Calculation of Induced Eddy Currents in Permanent Magnets of BLDC Machine

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Abstract - The aim of proposed paper is to present the calculation of induced eddy currents in permanent magnets (PM) of brush-less direct current (BLDC) machines. The study is carried out on outer rotor BLDC machine. The main source of these eddy currents is in reluctance variation due to stator geometry. The magnetic flux density distribution and penetration into the rotor are described. Fundamental analytical electromagnetic equations in conjunction with magneto static finite element method are used to calculate the eddy current distribution in a PM of outer rotor. Regarding the current distribution, the eddy current losses in PM are calculated. The final equation to calculate these losses in function of stator geometry is presented. On the other hand, time dependant (magneto transient) finite element method is also used to analyze this phenomenon. The induced eddy currents distribution in PM and losses due to them are calculated.

Key-Words: Eddy currents, Eddy current losses, Permanent magnets, BLDC machine, Magnetic flux density, Finite element method

1 Introduction

The study is carried out on the outer rotor BLDC machine (Fig.1). The permanent magnets are placed around the inner surface of rotor. Inside of rotor is stator. The stator is laminated to reduce induced eddy current due to excitation currents. These current are supplied in motor mode of operation by outer 6 – transistor inverter with conduction angle of 120°. The stator windings are connected in *wye*. Also, the stator lamination serves to diminish induces eddy currents due to permanent magnet change of polarizations at rotation. The reduction of hysteresis losses are mainly achieved by using low loss iron in stator design. Meanwhile, the permanent magnets are not laminated and are electrically good conductors. Any kind of magnetic flux density change induces the eddy currents and with them power losses. These losses are shown in supplementary heating of PM. With this, the temperature dependant working characteristic of PM (second quadrant of B-H curve) is moving toward coordinate origin point.

The paper outlines a case study on induced eddy currents in PM just due to stator geometry. Stator slot openings and special design of stator poles are causing the magnetic flux density changes in permanent magnets (Fig. 1). In such a study, the stator windings must not be connected to outer source and further more, no currents have to flow in these windings. To achieve these working conditions the BLDC machine must work in generator mode with no-load.

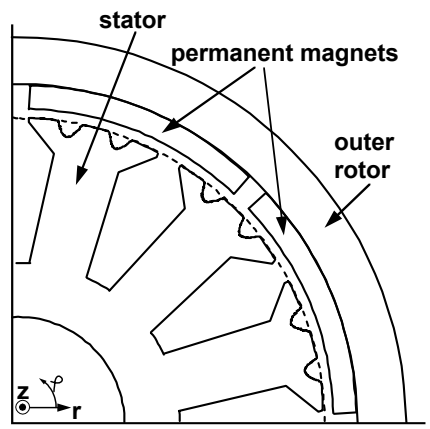

Fig.1. Outer rotor BLDC machine.

2 Analytical solution for losses in PM

To simplify the presentation of developed method the rotor coordinate system (Fig.1) was transformed into Cartesian (x, y, z) coordinate system. Basic coordinate x (coordinate φ in Fig.1) is around the rotor surface, coordinate y (coordinate $-r$ in Fig.1) is directed radial into rotor and coordinate z is in direction of rotor shaft.

Changes in magnetic flux density distribution in the air-gap due to stator surface design were carried out. On these bases the time dependant changes in magnetic flux density in PM were localized (Fig.2). This time dependant magnetic flux density due to rotor rotation is

the source of eddy current losses in PM mounted on inner rotor surface.

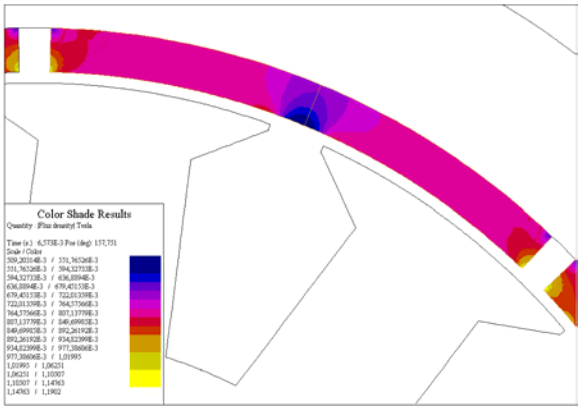

Fig.2: Magnetic flux density distribution in permanent magnet.

