WAVELET BASED BOUNDARY DETECTION

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Abstract: - The purpose of this paper is to develop an algorithm for denoising images corrupted with additive white Gaussian noise (AWGN) with a view to extract object's boundary. The noise degrades quality of the images and makes interpretations, analysis and segmentation of images harder. A pixel is said to be a boundary pixel if its deleted neighborhood contains at least one point from the object and one point from the object's complement. The discrete wavelet transform using scale correlation is a denoising approach that reveals boundary pixels more effectively than the simple wavelet decomposition. The detail coefficients in concordant bands are correlated and then synthesized after soft thresholding, which suppresses noise but signifies smooth intensity variations. The wavelet coefficients of noise have much trivial correlation than the wavelet coefficients of boundaries that propagate along the scale. Scale multiplication improves the localization accuracy significantly while keeping high detection efficiency. The combination of noise filtering coupled with boundary detection in a single algorithm enables disconnected boundary detection in a noisy scenario. Curve fitting or cubic spline can then augment the boundaries to estimate missing pixels.

Key Words: - Boundary detection, Edge detection, Multiscale, Multiresolution, Wavelet, Scale Correlation, Denoising.

1. Introduction

Almost all data contains some noise. For digital images, noise reduction is often a required step for many sophisticated exploring methods such as remote sensing. The normal representation of digital image is a matrix of pixels where each pixel measures the brightness of an object. Denoising is the process of reducing the noise in digital images. Denoising usually consists of three stages[1] :

1) Transform the noisy image to a new space, i.e. find a representation, which discriminate the image from noise.
2) Manipulate the coefficients in the new space, i.e., keep the coefficients where the signal to noise ratio is high, reduce the coefficients where the signal to noise ratio is low.
3) Transform the manipulated coefficients back to the original space.

Wavelet transforms as a powerful tool for recovering signals of considerable interest [2]. In fact, wavelet theory combines many existing concepts into global framework and hence becomes a powerful tool for several domains of application. In this paper wavelet scale correlation technique is envisaged after level-4 decomposition of the wavelet details coefficients and then synthesized after soft thresholding. There are two drawbacks for thresholding[3]. One is that a good choice of threshold is always adhoc and another is that specific distribution of signal and noise may not be well matched at different scales. The frequency domain filtering for boundary detection in a noisy scenario is inadequate due to Fourier's global behaviour. Noise manifests itself as fine-grained structure in the image and the wavelet transform provides a scale-based decomposition. Most of the noise tends to be represented by wavelet coefficients at finer scales. Discarding these coefficients would result in natural filtering out of noise on the basis of scale. Boundaries are found in the multiresolution image at concordant locations. The concordance of boundaries in adjacent subbands is useful in discounting spurious noise induced peaks that might show up in individual subbands. Boundaries propagate to certain coarser scales whereas noisy pixels are un-correlated with the scales, which are being exploited to extract the boundary map of a noisy image. If *f(x)* is additive white Gaussian noise then average number of local maximas at scale 2^{J+1} is half that of scales 2^{J}.

2. Existing Operators

Singularities in the signal can be extracted in spatial as well as highpass filtering of the signal.

2.1 Space Domain Techniques

Most spatial domain boundary detection techniques are the computation of a local derivative operator [4-6] with the first derivative operator considered as less sensitive to noise than the second derivative operator. Gradient, Laplacian and Robert are classical linear algorithms [7]. However these algorithms have an acute function on boundaries and are vulnerable to noise. Using Sobel mask to implement the derivative operator has the advantage of providing simultaneous differencing and smoothing effects and offers the opportunity to detect boundaries in the presence of noise. However the amount of noise in gradient images is still noticeable despite the smoothing effects of the Sobel mask. Sobel, Prewitt and Robert kernals find boundaries using the Sobel, Prewitt and Robert approximation respectively to the derivatives. It returns boundaries at those points where the gradient is maximum. The Laplacian of Gaussian method finds boundaries by looking for zero crossings after filtering with a Laplacian of Gaussian filter. The zero-cross method finds boundaries by looking for zero crossings after filtering. Canny method [6] finds boundaries by looking for local maxima of the gradient. The gradient is calculated using the derivative of a Gaussian filter. The method uses two thresholds, to detect strong and weak boundaries, and includes the weak boundaries in the output only if they are connected to strong boundaries.

