is to be used then these criteria are checked, internally by the algorithm provided.

2.3 Design Procedure

As was mentioned earlier, a robust design specification is one where the sensitivity

$S_o = (I + GK)^{-1}$, (25)

is at frequencies less than some low frequency below the crossover satisfies:

$$\bar{\sigma}\,(I + GK)^{-1} < 1/L \qquad \omega < \omega_L < \omega_c \quad (26)$$

where L is some high gain for $\omega < \omega_L < \omega_c$.

Typically, at low frequencies $\underline{\sigma}\,(GK) >> 1$, so the requirement becomes one of satisfying

$$\underline{\sigma}\,(GK) > L \qquad \omega < \omega_L < \omega_c \quad (27)$$

For the performance objectives, the closed loop response at frequencies higher than some frequency above the crossover frequency needs to satisfy

$$\bar{\sigma}\,((I + GK)^{-1} GK) < U \qquad \omega > \omega_H > \omega_c \quad (28)$$

where U is some low gain for $\omega > \omega_H > \omega_c$.

Typically, at high frequencies $\bar{\sigma}$(GK) << 1, so the requirement becomes one of satisfying

$$\bar{\sigma}\,(GK) < U \qquad \omega > \omega_H > \omega_c. \quad (29)$$

3 Problem Solution.

In order to understand how the gasifier system should be approached the following notes could be useful:

1) The system has to examined in terms of controllability, observability and stability

2) Any possible model order reduction should be carried out at the beginning of the design. The resulting reduced system should be check again as for (1)

3) The selection of the weights is very important in H∞. The better the weight selection the better the produced results.

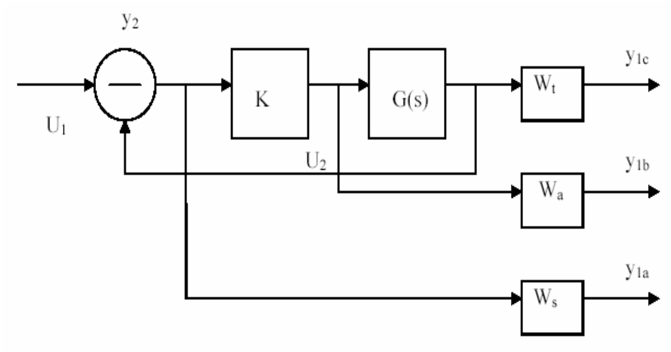

Fig.4: H∞ Design: Loss function for one degree of freedom

W_a is the weight that minimizes the actuator variations,

W_s is the weight that minimizes the noise rejection.

W_t is the weight that minimizes the error in the gain from the reference to input.

For the gasifier system only W_t and W_s are used and their value is presented at the next section. The loss function for the H∞ approach is given by:

$$\begin{bmatrix} Ws(s)*S(s) \\ Wa(s)*R(s) \\ Wt(s)*T(s) \end{bmatrix} \qquad (30)$$

where

$S(s) = (I + GK)^{-1}$ is the noise rejection and sensitivity function

$R(s) = (I + GK)^{-1}K$ the gain from output noise to actuator output

$T(s) = (I + GK)^{-1}GK$ is the complementary sensitivity function [10].

If up to this stage everything is calculated correctly, then the H∞ approach should provide an appropriate controller for the system.

4) Matlab provides efficient routines for both model order reduction and the H∞ controller calculation. [13]

4 Results.

It was known from the previous design [11] that if the bed mass loop were closed first then the rest of the system would be much easier to control. So this was exactly the thing to do, and of course, the weighting functions were appropriately designed in order for the H∞ algorithm to successfully pass all the relevant tests. The steps related to this design and the results of this study are presented below.

Step 1: The system was made square 4x4, the third and the fifth input were combined and the first two outputs were interchanged so as to control the bed mass loop first, by manipulating the char offtake.

Step 2: A simple input and output scaling was applied to the system of the form

$\text{Pre} = \text{diagonal}\left(\begin{bmatrix}-1 & 1 & 1 & 1\end{bmatrix}\right)$ and

$\text{Post}=\text{diagonal}\left(\begin{bmatrix}0.001 & 0.00001 & 0.001 & 0.1\end{bmatrix}\right)$

Step 3: A P.I controller was designed in order to close the first loop successfully. This controller is

$$\text{Pi(s)} = \frac{100*(\text{s}+0.001)}{\text{s}}$$

Step 4: Since the bedmass/char offtake loop was closed using this P.I controller extract the resulting system a 3x3 subsystem was extracted, using the following matrices:

$$\text{PRE} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{POST} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 5: Normalisation and alignment was applied to the new 3x3 subsystem at the frequency point of 0.0001 rad/sec [10].

Step 6: Set the weights as in (31).

Step 7: Use Matlab, to obtain an H∞ controller was produced.