Magnetic flux density distribution in permanent magnet only due to stator slot opening is shown in Figure 2. The colors in Figure 2 present the area of equal magnetic flux density. As it can be seen the flux is decreasing regarding permanent magnet depth, this is in function of air gap width and also depends by stator reluctance variation.

Magnetic flux density distributions in the air-gap of BLDC were calculated using finite element method (FEM) [1-3] in magneto static mode. The calculations were done for different consecutive rotor positions of BLDC rotor. The currents through windings were set to zero.

On the basis of coordinate transformation and space harmonic analysis of calculated magneto-static magnetic flux density distribution in the air-gap the time depending magnetic flux density y-component can be described by following equation:

$$B_y(x,t) = B_{dc} + \sum_{n=1}^{\infty} B_n \cos\left(\frac{8x}{R_m}n + \varphi_n + n\omega t\right) \tag{1}$$

where B_{dc} is DC component, B_n and φ_n are the amplitude and shift angle of n-th space harmonic, R_m is the inner radius of PM, ω is angular frequency. In this case the coordinate x is changing in limits between 0 and length of one permanent magnet.

The magnetic flux density distribution at different depths of PM is presented by following equation:

$$B_{PM,y}(x,y,t) = B_{dc} + \sum_{n=1}^{\infty}\left[B_n(y)\cdot \cdot\cos\left(\frac{8x}{R_m}n + \varphi_n + n\omega t\right)\right] \tag{2}$$

where $B_n(y)$ is function witch depends on change of magnetic flux density y-component into the permanent magnets. The Eq. 2 is further transformed into:

$$B_{PM,y}(x,y,t) = B_{dc} + \mathrm{Re}\left[\sum_{n=1}^{\infty} B_n(y)\cdot e^{j\left(\frac{8x}{R_m}n+\varphi_n+n\omega t\right)}\right] \tag{3}$$

and

$$B_{PM,y}(x,y,t) = B_{dc} + \mathrm{Re}\left[\sum_{n=1}^{\infty} B_n(y)^* \cdot e^{j(n\omega t)}\right] \tag{4}$$

The unknown function $B_n(y)^*$ can be found out by using differential equation [4]:

$$\frac{d^2 B_n(y)^*}{dy^2} - k^2 B_n(y)^* = 0 \tag{5}$$

The factor k is equal to:

$$k = (1+j)\sqrt{\frac{n\mu\gamma\omega}{2}} = (1+j)m \tag{6}$$

where m represents the reciprocal value of the penetration depth for n-th space harmonics:

$$\delta = \sqrt{\frac{1}{\pi\mu f_n}} \tag{7}$$

If the solution of differential equation (Eq. 5) is inserted into Eq. 4 the result is:

$$B_{PM,y}(x,y,t) = B_{dc} + \sum_{n=1}^{\infty}\left[B_n \cdot e^{my} \cdot \cdot\cos\left(my + \frac{8x}{R_m}n + \varphi_n + n\omega t\right)\right] \tag{8}$$

where coordinate y is changing in limits between 0 and thickness of the PM due to orientation of coordinate system. The analytical formulation (Eq. 8) of magnetic flux density distribution at different depths of PM (Fig. 7) obtained from known magneto-static magnetic flux density distribution in the air-gap of BLDC is then used to calculate the eddy current distribution in PM. Within further analyze the following assumptions are used:

- The induced eddy currents are flowing only in z-direction,
- The eddy current end effects are neglected.

