AN H infinity (H∞) DESIGN SCHEME FOR AN AIR BLOWN GASIFICATION CYCLE UNIT USING MATLAB

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Abstract: - The purpose of this paper is to use the H∞ design technique in order to produce an efficient controller for a real application, which concerns the design of a control scheme for an air blown gasification cycle unit plant (gasifier) for the production of environmentally clean energy. The resulting controller is designed for the 100% load operating point of the gasifier but at it will be shown that works equally well for the 50% load operating point.

Key-Words: - H∞, Gasifier, Multivariate Analysis, Open Loop, Close Loop, Stability, Robustness, Matlab.

1 Introduction.

The purpose of this paper is the development of a dynamic simulation model and a controller using the H∞ method for the plant (gasifier). This case study is based upon the data provided for the 100% load operating point of the gasifier. However the resulting controller is going to be tested on the 50% load operating point in order to show that its performance is satisfactory in both cases. The crucial aspect of the gasifier is that it is numerically ill-conditioned. This means that commercial software packages have a strong chance of failing to provide an accurate solution for the problem because their algorithms cannot handle such numerics.

2 Problem Specification.

Schematically the gasifier unit is described in Figure 1. A brief description of the gasifier is that it is a non-linear, truly multivariable system, having five inputs (coal, limestone, air, steam and char extraction) and four outputs (pressure, temperature, bed mass and gas quality) with a high degree of cross coupling between them.

The *control inputs* are:

WCHR char extraction flow (kg/s)
WAIR air mass flow (kg/s)
WCOL coal flow (kg/s)
WSTM steam mass flow (kg/s)
WLS limestone mass flow (kg/s)

The *disturbance input* is

PSINK sink pressure (N/m^2) (This represents the pressure upstream of the gas turbine that would vary according to the position of the gas turbine fuel valve.)

The *outputs* to be controlled are:

CVGAS fuel gas calorific value (J/Kg),
MASS bed mass (Kg),
PGAS fuel gas pressure (N/m^2),
TGAS fuel gas temperature (K).

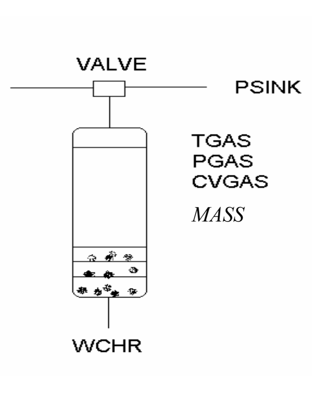

Fig. 1: The Gasifier

Note that:

1) The output vectors in the state – space model are ordered as given above.

2) Limestone absorbs sulphur in the coal so WLS should be at least set to a fixed ratio of WCOL. Nominally this should be to 1:10 limestone to coal. This leaves effectively 4 degrees of freedmome for the control design.

3) The data provided for the linear models provided (100%, 50%, 0% load) are completely open loop meaning that the bedmass controller is added solely for the validation of the simulation.

4) The order of the system under consideration is 25 and no model reduction has been applied to the data provided.

The proposed controller should be able to regulate the outputs bearing in mind that the input and output limits must not be exceeded.

For the output limits the following applies:

1) The CV fluctuation should be minimized, but must always be less than ±10 KJ/Kg

2) The pressure fluctuation should be minimized, but must always be less than ± 0.1 bar.

3) The bed mass should fluctuate by less than 5% of the nominal

4) Temperature fluctuation should kept to a minimum, but must always be less than ± 1 C. [12]

Table 1: Input limits

	Max (Kg/s)	Min (Kg/s)	Rate (Kg/s^2)
WCOL	10	0	0.2
WAIR	0	0	1.0
WSTM	6	0	1.0
WCHR	3.5	0	0.2

2.1 H∞ Optimal Control.

There are many process control problems where significant uncertainties exist in the system models, which therefore require robust control designs. Robust control design procedures enable good performance to be maintained even though significant modelling errors exist in the system description. [1], [2].

If a system has disturbance rejection robustness, the output will not be unduly influenced by the presence of disturbances. However, stability robustness is the most important requirement, since the final closed loop design should be stable despite modelling errors.

