Robust High Performance Servo Controller Design Technique Using Matlab/Simulink

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Abstract: - The aim of this paper is the design of a robust high performance phase lag controller. This controller is responsible for placing the lenses inside a military telescope. Controller's performance will be assessed upon its ability of regulating the input current inside a limit of *1.5 A* and its ability of setting the output response to a steady state of *1mm* in less than 0.5 seconds. Simulink was used in order to simulate the designed controller with and without the presence of noise disturbance. Finally there is a brief comparison of the simulation results between the designed phase lag controller and a phase lead one.

Key-Words: - Robust, Phase Lag, Phase Lead, Feedback, Simulink.

1 Introduction

Control theory describes the operation of feedback systems and can be applied to *drive* either simple systems, such as temperature regulators, or more complex ones like multivariate observers [1, 2]. A big range of control problems can be solved by using *feedback*. *Feedback* is the process of measuring the controlled variable and using this information to adjust its value. Examples of feedback control include missile autopilots [3, 4], telescopes and many more [5 - 7].

In this paper a robust high performance phase lag controller will be designed for a given model. The purpose of this controller is to place very accurately and very fast the lenses inside a military target locking telescope. The design technique will achieve a specification in terms of transient response, current limits and measurement of noise.

All the simulations were done in Matlb/Simulink, a powerful tool that has been extensively used over the past years in the area of the controller design technique [8 – 11], integrated AC/DC systems [12] and in combined artificial intelligence and high voltage engineering [13]. However it has been used for simulating other models outside of engineering such as economic ones [14].

2 Problem Formulation

Figure 1 presents the system model and it is actually concerning a servomechanism, i.e. a device that causes an output quantity to follow as close as possible the movement of an input quantity of the same kind.

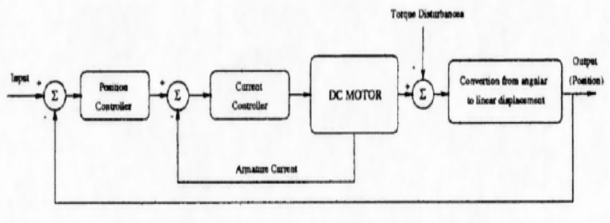

Figure 1. System Model

It is automatic and achieves its aim by subtracting, in some manner, the output from the input and transforming the *difference* into a force which tends to drive the output into conformity with input. This procedure seems quite simple but the practice is far from being the case. Difficulties arise from the fact that input and output are not always measurable due to fact of the *lags* present in a very mechanical system. Such *lags*, behind being difficult to remove, can produce highly undesirable effects, such as unregulated outputs.

This is exactly the problem of our system. The simultaneous regulation of the output response and the armature current inside specified boundaries. The initial though was to use two controllers. One of them to regulate the armature current and the other one to minimize the error between the input and the output. However for optimum system performance it was decided to design only one controller that would perform both tasks. At this point is essential to mention that for the positioning of the lenses inside the telescope responsible is a ball screw with a pitch of just 1.25mm diameter.

2.1 Phase Lag Controller

There are various types of controllers, namely phase lag, phase lead, phase lead – lag, P.I and P.I.D. P.I and P.I.D are used for improving the system's steady state errors and are often found in industrial process applications where the output response is quite slow.

Phase Lead – Lag is a combination of of a phase lead and a phase lag controller. Combines the features of both controllers, however is harder to design and costs more. For our application either a phase lead, or phase lag controller seems appropriate. A choice of the phase lag controller was done randomly. The phase lag controller belongs to the same class as the P.I controller. It can be regarded as generalization of the P.I controller. It introduces a negative phase into the feedback loop, which justifies its name. Its phase lag characteristic increases the overall phase lag (destabilizing factor) and its gain K reduces with frequency (stabilizing factor), [15, 16] Its transfer function is [15]

$$G(s) = K\frac{s+p}{s+q}, p > q > 0, \quad (1)$$

$$\arg G(s) = \arg(s+p) - \arg(s-p) < 0 \quad (2)$$

2.2 System's Transient Response

The damping factor ζ and the closed loop poles of the model will be calculated in this section.