2.2 Fourier Filters

Most of the signals in practice are time domain signals. When we plot time-domain signals, we obtain a time-amplitude representation of the signal. In many cases, the most important information is hidden in the frequency contents of the signal. Fourier transform and inverse Fourier transform are defined as two dimensional Discrete Fourier Transform (DFT) of a function *f(x,y)* and its Inverse Discrete Fourier Transform (IDFT) of size M x N is given by the expression

$$F(U,V) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\Pi(ux/M + vy/N)} \qquad (1)$$

$$f(x,y) = \sum_{U=0}^{M-1}\sum_{V=0}^{N-1} F(u,v)e^{j2\Pi(ux/M + vy/N)} \qquad (2)$$

Fourier analysis being global is also not good in detecting sharp intensity variations amongst neighboring pixels.

2.3 Multiresolution Analysis

Multiresolution theory [9] incorporates and unifies techniques from a variety of disciplines including subband coding from signal processing, quadrature mirror filtering from digital speech recognition and pyramidal image processing. It is concerned with the representation and analysis of signals or images at more than one resolution [10]. The appeal of such an approach is obvious feature that might go undetected at one resolution may be easy to spot at another. Wavelet analysis is well suited to isolate sharp transients in a signal, a task at which Fourier analysis is not so pleasing.

2.4 Noise Model

Additive White Gaussian noise (AWGN) is inherent in images. The principal sources of noise in digital images arise during image acquisition or transmission. Noise is referring to stochastic variations as opposed to deterministic distortions such as shading or lack of focus. Noise present in an image is due to wide range of sources e.g. variations in the detector sensitivity, environmental variations, the discrete nature of radiation and transmission or quantization errors, *etc.* It is also possible to treat irrelevant scene details as if they are image noise (*e.g.* surface reflectance textures). In terms of a spatially sampled image, uncorrelated noise is the random gray level variations within an image that has no spatial dependence from pixel to pixel. In other words, the gray level of a pixel due to uncorrelated noise does not depend on the gray levels of its neighboring pixels. All the working in this paper has been done with the assumption that the noise is uncorrelated with the spatial coordinates and the pixel's intensity value. The wavelet coefficients of noise have much weaker correlation between scales than the wavelet coefficient of boundaries.

Wavelet based transformation for a multiresolution [9] approach simplifies both the mathematics and physical interpretation, is still an active area of research for the satellite imaging, medical imaging or in the field of fault detection in PCBs/equipment etc. It is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies [10]. This makes sense in images as they have high frequency components for short duration and low frequency components for long duration. This innovative approach will be employed for boundary detection of noisy images where conventional filters fall short.

Spatial and Wavelet based techniques have different advantages and disadvantages. The spatial domain approach is more successful in detecting and localizing the weak boundaries but can produce false boundaries in response to noise. Although wavelet based boundary detector is less sensitive to noise induced images, it is not as accurate as the spatial domain method in localizing the boundaries.

The purpose of this paper is to develop an algorithm for detection of connected and disconnected boundaries in an image such that it incorporates an efficient technique for noise elimination vis-à-vis existing conventional operators [8]. The proposed method favours boundaries that exist at multiple scales and suppresses boundaries that exist at fine scales. Wavelet transform has the advantage of locally analysing both in spatial as well as frequency domain. Multiscale boundary detection methods [9-11] have other advantages. In this paper scale multiplication based boundary detection scheme is worked out by multiplying four adjacent subbands as a product function. Execution time is little longer for complex images due to convolution operation between the mirror filters and the whole image running at the background. In this paper an effort has been made to determine boundaries at the local maxima in the product function after thresholding. Unlike many multiscale boundary detectors, where the boundary maps were found at several scales and then synthesized together. Scale multiplication achieves better results than either of the two scales, especially on the localization performance. An efficient integrated boundary map will be evaluated. Significant improvement is attained through this technique vis-à-vis existing methods. Experiments on benchmark images have been made using classical spatial domain filters, frequency domain filters and wavelet filters with wavelet scale correlation boundary detection algorithms.