The induced eddy current density as consequence of time and space dependant magnetic flux density can be found out by:

$$\operatorname{rot}\vec{J} = -\gamma \frac{\partial \vec{B}}{\partial t}, \tag{9}$$

where γ is specific electrical conductivity of material, in this case permanent magnet.

After solving Eq. 9 the induced eddy current density distribution across PM surface in z-direction is:

$$J_z(x,y,t) = \sum_{n=1}^{\infty}\left[\frac{\gamma R_m \omega}{8} B_n \cdot e^{my} \cdot \cdot \cos\left(my + \frac{8x}{R_m} n + \varphi_n + n\omega t\right)\right] + C \tag{10}$$

$$C = \sum_{n=1}^{\infty} C_n \tag{11}$$

Because the permanent magnets are electrically isolated between each other, the total sum of eddy current density over one magnet surface must be zero. In this way it is possible to determine the constant C.

$$\int_0^{l_m}\int_0^{-d_m} J_z(x,y,t)\,dxdy = 0 \tag{12}$$

where l_m, d_m are the length and thickness of the PM respectively.

On the basis of known eddy current density distribution (Eq. 10, Fig. 6) the eddy current losses in PM are calculated by:

$$P_{PM} = \frac{1}{\gamma}\int_0^{l_m}\int_0^{-d_m}\int_{-h_m/2}^{h_m/2} J_z^2\,dxdydz \tag{13}$$

where h_m is the high of the PM.

3 Numerical solution for losses in PM

Using finite element method with magneto-transient analyzes [3] all upper described working conditions were achieved by the external electric circuit (Fig. 3) connected to finite element model of BLDC. These circuit allows that BLDC machine runs in generator mode of operation with open winding (load is in range of few kΩ).

Normally the eddy currents are not included in 2D finite element calculations. To take into account eddy currents the solid conductors (Fig. 4) are added to the external 3-phase circuit (Fig. 3) connected with PM in finite element model of BLDC. Using the solid conductors in calculation of eddy currents, the end effect is neglected, because the FEM assumes that solid conductors are infinitely long. Using FEM model of BLDC connected with described external electric circuit the magneto-transient calculations of eddy currents in PM were done.

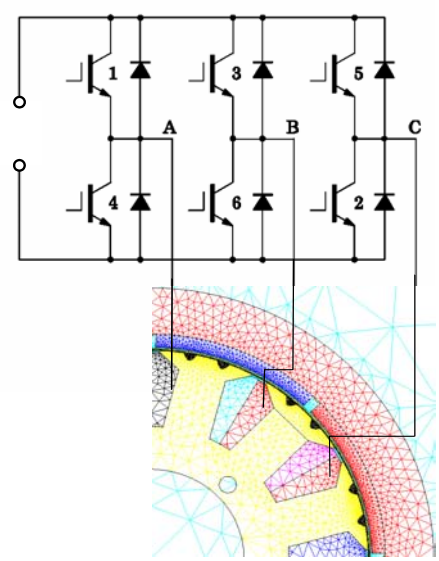

Fig.3: Finite element model coupled with external 3-phase circuit for generator mode operation at no load.

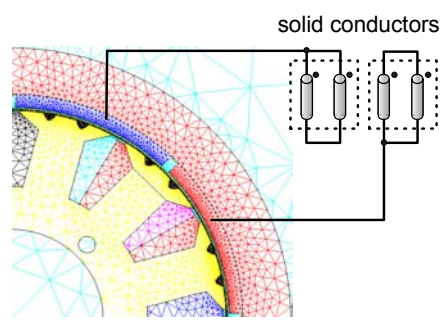

Fig.4: Finite element model coupled with external solid conductors for eddy current calculation in PM.