First of all we make an assumption that the percentage overshoot $M_p = 5$, *or 5%* or 0.005.

$$M_p = e^{[-\pi\zeta/\sqrt{1-\zeta^2}]} \therefore 0.005 = e^{[-\pi\zeta/\sqrt{1-\zeta^2}]} \therefore \zeta \simeq 0.7 \quad (3)$$

Also from the specification provided for the long range movement the required accuracy is 5μ for *500mm* distance. Thus the bandwidth (*B/W*) ω_n is:

$$e^{-(\zeta\omega_n)} = \frac{5x10^{-6}}{50x10^{-3}} = 0.01 \therefore \omega_n = 26.28 \text{ rad/sec.} \quad (4)$$

The closed loop poles are located at $s_{1,2} = -\sigma \pm i\,\omega_d$

$$\sigma = \zeta\omega_n \therefore \sigma = 0.7 \, x \, 26.28 \approx 18.4 \quad (5)$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 26.28\sqrt{1-0.7^2} \quad \therefore \quad \omega_d = 18.8 \quad (6)$$

This means that

$$s_{1,2} = -\sigma \pm j\omega_d = -18.4 \pm \ i\ 18.8 \quad (7)$$

Both closed loop poles are at the left hand plane so the system under consideration is stable. Finally we can conclude that the system's maximum phase margin (*PM*) is equal to $100\zeta = 70^o$.

2.3 System's Transfer Function

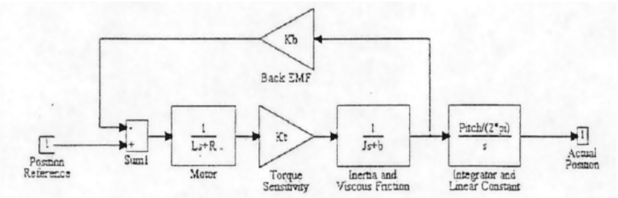

Figure 2. Bruhless DC Motor with Liner Position Output

The next step is to evaluate the transfer function of the DC motor (Figure 2). Using the general rule evaluating a closed loop transfer function we get:

$$G(s) = \frac{y(s)}{y_{ref}(s)} =$$

$$\frac{[K_t/(L\cdot s+R)\cdot(j\cdot s+b)]}{1+[K_b\cdot K_t/(L\cdot s+R)\cdot(j\cdot s+b)]}\cdot\frac{Pitch}{2\pi\cdot s} = \quad (8)$$

$$\frac{K_t\cdot Pitch}{2\pi\cdot s\cdot[s^2\cdot(L\cdot j)+s(L\cdot b+R\cdot j)+(R\cdot b+K_b\cdot K_t)]}$$

The nominal values of the transfer function were given and are:

L = 0.15 x 10^{-3} H,
b = 4 x 10^{-4} Nmm (rad/sec),
j = 0.247 kg$(mm)^2$,
Kb = Kt = 8.48 NmmA^{-1},
Pitch = 1.25 mm and
R = 2.53 Ω.

By plugging these values at (8) and by factorizing the denominator we get:

$$G(s) = 1.69/(s+16748)\cdot(s+122) \quad (9)$$

3 Controller Design

3.1 Evaluation of the Phase Lag Controller

For simplicity in calculations without big error the pole *-16748* can be ignored. This is because this pole is *"very slow"* compared to the *-122* one.

So (9) can be simplified to [16]:

$$G(s) = 1.69/s\cdot(s+122) \quad (10)$$

If the closed loop poles and the transfer function of a system are known then a controller can be designed using the *pole assignment* method.

The general expression of the desired controller is:

$$K(s) = K\cdot\frac{(s+B)}{(s+A)} \quad (11)$$

So

$$G(s)\cdot K(s)=K\cdot\frac{1.69\cdot(s+B)}{s\cdot(s+122)\cdot(s+A)} \tag{12}$$

The *characteristic equation* is:

$$s\cdot(s+122)\cdot(s+A)+1.69\cdot K\cdot(s+B)=0\ \therefore \tag{13}$$

$$s^3+s^2(A+122)+s\cdot(122\cdot A+1.69\cdot K)+1.69\cdot K\cdot B=0$$

The new desired polynomial is:

$$\begin{aligned}&(s+18.4-18.8i)(s+18.4+18.8i)(s+R)=\\&s^3+s^2(36.8+R)+s\cdot(36.8\cdot R+692)+692\cdot R\end{aligned} \tag{14}$$

There is also one more equation to be obtained from the data provided by the manufacturer. From this data the *medium range movement*, i.e. a movement between *0.1mm* to *1mm,* time less than or equal to *0.02 sec* is needed strictly following a ramp profile to an *accuracy* of *±20μm* with a current consumption of not more that *1.5 A*. This means that for ***1mm*** movement ***20msec*** time is needed. This implies the 20th ramp [17], i.e.:

$$K_v=\frac{1}{20\cdot(20\mu m)}=2500$$

$$\begin{aligned}&K_v=\lim_{s\to 0}[s\cdot K(s)\cdot G(s)]\ \therefore\\&s\cdot K(s)\cdot G(s)=sK\frac{1.69\cdot(s+B)}{s\cdot(s+122)\cdot(s+A)}=2500\therefore\end{aligned} \tag{15}$$