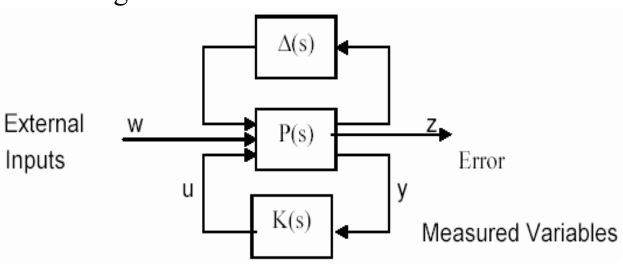

Fig.2: Standard representation of inaccurately known plant under feedback control

The uncertainty inherent in a plant model can be obtained by the representation of Figure 2, where P(s) is the plant to be controlled, K(s) is the designed controller and Δ(s) is the perturbation. It should be noted that the external inputs "w" is a vector of all the signals entering the system, and the "error" (z) is a vector of all the signals required to characterise the behaviour of the closed loop system. Both of these vectors may contain elements, which are abstract in the sense that they may be defined mathematically, but do not represent signals that actually exist at any point in the system. u is the vector of control signals, and y is the vector of measured outputs.

However, Figure 2 can be simplified as in Figure 3 if the perturbation Δ(s) is not shown.

P(s) is derived from the *nominal* plant model. However, it may also include weighting functions, which depend on the design problem that is being solved. Suppose that P(s) is partitioned as

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{12}(s) \end{bmatrix} \quad (1)$$

so that

$$z = P_{11}w + P_{12}u, \quad (2)$$

$$y = P_{21}w + P_{22}u. \quad (3)$$

Then, u and y can be eliminated using $u=Ky$, to obtain

$$z = [\, P_{11} + P_{12}K(I - P_{22}K)^{-1} P_{21}]w. \quad (4)$$

For convenience, expression (4) can be written as follows: $z = F_1(P,K)w$.

By suitably defining w and z (or, equivalently, P), it is possible to put a number of practical design problems into the form

$$\text{minimise } \|F_1(P,K)\|_\infty$$

where the minimisation is over all realisable controllers K(s) that stabilise the closed loop system, and $\|\cdot\|_\infty$ is defined as $\|G\|\infty = \sup_\omega \overline{\sigma}(G(j\omega))$.

This is known as the H∞ - optimisation problem**.**

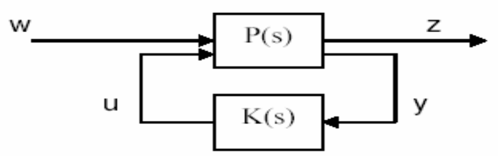

Fig.3: Representation of inaccurately known plant under feedback control without perturbation Δ.

2.2 Solution of the H∞ problem.

2.2.1 Equivalence to the model matching problem.

It was shown earlier that the general H∞ could be formulated as minimize $\|F_1(P,K)\|_\infty$ over the stabilising compensator K.

Combining P and J of Figure 4 into a transfer function T and assuming that $P_{22} = G$, the following conclusions can be deducted:

$$z = F_1(P,K)w = F_1(T,Q)w = [T_{11} + T_{12}Q(I - T_{22}Q)^{-1}T_{21}]w. \quad (5)$$

Here, $T_{22} = 0$ (for the scope of this research) so the previous expression can be simplified to

$$z = [T_{11} + T_{12}QT_{21}]\mathrm{w}. \tag{6}$$

So it can be now said that all that is needed is to

$$\min_{Q\in\infty} \left\| T_{11} + T_{21} \right\|_{\infty} \tag{7}$$

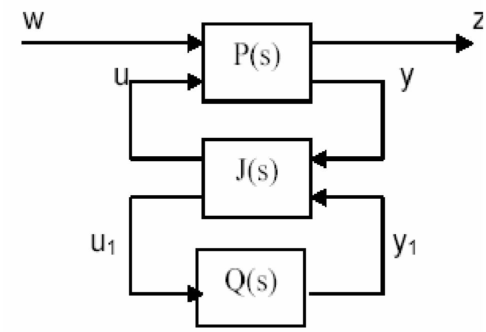

Fig.4: Another standard representation of inaccurately known plant under feedback control

This is sometimes known as the model - matching problem because to solve it we need to choose Q such that $T_{12}QT_{21}$ "matches" the model – T_{11} as well as possible [3], [4].