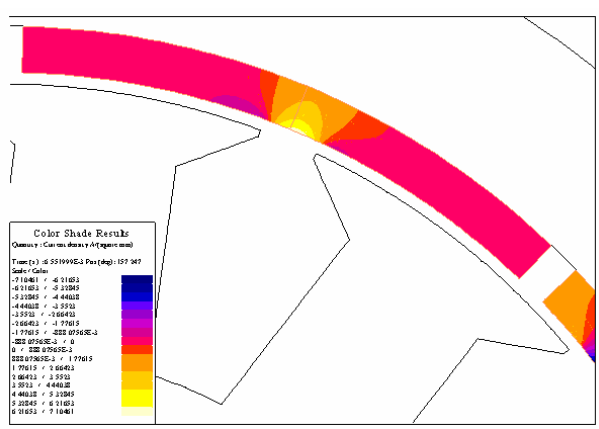

Fig.5: Eddy current distribution on PM surface only due to slot opening at 4000 rpm

The Figure 5 shows the eddy current distribution on PM magnet cross-section as consequence of magnetic flux density changes only due to slot opening at rotor

angular velocity of 4000 rpm. In a different way the same case is shown on Figure 6.

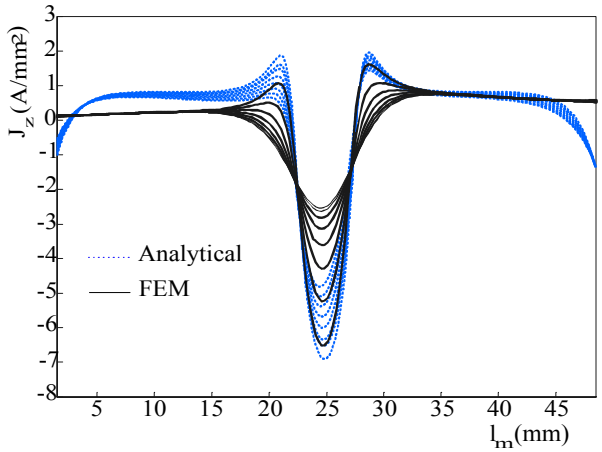

Fig.6: Analytical and FEM current density distribution results in PM only due to slot opening at 4000 rpm

The analytical distribution of eddy current density on the surface of one PM calculated using Eq. 10 and that obtained from finite element method considering only slot opening as the source of eddy currents are shown on Figure 6. Rotor is in the position as in the Figure 5. The curves represent the eddy current density across the PM line path at different depths as a parameter (depth of PM is 4 mm). The highest values of induced eddy currents in PM are beside the air-gap in center line with slot opening. As shown the analytical method of eddy current distribution gives reasonable good results. The main problem lies in accuracy of analytical formulation of magnetic flux density distribution (Eq. 8) at different depths of PM shown on Figure 7.

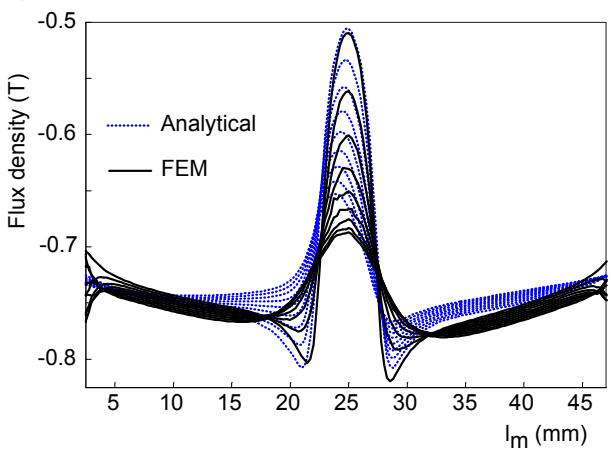

Fig.7: Analytical and FEM magnetic flux density distribution results in PM only due to slot opening at 4000 rpm

On Figure 7 are compared the time and space dependant analytical distribution of magnetic flux density on the surface of one PM calculated using Eq. 8 and magnetic flux density obtained from magneto-transient finite element calculation. Rotor is in the position as in the Figure 5 and it rotates with 400 rpm. The PM depths where the magnetic flux density is calculated are the same to those used for eddy current calculation (Fig. 6). Further in the contents, only eddy current losses in PM calculated by magneto-transient FEM coupled with external circuit will be presented .