$$(s\to 0)\ 1.69\cdot K\cdot B=305\cdot 10^3\cdot A$$

By comparing the coefficients of (13) and (14) And knowing that K_v is 2500 (Eq 15), we get a system of five simultaneous equations: (16)

$$\begin{aligned}&i)\ s^3:1=1\\&ii)\ s^2:122\cdot A=36.8+R\\&iii)\ s^1:122\cdot A+1.69\cdot K=36.8\cdot R+692\\&iv)\ s^0:1.69\cdot K\cdot B=692\cdot R\\&v)\ 1.69\cdot K\cdot B\ =305\cdot 10^3\cdot A\end{aligned} \tag{16}$$

The solution of this system gives the following results:

A = 0.194
B = 15.51
K = 2.257* x *10³
R = 85.5

So finally the controller is:

$$K(s)=2257\frac{(s+15.51)}{(s+0.194)} \tag{17}$$

4 Simulation and Results

In order to access the performance of the designed controller we performed a series of simulations. All the simulations were done using Simulink. Firstly the designed controller was tested without the presence of noise in order to evaluate its capabilities in an ideal situation. For the second simulation we have added a *Band Limited White Noise* in order to access the performance of the designed controller under more realistic conditions. As the results of the next sections show, the performance of the designed controller was equally successful for both cases.

4.1 Simulation without presence of noise

The Simulink model used is depicted in Figure.3. As it was mentioned before the nominal values for the transfer function and the various variables were provided to us by the manufacturer.

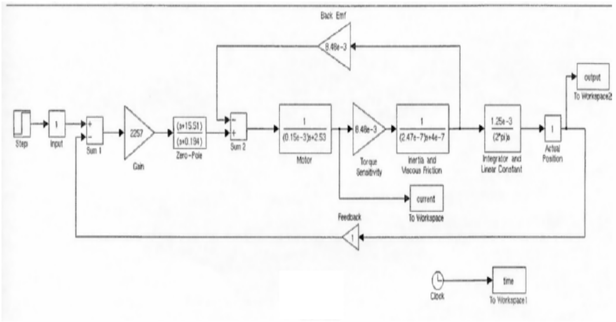

Figure 3. Simulation model without noise

Figures 4 and 5 show the output of the system and the current regulation respectively

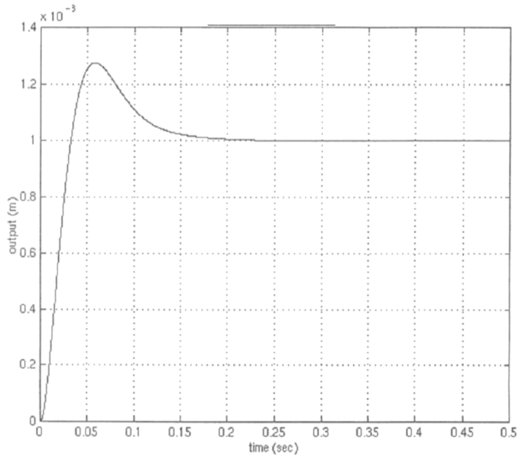

Figure 4. Output response without noise

As it can be seen form figures 4 and 5 both output response and the current regulation are inside the specified limits, which is *1mm* for the output to reach *steady state* and maximum peak to peak current not more than 1.5 A. As it can be seen the output reaches its steady state within 0.15 sec, quite fast, and the maximum peak to peak current is about 1.1 A.

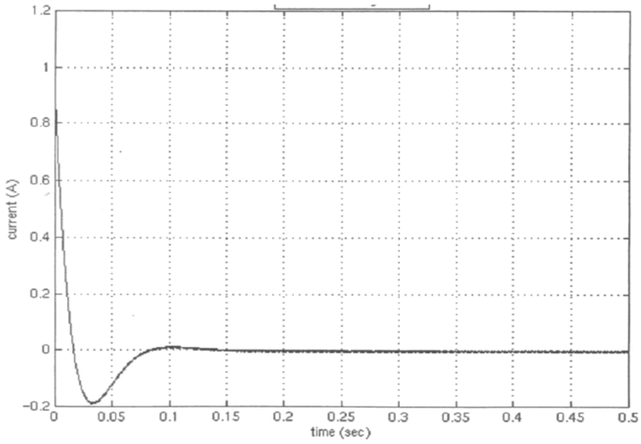

Figure 5. Current regulation without noise

4.2 Simulation with presence of noise

To make things more realistic we have also investigated the controller's behavior under noise disturbances. For this reason we added a *Band – Limited White Noise* into the feedback loop with the following characteristics:
Power Strength: 4 x 10^{-12}
Sample Time: 0.001 sec

In order to have a successful simulation under noise conditions these two parameters have to be balanced in a way. So for a noise disturbance with standard deviation σ of about *2μm* a maximum power of 4×10^{-12} is reasonable ($\approx\sigma^2$) and the specified sampling time is acceptable as well [17].