2.2.2 Equivalence to Hankel Approximation

From now on $X^*(s)$ will be used to denote $X^T(-s)$. Moreover, the argument (s) will be dropped for convenience reasons. Assuming that the following relationships are valid

$$T_{12}\, T^*_{12} = I \text{ and } T^*_{21}\, T_{21} = I \tag{8}$$

it is easy to show that, if X and Y are all-pass (or inner) and $\sigma(.)$ denotes any singular value of (.), then $\sigma(XAY) = \sigma(A)$, and that, for any $\sigma(A^*) = \sigma(A)$. Hence it can be said that

$$\left\| T_{11} + T_{12}QT_{21} \right\|_{\infty} = \left\| T_{12}(T^*_{12}T_{11}T^*_{21} + Q)T_{21} \right\|_{\infty} = \left\| T^*_{12}T_{11}T^*_{21} + Q \right\|_{\infty} = \left\| T_{21}T^*_{11}T^*_{12} + Q \right\|_{\infty} \tag{9}$$

Note:
All pass transfer function: $G(s) = G_o(s)*A(s)$, $A(s) = (s-a)/(s+a)$, where $|A(s)| = 1$ while its phase decreases.

Since $Q \in H\infty$ and $T_{ij} \in H\infty$, Q^* has only unstable poles while $T_{21}T^*_{11}T_{12}$ potentially has both stable and unstable poles. It can be shown, by a rather complicated analysis, that $T_{21}T^*_{11}T_{12}$ in fact has only stable poles [5].

If R is defined likewise as

$$R = T_{21}T^*_{11}T_{12} \tag{10}$$

then

$$\min_{Q\in H_\infty} \left\| T_{11} + T_{12}QT_{21} \right\| = \min_{Q\in H_\infty} \left\| R + Q \right\| \tag{11}$$

which converts the model – matching problem into the problem of approximating a stable transfer function R by an unstable one ($-Q^*$). This is known as the *Hankel approximation problem*, or the *Nehari extension problem*. [6]

3 Problem Solution.

In order to understand how the gasifier system should be approached the following notes could be useful:

1) The system has to examined in terms of controllability, observability and stability

2) Any possible model order reduction should be carried out at the beginning of the design. The resulting reduced system should be check again as for (1)

3) The selection of the weights is very important in H∞. The better the weight selection the better the produced results.

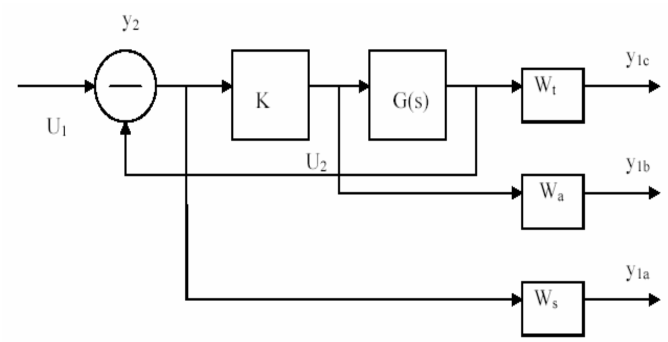

Fig.5: H∞ Design: Loss function for one degree of freedom

W_a is the weight that minimizes the actuator variations,
W_s is the weight that minimizes the noise rejection.
W_t is the weight that minimizes the error in the gain from the reference to input.

For the gasifier system only W_t and W_s are used and their value is presented at the next section. The loss function for the H∞ approach is given by:
where
$S(s) = (I + GK)^{-1}$ is the noise rejection and sensitivity function

$R(s) = (I + GK)^{-1} K$ the gain from output noise to actuator output
$T(s) = (I + GK)^{-1} GK$ is the complementary sensitivity function [10].

$$\begin{bmatrix} Ws(s) * S(s) \\ Wa(s) * R(s) \\ Wt(s) * T(s) \end{bmatrix} \quad (12)$$

If up to this stage everything is calculated correctly, then the H∞ approach should provide an appropriate controller for the system.

4) Matlab provides efficient routines for both model order reduction and the H∞ controller calculation [7], [8], [9].

4 Results.

Step 1: The system is firstly reduced to a 15th order model and then made square 4x4, (the third and the fifth input were combined and the first two outputs were interchanged so as to control the bed mass loop first, by manipulating the char offtake) [5].