4. Boundary Detection Using Wavelet

Boundary detectors are actually discritized wavelet functions and convolution with these operators gives the wavelet transform of the image at certain scale. Approximation of continuous wavelet model with dyadic discretization results in classical boundary detectors. The scale of the wavelet can be adjusted to detect boundaries of different level of scale. Coarser scale results in undetected boundaries and fine scale results in noisy and discontinuous boundaries. For coarser scale the coefficients of wavelet transform increase for step boundaries and decrease for dirac and fractal boundaries. The scale of boundary detector is adjustable to control the boundary's significance in contrast to classical operators. A larger scale wavelet can be used at positions where the wavelet transform decreases rapidly across scales to remove the effect of noise while using a smaller scale wavelet at positions where the wavelet transform decrease slowly across scale to preserve the precise position of the boundary. Wavelet filters of large scales are more effective for removing noise, but at the same time increase the uncertainty of the location of boundaries. Wavelet filters of small scales preserve the exact location of boundaries but cannot distinguish between noise and real boundaries.

Boundary detection in noisy scenario can be treated as an optimal linear filter design problem [12-13]. Most image processing algorithms use Quadrature Mirror Filter pair (QMF) to perform multiresolution analysis [14 16]. Such analysis of images with QMF has been used to exploit both detailing and smoothing capabilities of wavelets to get the detail images. The scaling function and the wavelets in one-dimensional space can be given by the following general formula:

$$\varphi_{a,b}(x) = (a)^{-1/2}\varphi\left(\frac{x-b}{a}\right)\ldots a \succ 0, b \in R \quad (3)$$

$$\psi_{a,b}(x) = (a)^{-1/2}\psi\left(\frac{x-b}{a}\right)\ldots a \succ 0, b \in R \quad (4)$$

Where, $\varphi_{a,b}(x)$ is the family of scaling function at scale *a* and translated by *b*, $\psi_{a,b}(x)$ is the family of wavelets at scale *a* and translated by *b*, *a* is the scaling factor, b is the translation desired, and φ and ψ are $\varphi_{0,0}$ and $\psi_{0,0}$ respectively.

In two-dimensional spaces one scaling function and three wavelets are needed. The scaling function and the three wavelet functions are defined as

$$\varphi(x,y) = \varphi(x)\varphi(y) \quad (5)$$

$$\psi^1(x,y) = \varphi(x)\psi(y) \quad (6)$$

$$\psi^2(x,y) = \psi(x)\varphi(y) \quad (7)$$

$$\psi^3(x,y) = \psi(x)\psi(y) \quad (8)$$

The scale is not varied to avoid lower resolution on account of increasing scale. What has been done is to perform one scale decomposition and obtain the boundaries at that level by extracting the detailing images and then proceed on to further analysis with the lowpass residue. With this methodology better boundaries can be obtained using Orthogonal and biorthogonal wavelets.

The horizontal ψ^1, vertical ψ^2 and diagonal ψ^3 components are nothing but the gradient of the image along the x, y and diagonal directions. Following this the magnitude of the image is taken at every level of decomposition, which on thresholding gives the boundaries at that level of decomposition[9]. Thresholding has been done following common criteria for both wavelet and conventional operators so as to facilitate criteria for comparison. With every subsequent level of decomposition the high frequency details go away. This approach has shown promising results in comparison to conventional operators as it offers a lot of flexibility in the form of multilevel decomposition. Depending on the requirement of details desired, the level of decomposition might be carried out. With this approach even boundaries of noisy images have been obtained successfully and shown with experiments.

5. Wavelet Decomposition

The boundaries of the input image are extracted at different scales through multilevel decomposition of the image as following: -

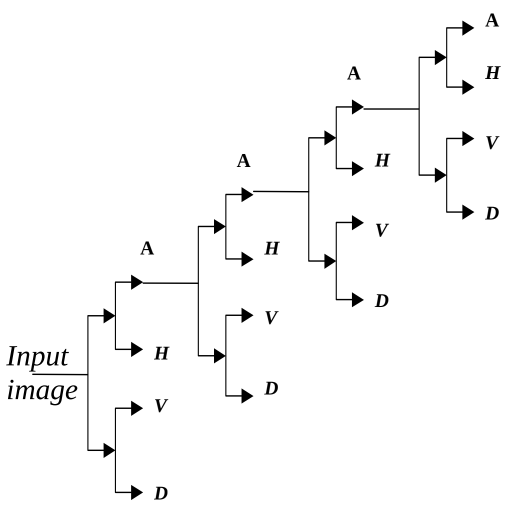


*Figure 1. Wavelet decomposition at level 4. **A** denotes approximations, **H** horizontal details, **V** vertical details and **D** diagonal details*