Step 8: All of the scaling matrices produced during this procedure for the 3x3 subsystem were augmented in order to become 4x4, matrices and the original 4x4 gasifier was cascaded with them. The resulting closed – loop system, shown in Figure 5 was simulated and the step responses of this system were obtained, Figure 6.

$$\text{W}_{\text{input}}(3x3) = diagonal\begin{bmatrix} \frac{0.0001}{s+0.0003} & \frac{0.002}{s+0.005} & \frac{0.0001}{s+0.05} \end{bmatrix}$$
$$\text{W}_{\text{output}}(3x3) = diagonal\begin{bmatrix} \frac{s+1*10^{-6}}{s+1} & \frac{s+1*10^{-5}}{s+0.1} & \frac{s+1*10^{-8}}{s+1} \end{bmatrix}$$
$$(31)$$

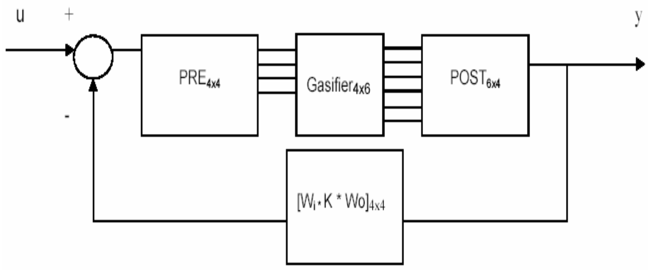

Fig.5: Resulting Closed Loop System

5 Conclusions.

As can be seen from Fig.10, the performance of the system in the time domain is very good and satisfies the problem's specifications. The system is now diagonal dominant and the elements of the main diagonal reach their steady state values rapidly. Moreover, most the interaction has been eliminated, since all of the off diagonal elements settle at the zero level well before the two hours time limit.

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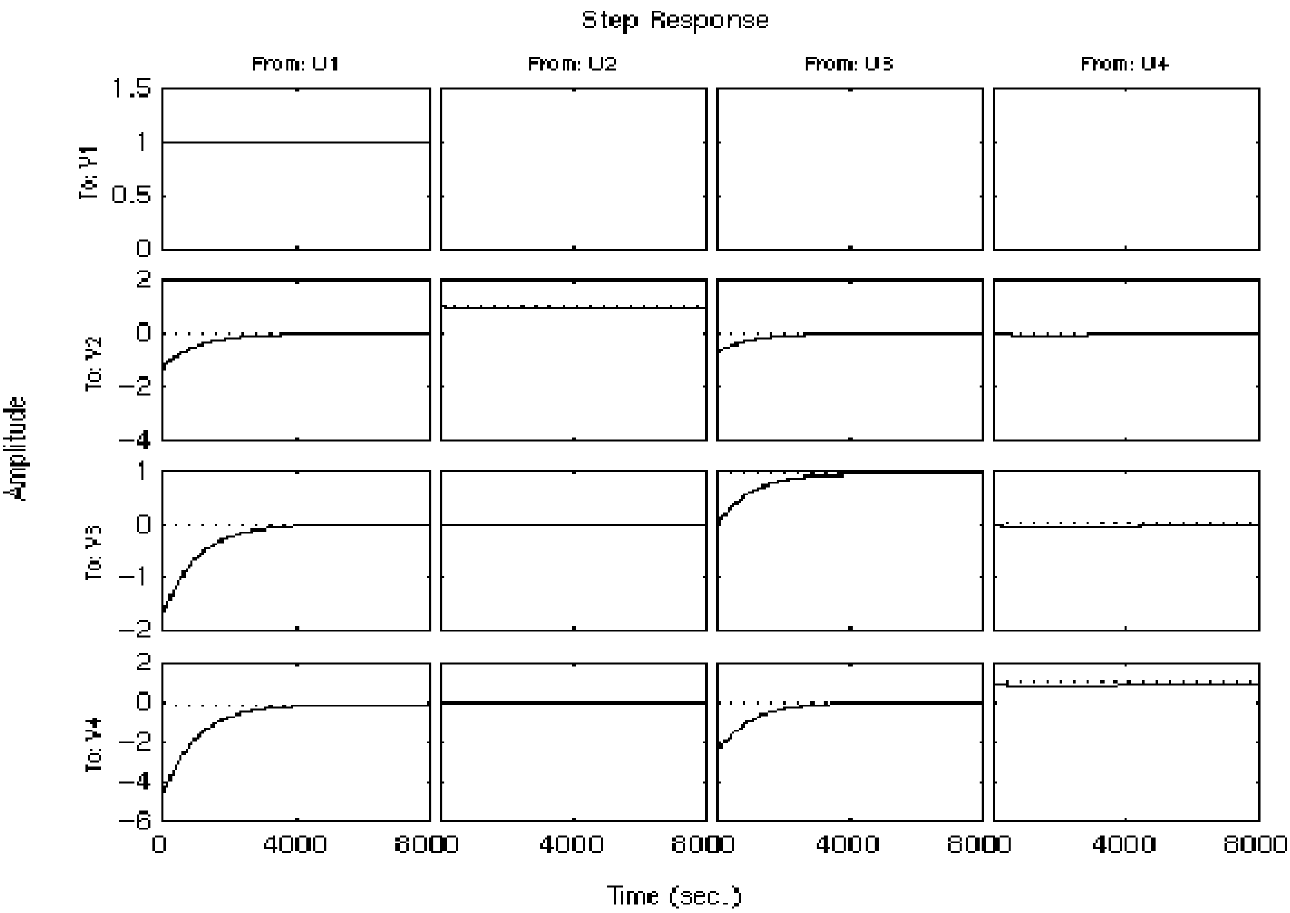

Fig 6: Step Responses of the Full Gasifier Model (The first loop was closed first using a P.I controller)