Step 2: A simple input and output scaling was applied to the system of the form

$$Pre = diagonal([-1 \quad 1 \quad 1 \quad 1])$$ and

$$Post = diagonal([0.001 \quad 0.00001 \quad 0.001 \quad 0.1])$$

Step 3: Apply normalisation at the frequency point of 0.001 rad/sec and alignment at the frequency point of 0.0001 rad/sec. The normalisation routine has produced again non diagonal matrices, so a change in the outputs has happened again. The first output controls Pressure, the second loop controls Fuel Gas, the third loop controls Temperature, and the last loop controls the Bed Mass [10].

Step 4: Weights had now to be selected in order to define the appropriate functions to be minimised. The new weights are:

$$W_{input}(4x4) = diagonal\left[\frac{0.001}{s+0.001} \quad \frac{0.0001}{s+0.0003} \quad 1 \quad \frac{0.00001}{s+0.000001}\right]$$

and

$$W_{output}(4x4) = diagonal\left[\frac{s+1*10^{-4}}{s+1} \quad \frac{s+1*10^{-5}}{s+1} \quad 1 \quad \frac{s+1*10^{-6}}{s+0.01}\right]$$

Step 5: The H∞ controller was now calculated and applied directly onto the full order gasifier model. The details of the resulting controller were checked. This is because with the H∞ method an unstable controller may be produced but the closed loop stability of the resulting system is guaranteed.

However, if a controller is unstable (a not impossible occurance since non minimum phase zeroes in the plant are present), it would not be possible to apply this to the plant because it can not start working in a stable way. So, the resulting shaped controller has 21 states and its poles were all stable as Table 2 indicates

Table 2: H∞ controller poles

-3.0463e+02	-1.2223e-02
-1.0070e+00 + 1.1807e-01i	-3.3385e-03
-1.0070e+00 - 1.1807e-01i	-1.0000e-03
-1.6952e-01 + 1.6738e-01i	-7.9077e-04 + 4.0754e-04I
-1.6952e-01 - 1.6738e-01i	-7.9077e-04 - 4.0754e-04I
-1.6385e-01	-6.8506e-04 + 1.9995e-04I
-1.0642e-01	-6.8506e-04 - 1.9995e-04I
-5.6740e-02	-1.0000e-06
−4.7260e-02	-3.2543e-04
-2.9608e-02	-1.9450e-04
	-3.0000e-04

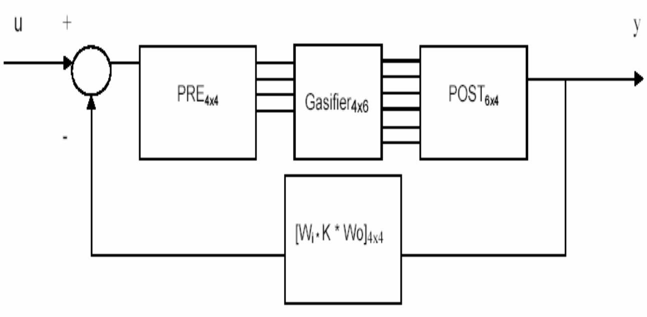

Fig.6: Resulting Closed Loop System

Figures 7 indicates that the results for the 100% load case are satisfactory. Once again the H∞ method has produced an effective controller. The system is diagonal dominant and all the diagonal elements reach their steady state values inside the time limit. Only the fourth output seems to be quite slow compared to the other two, but it also reaches its steady state value inside the specifications. Moreover, as is obvious the interaction is very much reduced and even the interaction in the elements (4,1) and (4,2) reach the zero value very short after 2 hours limit.

This may simply be a matter of just a bit of more fine tuning of the weighting functions. However, this could be a suggestion for further work.

Figure 8 shows the H∞ controller behaviour for the 50% load case. Note that the controller and weights used for the 50% load were exactly the same as the ones in Step 4 for the 100% load case.

It is obvious that the produced controller works equally well for both cases.

Finally it should be noted that H∞ controller was produced for the full gasifier system (25th order) This may not be considered as an advantage but it shows the effectiveness of this approach. At the time that this research was completed it was not known of any other method capable of handling this system in this way, especially when applying the calculated controllers to the full order gasifier model.

5 Conclusions.

The results show that the task of this paper was successfully tackled. However there are a few points that should be pointed out. First of all the singular values of the system must be examined, because they indicate the numerical condition number of the system. In our case they were too small, so some sort of scaling had to be applied.