6. Proposed Algorithm

A pair of QMF is applied on the gray-level image. On the magnitude image so obtained thresholding is performed to obtain the boundary detected image at level-1. High Frequency details H at level-1 are extracted and used to get the magnitude image of horizontal and vertical details. Lowpass residue of level-1 is taken for analysis to get 2nd level decomposition. Steps 1, 2, and 3 are performed on level-2 magnitude image to obtain boundary detected image at level-2. Lowpass residue is carried over from previous level to iterate up to level-4. Above algorithm is an iterative process. Image being passed on to the following stage every time gets smoothed as the high frequency details are extracted at every level. This scheme is especially very useful in getting boundaries of a noisy image. In this paper the results so attained are through level four decomposition of the image by wavelets. The values of the wavelet coefficients are thresholded. The threshold is taken as four times the mean value of the matrix.

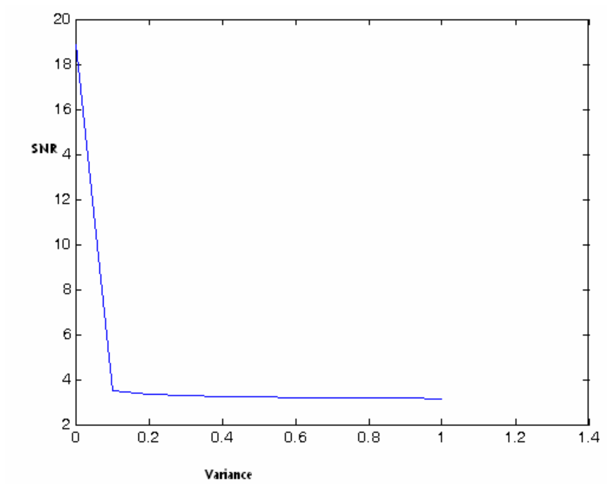


Figure 2. SNR when Gaussian noise induced in the Cameramans image for zero mean and varying variance

Each of the detail component after synthesizing to original size of the image is multiplied by its all four dimensional concordant component such that

$$D^H = \psi_v^H * \psi_{v+1}^H * \psi_{v+2}^H * \psi_{v+3}^H \quad (9)$$

$$D^V = \psi_v^V * \psi_{v+1}^V * \psi_{v+2}^V * \psi_{v+3}^V \quad (10)$$

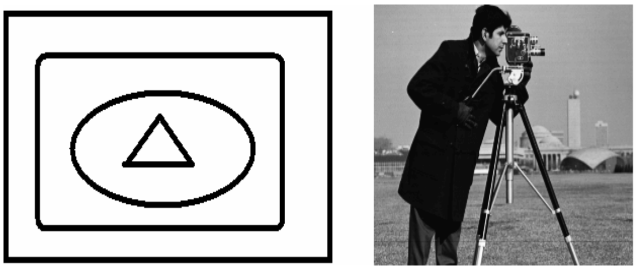
$$D^D = \psi_v^D * \psi_{v+1}^D * \psi_{v+2}^D * \psi_{v+3}^D \quad (11)$$

$$E = D^H + D^V + D^D \quad (12)$$

where $v \in L^2$, D represents respective detail coefficients. Superscript H, V and D represent horizontal vertical and diagonal details while E

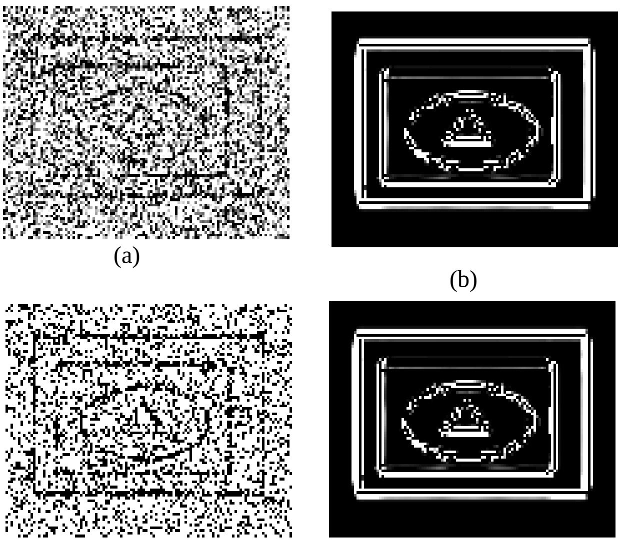
represents the combined boundary map.