Figure 6 shows the S*imulink model* and Figures 7 and 8 show the output of the system and the current regulation respectively

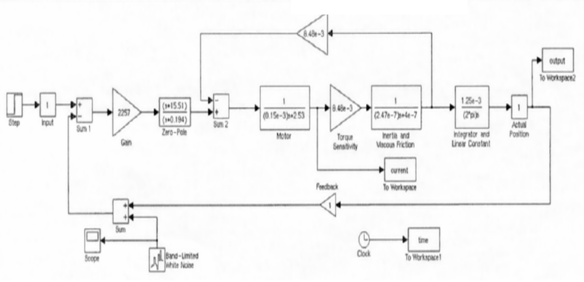

Figure 6. Simulation model with noise

Figure 7 shows that the output is almost unaffected from this external disturbance. The noise indicates its presence since on the output waveform appears a few signs of oscillations.

The output of the current though has changed significantly (Figure 8). The basic shape is the same but the presence of noise is very obvious. Looking closer however we can see that the maximum peak to peak variation of the current is about 1.4 A which is inside the specifications. This means that the controller is capable of handling noise disturbances (until certain noise level) quite effectively.

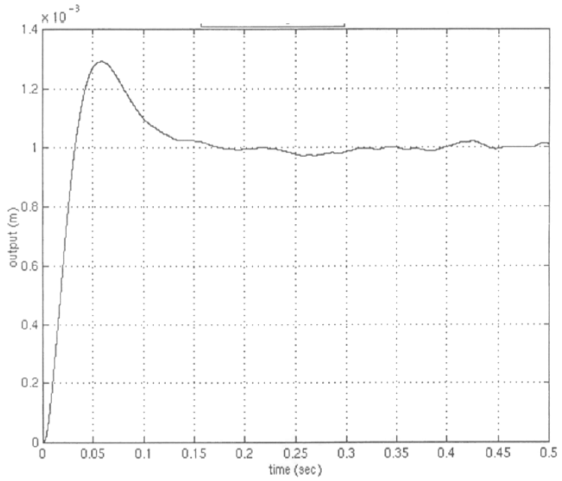

Figure 7. Output response with noise

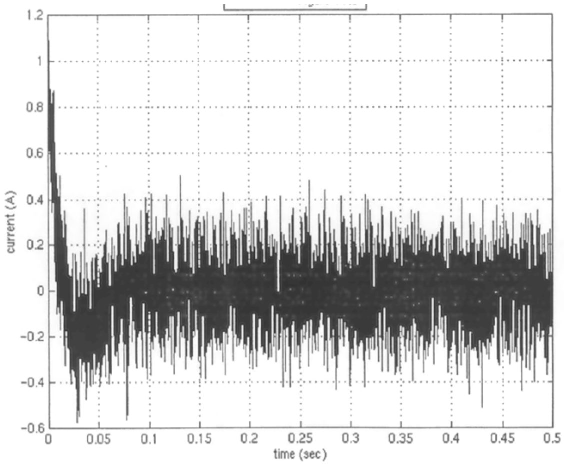

Figure 8. Current regulation with noise

5 Conclusion

In this paper a robust phase lag controller was designed and its behaviour was simulated using Matlab/Simulink. The manufacturer had specified that an optimum controller should be able to provide an output steady state response of 1mm and a current regulation of not more than 1.5 A peak to peak variation. Both of these tasks were met so the controller's performance can be regarded as a successful one. Different values of controller gains *K* give different results, (Table 1).

It should be noted at this stage that the controller could have been a phase lead one instead of a phase lag. Out of scientific interest the designed controller was reversed (i.e. a pole became zero and a zero became a pole) in order to investigate the system's behavior.

The simulation results showed that the

armature current response remained unchanged. This means that the current in this model can be regulated successfully with a phased lead or a phase lag controller.

Controller Gain K	**Output Response**	**Current Response**
10^6	Pure Noise	Unregulated
10^4	Oscillatory	Peak to Peak value outside the specified limits
10^3	**Acceptable**	**Regulated as desired**
10^2	Too slow	Not regulated

Table1. System's Response for various controller gains

The output response was changed significantly though. It was faster with much less overshoot but the final steady state value (*0.32 mm*) was much less than *1mm* i.e. equal to the input. As a future extension to this problem we can add the investigation of the system not only in the presence of noise but in the presence of vibrations as well.

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