Because the gasifier's order was too high a model order reduction was applied. Amongst the various ways of doing for very complicated systems like this one it might be better to use commercial software, such as MatLab that provide efficient model order reduction algorithms. It should be kept in mind, however, not to push the system order too low. This is because by producing a very low order model some of the significant dynamics of the system can be lost, so the resulting controller would work well for the reduced system but it would be insufficient for the full order model.

Finally the selection of the weights is another important aspect. These should be selected as high pass and low pass filters intersecting at a desired bandwidth frequency. The correct selection of the weights means that all the H∞ design criteria will be met, and so the controller will be reliably calculated.

Although an acceptable solution for this problem was achieved there are still some possibilities for improvement. The system can be considered diagonal dominant and the elements of the main diagonal reach their steady state values almost inside the specified time limit. Some further fine tuning could be applied to the existing controller, in order to eliminate the existing interaction

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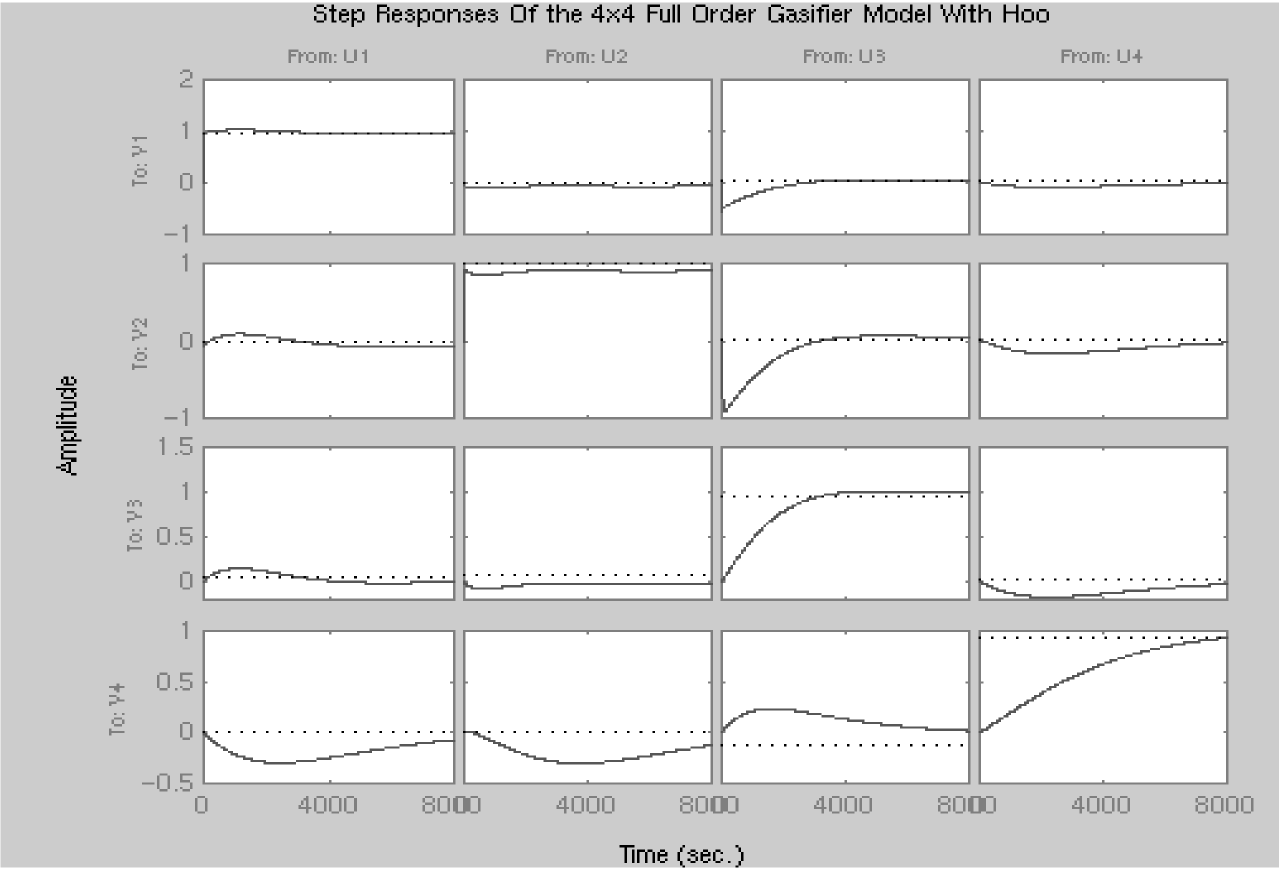