The boundary structure remains in contact in the fine scale where as noise is eliminated as per the scale variations. The directional boundaries are taken at various resolutions and the image matrices so attained are interpolated to the original dimension of the image to execute matrix multiplication. In the process the directional details are enhanced and isolated noisy pixels are eliminated due to its non-existence in lower dimensional space. The process results in enhanced horizontal, vertical and diagonal details. The three images are synthesized together to obtain the augmented boundaries of the image.



(a) (b)

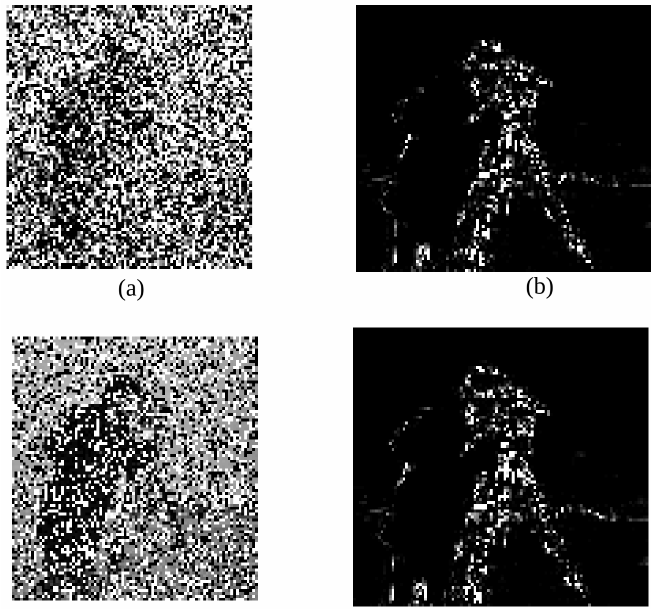
Figure 3. Geometric shapes(a) and Cameraman(b) images used to illustrate experimental results.



(a) (b) (c) (d)

Figure 4. Boundaries extracted by wavelet scale correlation upto 4th level scale correlation from Geometric Shapes Image. (a) Gaussian noise induced with μ=0 & σ=1. (b) Boundaries detected from (a). (c) Salt & Pepper noise with 0.5 noise density. (d) Boundaries detected from (c).

Experiments are conducted using Haar(db1) to Daubechies(db8) and Bior(3.7) which revealed analogous results with classical operators for high signal to noise ratio (SNR) images. Results for Uniform noise in the image are trivial due to wavelets in built approximating and detailing characteristics. Figure-2 demonstrates SNR for Gaussian noise with varying mean and variance induced to Cameraman image of figure-3b for experimental purpose. Adequate results were achieved for low SNR where spatial as well as frequency domain operators fall short to give any significant intelligence about the boundary map.



(a) (b) (c) (d)

Figure 5. Boundaries extracted by wavelet scale correlation upto 4th level scale correlation from noisy Cameraman Image (a) Gaussian noise induced with μ=0 & σ=1.in figure 3(b). (b) Boundaries detected from (a). (c) Salt & Pepper noise with 0.5 noise density. (d) Boundaries detected from (c).

The proposed scheme although some of the boundary pixels were found missing, outperformed and gave adequate results. Figure 4(a) is Geometric Shapes image with Gaussian noise of zero mean and one variance. Figure 4(c) is Geometric Shapes image with 0.5 density Salt & Pepper noise. The boundary map by wavelet scale correlation figure 4(a) and 4(c) are shown in figure 4(b) and 4(d) respectively.

5. Conclusion

In this paper novel boundary detection technique has been developed. Comparison of proposed algorithm with classical operators in noisy environment is performed. A database is established and is subjected to psychovisual comparison. Boundary detection by proposed algorithm is found to be better than conventional operators. The proposed method favors boundaries that exist at multiple scales and suppress at finer scales. Orthogonal and biorthogonal wavelets are tried. Lesser the length of wavelet coefficients better is the detection results. db1 gave the best results with in the wavelets for detection for the test images due to its inbuilt feature of capturing high frequency contents of the image while being computationally analogous with classical operators.. The noise is highly un-correlated amongst the subbands. Scale correlation of adjacent bands at concordant locations depletes the noise pixels and reveals the object's boundary and has outperformed the conventional operators for low SNR images.

AWGN in the images are generated through computer and assumed to be uncorrelated with image's spatial coordinates and its intensity values. However following needs to be worked out:

1. Figure of merit.
2. The noise correlation with the image.
3. Application on images corrupted with real noise.
4. Curve fitting for disconnected boundaries.

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