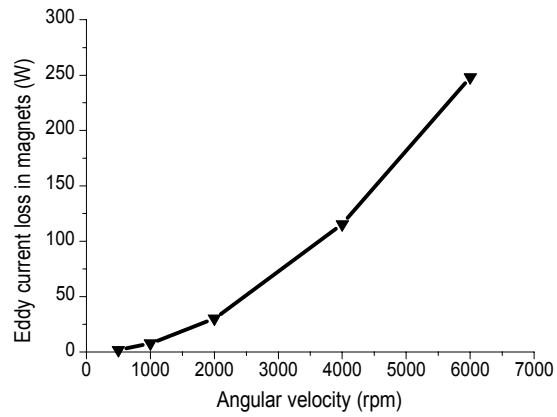

Fig.8: Total eddy current losses in PM calculated by FEM only due to slot opening

The total eddy current loss in PM at different rotor angular velocities when only slot opening (Fig. 2) is considered is shows on Figure 8. When constructing BLDC machines for high velocities (few 1000 rpm) attention on eddy currents losses in PM must be done. For diminution of eddy current losses the radial or axial segmentation of PM is proposed in [5-6].

The special design of stator poles (Fig. 9) induces additional eddy currents in PM. Figure 9 shows the eddy current distribution over PM surface due to slot opening and special stator pole design. Because of special stator pole design supplementary eddy current losses in PM give evident rise to total eddy current losses in PM at high velocities as shown in Figure 10.

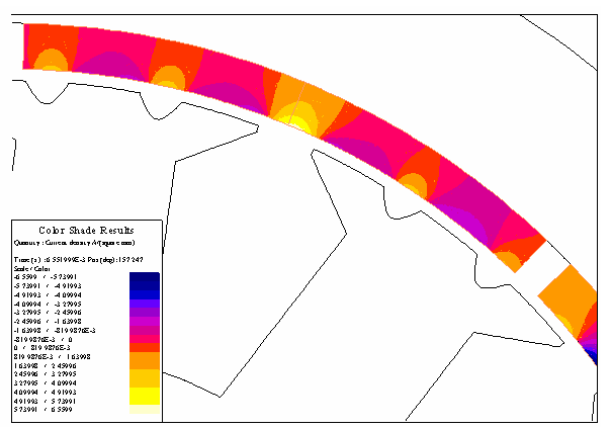

Fig.9: Eddy current distribution on PM surface due to slot opening and special stator pole design at 4000 rpm

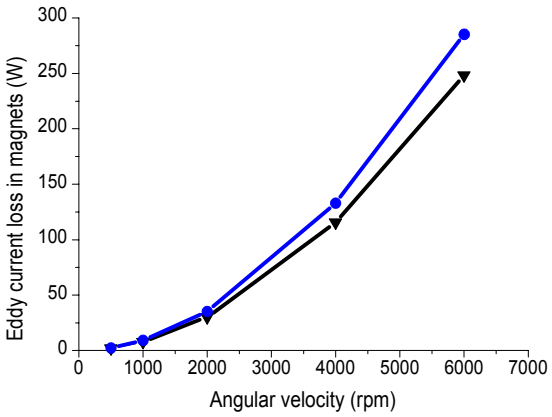

Fig.10: Total eddy current losses in PM calculated by FEM only due to slot opening (triangular), due to slot opening and special stator pole design (spot)

4 Conclusion

The proposed analytical method for eddy current losses calculation in PM gives a reasonable good results but it must be further analyzed for more precise results. More geometric parameters of BLDC design must be introduced in to analytical formulations, especially those which have influence on reluctance variation. The eddy current losses in PM obtained from magneto-transient FEM coupled with external circuit indicate the supplementary heating of PM and BLDC machine at high speeds even when it operates at no load.

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