Fig.7: H∞ controller applied to the full order gasifier. 100% load case

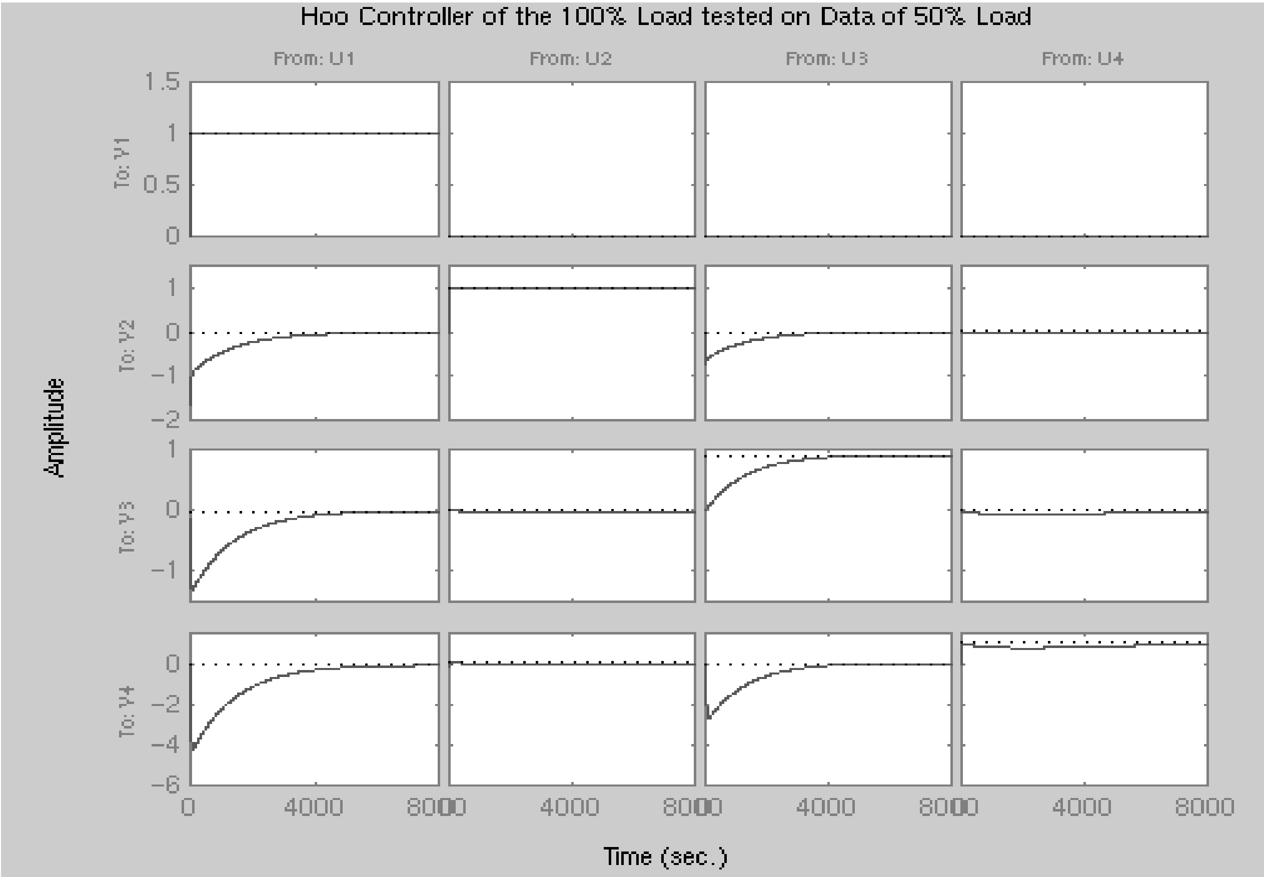

Fig.7: H∞ controller, calculated for the 100% load case, applied to the full order gasifier at 50% load case.